PROBLEM 1

Data

Strain gauges (SG):

R_S = 160 Ω resistance
G = 2,4 Gauge factor
P_{dmax}= 4 μW maximum power dissipation admitted in in each sensor
Temperature Coefficient $\alpha = \frac{\Delta R_S}{R_{S0}} = 4 \cdot 10^{-3}\ C^{-1}$

Uncontrolled temperature mismatch between sensors in the range $\Delta T_{max} \approx \pm 0{,}1°C$, quasi-stationary, i.e. with variation over long time intervals >>10s

Differential Preamplifier:

Wide band f_p > 100MHz

$\sqrt{S_V}$ = 12 nV/Hz$^{1/2}$ **(unilateral)** wide band density of voltage noise referred to the preamp differential input; a 1/f noise component is also present with corner frequency f_{cv} = 100 kHz

$\sqrt{S_i}$ = 5 pA/Hz$^{1/2}$ **(unilateral)** wide band density of current noise referred to the preamp differential input; a 1/f noise component is also present with corner frequency f_{ci} = 100 kHz

Strain to be measured

$\varepsilon(t) = \varepsilon_S \cos(\omega_m t)$

Fundamental component of oscillatory strain of extension-contraction, caused by moto cycle motor rotating at 6000rpm (revolutions per minute) i.e. with frequency f_m*=100Hz*

The amplitude of the oscillation is almost constant, i.e. slowly varies over time intervals >10s

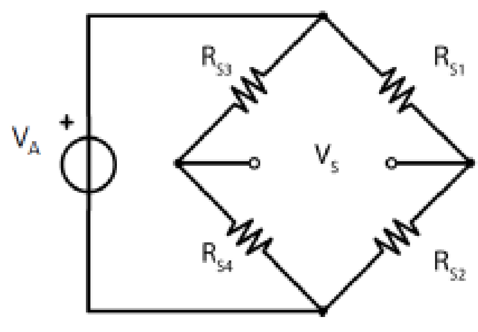
A sinusoidal reference electrical signal is available with the frequency and phase of the motor rotation

(A) Configuration, transduction gain and effect of the temperature mismatch

- **Configuration**

  standard Wheatstone bridge configuration with four equal SG

  (R_{S1} , R_{S2} , R_{S3} , R_{S4}) and only one sensor subject to strain (R_{S2}).



- **DC supply voltage V_A of the bridge**

  the DC voltage supply V_A is limited by the allowed power dissipation in a sensor

  $$\frac{(V_A/2)^2}{R_S} \le P_{d,\max} \qquad \text{hence} \quad V_A \le 2\sqrt{P_{d,\max} R_S} = 50{,}5mV \quad \text{and we adopt} \quad V_A = 50mV$$

- **Strain-to-voltage transduction factor**

  Small strain → small variations ΔR_S of resistance R_{S2} →small variations of bridge output , can be computed with first-order approximation

  $$V_S = \frac{V_A}{4} \cdot \frac{\Delta R_S}{R_S}$$

  Strain $\varepsilon = \dfrac{\Delta l}{l}$ → resistance variation $\Delta R_S = G\varepsilon R_{S0}$ → output signal V_S

  $$V_S = \frac{V_A}{4} \cdot \frac{\Delta R_S}{R_{S0}} = \frac{V_A}{4} \cdot G\varepsilon = 30mV \cdot \varepsilon$$

  Transduction factor

  $$\frac{dV_S}{d\varepsilon} = \frac{V_A}{4} \cdot G = \frac{30mV}{strain} = \frac{30nV}{\mu strain}$$

- **Effect of temperature mismatch of the sensors**

  Mismatch ΔT between active sensor (R_{S2}) and compensation sensor (R_{S1}) temperature causes a difference $\Delta R_S = \alpha\, \Delta T \cdot R_{S0}$ between these resistances, therefore a bridge output signal

  $$V_{ST} = \frac{V_A}{4} \cdot \frac{\Delta R_S}{R_S} = \frac{V_A}{4} \cdot \alpha \cdot \Delta T$$

