

**Problem 1**

<p><b>Signal:</b>                  rectangular pulse with  <math>T_p = 500 \mu s</math> duration  <math>V_p</math> amplitude, to be measured</p>	<p><b>Noise:</b>  <math>\sqrt{S_{v,u}} = 40nV/\sqrt{Hz}</math> (unilateral) white                  with noise band limit <math>f_n = 2,5MHz</math></p>
<p><math>f_s = 1/T_s</math> sampling frequency (selectable)  <math>T_s = 1/f_s</math> sampling interval</p>	

The amplitude of the signal above specified must be measured in presence of the noise above reported.

A) Select a practical filter that well approximates the optimum filter for the measurement required. Calculate the corresponding S/N equation and evaluate the minimum measurable amplitude. Compare with the measurement done without filtering, obtain the equation of the factor of improvement, evaluate it quantitatively and explain it intuitively.

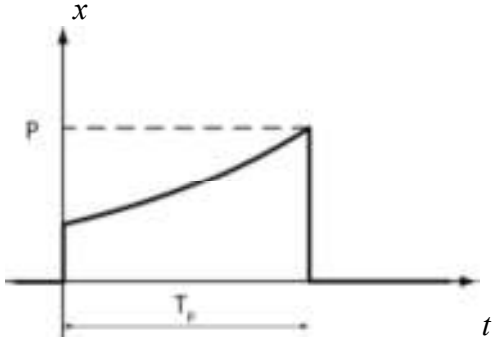
B) Consider now to operate with discrete-time filtering, taking samples of the input waveform spaced by a constant interval  $T_s$  and elaborating the sample values. Explain how a filtering of the same kind as that seen in (A) can be implemented with discrete-time filtering. Obtain the corresponding S/N equation and the minimum measurable pulse amplitude, showing how the result depends on the number of samples taken of the signal

C) Employing an approximate representation of the noise (triangular autocorrelation function in time or rectangular spectrum in frequency), obtain the equation that describes the dependence of the result of the discrete-time filtering seen in (B) on the sampling frequency  $f_s$  and compare it with the result of the continuous-time filtering seen in (A). On the basis of this analysis decide what sampling frequency  $f_s$  is advisable for the measurement.

D) Take now into account more detailed information available about the noise, namely that the band is limited by a single-pole low-pass filtering with time constant  $T_n = 100ns$  (i.e. frequency  $f_p = 1/2\pi T_n$ ), and therefore the actual noise autocorrelation function and the spectrum are known in detail. On this basis, revise the analysis of how the result of the discrete-time filtering depends on the sampling frequency  $f_s$  and compare it with the result of the continuous-time filtering seen in (A). Revise accordingly the conclusion drawn in (C) about the advisable sampling frequency.

*(NB: see text also on the other side of the sheet)*

**Problem 2**

<p><b>Pulse signal:</b>  <math>x(t) = Pe^{(t-T_p)/T_D}</math> in <math>0 \leq t \leq T_p</math>  <math>x=0</math> elsewhere  <math>T_p = 10\mu s</math> and <math>T_D = 10\mu s</math>  P amplitude, to be measured  In (A) and (B): single pulses  In (C): repetitive pulses with repetition interval <math>T_R=100\mu s</math>  An auxiliary signal is available, synchronous with pulse onset</p>	
<p><b>Noise:</b> white with wide band <math>&gt; 50\text{MHz}</math>  <math>\sqrt{S_{v,u}} = 20\text{ nV}/\sqrt{\text{Hz}}</math> (unilateral)</p>	

The amplitude of a pulse signal that has the waveform above shown (a truncated rising exponential) must be measured in presence of white noise with wide band as specified.

A) Explain how the noise can be filtered employing a Gated Integrator. Calculate the Signal-to-noise ratio  $(S/N)_G$  thus obtainable and compute the value of the minimum measurable pulse amplitude  $P_{min,G}$

B) Explain the optimum filtering for the measurement of the pulse signal. Calculate the optimum signal-to-noise ratio  $(S/N)_o$ . Compute the value of the minimum pulse amplitude  $P_{min,o}$  measurable employing the optimum filter. Calculate the ratio  $P_{min,G} / P_{min,o}$  between the minimum measurable amplitude with GI and that with optimum filtering.

You are informed that in this case the optimum filtering can be simply accomplished in reality with a variable-parameter passive filter: point out and explain the circuit that can be employed for implementing in reality such a filter.

C) Consider now a case where the pulse signals are repetitive with repetition interval  $T_R=100\mu s$  and have all the same amplitude P. The repetition makes possible to enhance the measurement by exploiting the information redundancy, that is, by exploiting more than one pulse in the measurement. In the present case it is required to obtain such enhanced measurement by means of a simple variable-parameter passive analog filter that performs a twofold action, namely:

- acquisition of each pulse with optimum weighting
- exploitation of more than one pulse for the amplitude measurement.

Analyze and discuss how this action can be obtained by employing as filter

- (1) a Boxcar Integrator (BI) with a suitably modified parameter
- (2) a Ratemeter Integrator (RI) with a suitably modified parameter.

Taking as reference the optimum measurement of a single pulse, evaluate the factors of (S/N) improvement that can be gained by exploiting the redundancy in the two cases considered, that is, with the BI and with the RI.