  A strain ε causes a difference $\Delta R_S = G\varepsilon \cdot R_{S0}$ and therefore an output V_{SS}

  $$V_{SS} = \frac{V_A}{4} \cdot G\varepsilon$$

  A temperature difference ΔT simulates the presence of a strain ε_T that produces the same output V_{SS}=V_{ST} , that is, a fake strain

  $$\varepsilon_T = \frac{\alpha \cdot \Delta T}{G}$$

  The temperature mismatch can thus generate errors up to

  $$\varepsilon_{Tm} = \pm \frac{\alpha \cdot \Delta T_{\max}}{G} = \pm 1{,}66 \cdot 10^{-4}\, strain = \pm 166\ \mu strain$$

The fake strain signal generated is ruled by the mismatch ΔT, hence it is an almost constant signal, with frequency spectrum well below 0,1 Hz.

(B) Measurements with strain gauges operating with DC voltage supply

B1 – Measurement of the oscillating strain

The equivalent source resistance between the output terminals of the Wheatstone bridge is equal to the resistance R_S of one SG. We evaluate the voltage noise due to the preamp current noise and the noise of the source resistance

$R_S\sqrt{S_i} = 0,8nV/\sqrt{Hz}$ with 1/f corner frequency f_{cv} = 100 kHz

$\sqrt{4kT\cdot R_S} \approx 1,6\,nV/\sqrt{Hz}$ without appreciable 1/f noise component

We recognize that the preamp voltage noise

$\sqrt{S_V} = 12nV/\sqrt{Hz}$ with 1/f corner frequency f_{cv} = 100 kHz

is dominant in the total noise $S_n(f)$ and the other contributions are negligible

$\sqrt{S_n} \approx \sqrt{S_V} = 12\,nV/\sqrt{Hz}$ with 1/f noise component with corner frequency f_{cv} = 100 kHz

The signal to be measured is the fundamental component of the strain oscillation, which is at the revolution frequency of the motor

$$f_m = 6000rpm = 100Hz$$

An auxiliary signal with same frequency and phase as the signal is available (obtained by monitoring the rotation of the motor).

$$r(t) = B\cos\omega_m t$$

The signal amplitude V_S varies slowly, over time intervals >10s,

$v(t) \approx V_S\cos(\omega_m t)$ with slowly varying V_S

In the frequency domain, the signal is a line with narrow bandwidth $\Delta f_S < 1Hz$, centered at the oscillation frequency f_m.

For measuring the amplitude V_S we use a Lock-in Amplifier (LIA) employing as reference the auxiliary signal. For the low-pass filter in the LIA, we select a band-limit f_L greater than the signal bandwidth $\Delta f_S < 1Hz$

$$f_L = 1Hz$$

The S/N can be computed as the ratio of the full power $V_S^2/2$ of the input signal (in phase with the reference) to half of the input noise power in the passband of width $\Delta f = 2f_L$ defined by the LIA low-pass filter and centered on the oscillation frequency f_m (the noise components have random phases, on the average half of them are rejected by the phase selection)

$$\left(\frac{S}{N}\right)_m^2 = \frac{V_S^2/2}{S_n(f_m)\Delta f/2} = \frac{V_S^2}{2S_n(f_m)f_L} \qquad (S_n \text{ unilateral density})$$

The S/N can also be computed as the ratio of:
(1) the amplitude V_S of the signal brought back in the baseband at f=0 (by the frequency-domain convolution with the reference signal)
(2) the input spectral density S_n around f_m brought back around f=0 in the baseband (by the frequency-domain convolution with the spectrum of the reference signal) and filtered by the lowpass band-limit f_L of the LIA low-pass filter. We note that the convolution doubles the spectral density $2S_n$; this effect is commonly denoted "spectrum folding"

$$\left(\frac{S}{N}\right)_m = \frac{V_S}{\sqrt{2S_n(f_m)f_L}} \qquad (S_n \text{ unilateral density})$$

The minimum measurable signal amplitude (at S/N=1) is therefore

$$V_{S\min} = \sqrt{2}\sqrt{S_n(f_m)}\sqrt{f_L}$$

The presence of the 1/f noise component makes the noise density at f_m much higher than the wide-band noise density S_V

$$\sqrt{S_n(f_m)} = \sqrt{S_V + S_V\frac{f_C}{f_m}} \approx \sqrt{S_V\frac{f_C}{f_m}} \approx 31{,}6\sqrt{S_V} = 380\,nV/\sqrt{Hz}$$

Therefore, the minimum signal is fairly high

$$V_{S\min} = \sqrt{2}\sqrt{S_n(f_m)}\sqrt{f_L} \approx 535nV$$

and corresponds to a fairly high minimum measurable amplitude of the oscillating strain

$$\varepsilon_{\min} = \frac{V_{S\min}}{\frac{dV_S}{d\varepsilon}} = 17{,}8\mu strain$$

B2 – Effect of the temperature mismatch on the measurement

In this measurement the temperature mismatch of the SG does not cause problems. The mismatch produces at the output of the Wheatstone bridge an almost stationary spurious signal (with frequency spectrum below 0,1Hz) which is out of the frequency band admitted by the LIA and therefore is not present in the LIA output.

(C) Measurements with strain gauges operating with high-frequency AC voltage supply

C1 – Measurement of the oscillating strain

In the SG operation with DC supply voltage the minimum measurable signal is fairly high because the contribution of the 1/f noise is very strong at the frequency of the signal f_m. In order to obtain better results, we can shift the signal at a higher frequency where the 1/f noise is lower, preferably above the corner frequency f_{cv} = 100 kHz in order to have 1/f noise lower than the wide-band noise. This must be obtained at an early stage of the signal processing chain, where 1/f noise has not yet been added to the signal; in our case, the frequency must be shifted before the preamplifier. Therefore, we use for the bridge supply a sinusoidal voltage at a frequency f_h higher than f_{cv} , for instance

$$f_h \approx 10 f_{cv} = 1MHz$$

$$v_a(t) = V_{AC}\cos(\omega_h t)$$

Since the power dissipation is ruled by the effective amplitude of the AC voltage waveform, we can use amplitude V_{AC} of the AC voltage supply higher by a factor $\sqrt{2}$ than the DC voltage supply V_A

$$V_{AC} = \sqrt{2}\cdot V_A \approx 70mV$$

We have consequently a higher transduction factor

$$\frac{dV_S}{d\varepsilon} = \frac{V_{AC}}{4}\cdot G = \frac{42mV}{strain} = \frac{42nV}{\mu strain}$$

The bridge output signal arises from the AC voltage supply at frequency f_h modulated by the mechanical oscillation at frequency f_m , therefore it has components at the high frequencies $f_h + f_m$ and $f_h - f_m$, where only the wide-band noise S_V is significant

We bring the signal back to the frequency f_m = 100Hz in the baseband with a LIA employing as reference the supply voltage of the bridge $v_a(t)$. The signal must be available at the LIA output, hence the LIA low-pass filter must have band-limit f_{L1} quite higher than f_m . We adopt f_{L1}=1kHz and thus obtain

$$\left(\frac{S}{N}\right)_m = \frac{V_S}{\sqrt{2S_n(f_h)f_{L1}}} = \frac{V_S}{\sqrt{2S_V f_{L1}}}$$

At the LIA output we have an oscillating signal and the minimum measurable amplitude is

$$V_{S\min} = \sqrt{2S_n(f_h)}\sqrt{f_{L1}} \approx \sqrt{2S_V}\sqrt{f_{L1}} = 535nV$$

This corresponds to a still fairly high minimum measurable amplitude of the oscillating strain

$$\varepsilon_{\min} = \frac{V_{S\min}}{\frac{dV_S}{d\varepsilon}} = 12,7\mu strain$$

In the following, for the sake of clarity we will denote as LIA1 the Lock-in Amplifier employed in this Sec. C and as LIA2 the other LIA employed in Sec. B.

C2 – Effect of the temperature mismatch on the measurement

In this case, the spurious quasi-stationary signal due to the temperature mismatch is present in the LIA1 output: it is modulated to f_h by the AC supply of the bridge, demodulated to $f=0$ by the LIA1 and passed by the LIA1 low-pass filter to the LIA1 output. Therefore, if the output of the LIA1 is measured without further filtering, a strong error due to the temperature mismatch is added and sets the actual limit to the measurement

$$\varepsilon_{T\mathrm{m}} = \pm \frac{\alpha \cdot \Delta T_{\max}}{G} = \pm 1,66 \cdot 10^{-4}\, strain = \pm 166\ \mu strain$$

C3 – Improving the measurement of oscillating strain by further filtering

At the LIA1 output, the fake signal due to the temperature mismatch and the signal due to oscillating strain can be clearly distinguished. The oscillating strain has narrow spectrum centered at f_m=100Hz, while the fake signal has a low-frequency spectrum in the range below 0,1Hz. The error due to temperature mismatch can be avoided (or at least strongly reduced) by applying to the LIA1 output a further filter that eliminates the fake signal by rejecting the low frequency components below 0,1Hz. By suitably selecting this filter, it is also possible to reduce the noise and the corresponding minimum measurable amplitude. Here are some examples of the filter.

a) <u>High pass filter with band limit about 10Hz</u>
It eliminates the fake signal and the associated error and brings just a minor reduction of the filtering bandwidth. The noise reduction is practically negligible, but we get now the result computed in Sec. C1 without the mismatch error

$$V_{S\min,H} = \sqrt{2S_n\left(f_h\right)}\sqrt{f_{L1}} \approx \sqrt{2S_V}\sqrt{f_{L1}} = 535nV$$

that is $$\varepsilon_{\min,H} = \frac{V_{S\min,H}}{\frac{dV_S}{d\varepsilon}} = 12,7\mu strain$$

b) <u>Resonant filter tuned at f_m = 100Hz</u>
It eliminates the fake signal and the associated error and significantly reduces the filtering bandwidth. For a filter tuned at 100Hz a quality factor Q≈5 is currently available and gives a noise bandwidth
$\Delta f_R = \pi f_m / 2Q \approx 30Hz$
We get then

$$V_{S\min,R} = \sqrt{2S_n\left(f_h\right)}\sqrt{\Delta f_R} \approx \sqrt{2S_V}\sqrt{\Delta f_R} \approx 93nV$$

that is $$\varepsilon_{\min,R} = \frac{V_{S\min,R}}{\frac{dV_S}{d\varepsilon}} = 2,2\mu strain$$

c) <u>LIA2 as seen in Section B, but now employed to filter the output of LIA1</u>.
LIA2 eliminates the fake signal and the associated error and sets a very narrow filtering bandwidth that strongly reduces the noise. For the low-pass filter in the LIA2, we select again a band-limit f_{L2} just greater than the signal bandwidth $\Delta f_S < 1Hz$.

$f_{L2} = 1Hz$

We must take into account, however, that the LIA2 receives at the input the noise spectrum doubled by the spectrum folding in LIA1 and therefore we get

$$V_{S\min,L} = \sqrt{2}\sqrt{2S_V}\sqrt{f_{L2}} \approx 24nV$$

that is $$\varepsilon_{\min,L} = \frac{V_{S\min,L}}{\frac{dV_S}{d\varepsilon}} = 0,6\mu strain$$

(D) <u>Situation in measurements of constant strain with the apparatus above discussed</u>

The measurements of stationary strain with the apparatus above considered are subject to tougher problems and limitations than the measurements of oscillating strain.

Measurements with DC supply voltage are plagued by high 1/f noise and are fully sensitive to the errors due to sensor temperature mismatch. They are definitely not advisable.

Measurements with AC voltage supply avoid the 1/f noise contribution, but they are still fully sensitive to the errors due to sensor temperature mismatch. In fact, both the true strain signal and the fake signal due to temperature mismatch are stationary and cannot be distinguished by a selection in frequency. They are both modulated to f_h by the AC supply of the bridge, demodulated to $f=0$ by the LIA1 and passed by the LIA1 low-pass filter to the LIA1 output.

In conclusion, if the strain to be measured is stationary the limit to its measurement is set by the sensor temperature mismatch, which is the major cause of error. For instance, we have seen in the quantitative case dealt with in Sec. C that this limit would be

$$\varepsilon_{T\mathrm{m}} = \pm\frac{\alpha \cdot \Delta T_{\max}}{G} = \pm 1,66 \cdot 10^{-4} strain = \pm 166\ \mu strain$$

which is much higher of the limit due to the properly filtered noise

$$\varepsilon_{\min} = \frac{V_{S\min}}{\frac{dV_S}{d\varepsilon}} = 12,7\mu strain$$