PROBLEM 2

Problem Data

OPTICAL SIGNAL:

wavelength λ= 600nm,

rectangular pulse waveform with duration T_F=10 μs, optical power P

PHOTOMULTIPLIER TUBE:

Gain $G=2\cdot 10^6$,

Excess Noise Factor F=2

Quantum detection efficiency η=0,05 at λ= 600nm

Radiant Sensitivity (Spectral Responsivity) $S_R = \eta \frac{\lambda[\mu m]}{hc/q} = 24\, mA/W$ at λ= 600nm

Cathode Dark current $I_{Dk}= 10^{-15}$ A = 1 fA

Cathode current due to environment light in the day is $I_{Ek}=2\ 10^{-12}$ A = 2 pA

Cathode current due to environment light in the night is negligible

Photocurrent at the cathode I_p

PREAMPLIFIER

Load Resistance connected to the PMT anode $R_L=1k\Omega$

Load Capacitance connected to the PMT anode C_L=2 pF

Current Noise spectral density (unilateral) at Amplifier input $\sqrt{S_{iA}} = 1\, pA/\sqrt{Hz}$

Voltage Noise spectral density (unilateral) at Amplifier input $S_{vA} = 1\, nV/\sqrt{Hz}$

PIN PHOTODIODE:

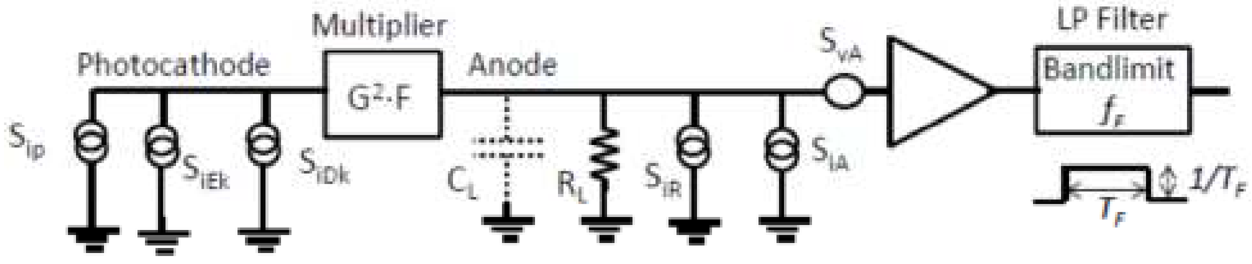
Quantum detection efficiency η_d=0,60 at λ= 600nm

Radiant Sensitivity (Spectral Responsivity) $S_{Rd} = \eta_d \frac{\lambda[\mu m]}{hc/q} = 288\, mA/W$ at λ= 600nm

Dark current $I_{Dd}=2\cdot 10^{-12}$ A = 2 pA

Current due to environment light in the day is $I_{Ed}=2\cdot 10^{-12}$ A = 2 pA

(A) Noise referred to the PMT photocathode



Circuit Diagram for Noise Evaluation

Dark-current I_{DK} and environment daylight current I_{Ek} are specified at the cathode and we evaluate their shot noise directly at the cathode

$I_{Dk} = 10^{-15}$ A $\qquad \sqrt{S_{iDk}} = \sqrt{2qI_{Dk}} \approx 1{,}8 \cdot 10^{-17}\ A/\sqrt{Hz}$

$I_{Ek} = 2\ 10^{-12}$ A $\qquad \sqrt{S_{iEk}} = \sqrt{2qI_{Ek}} \approx 8 \cdot 10^{-16}\ A/\sqrt{Hz}$

we verify that in measurements performed in daylight the contribution of the environment light noise is dominant over that of the dark current noise.

As concerns the noise of the electronic circuitry, first we evaluate the total current noise density at the PMT output (i.e. the preamplifier input)

$$\sqrt{S_{iU}} = \sqrt{S_{iA} + S_{vA}/R_L^2 + 4kT/R_L}$$

which since $\sqrt{S_{iA}} = 1\ pA/\sqrt{Hz}$, $\quad \sqrt{S_{vA}}/R_L = 1\ pA/\sqrt{Hz}$, $\quad \sqrt{4kT/R_L} = 4\ pA/\sqrt{Hz}$

gives $\qquad \sqrt{S_{iU}} \approx 4{,}2\ pA/\sqrt{Hz}$

Second, we refer this noise density to the cathode by dividing it by the PMT gain G and the excess noise factor $\sqrt{F}$

$$\sqrt{S_{iUk}} = \sqrt{S_{iU}}/G\sqrt{F} \approx 1{,}5 \cdot 10^{-18}\ A/\sqrt{Hz}$$

The high value of G makes the noise of the electronic circuitry negligible not only with respect to the environment-light noise, but also with respect to the dark-current noise.

(B) Measurements with PMT in daylight

B1 - Filtering

The band-limits set to the noise at high frequency by the low-pass filtering $R_L C_L$ circuit and by the preamplifier bandwidth are much higher than the band-limit of the rectangular signal, therefore all the noise component can be considered white.

With white noise the optimum filter is simply the matched filter, with weighting function equal to the signal. We employ a Gated Integrator with gate equal to the pulse duration T_F and consider to use a normalized GI with noise band-limit

$$f_F = 1/2T_F$$

B2 – Minimum amplitude limited by the stationary background noise

In measurements performed in daylight, the environment-light noise S_{iEk} is by far the dominant component. Therefore, the total stationary background shot noise density S_{iB} is practically equal to S_{iEk}

$$\sqrt{S_{iB}} = \sqrt{S_{iDk} + S_{iEk} + S_{iUk}} \approx S_{iEk}$$

Let us first consider only the stationary background noise and neglect the noise due to the fluctuations of the photocurrent. We get

$$\frac{S}{N} = \frac{I_p}{\sqrt{F \cdot S_{iEk} f_F}} = \frac{I_p}{\sqrt{2qI_{Ek}\dfrac{F}{2T_F}}} = \frac{I_p}{\sqrt{\dfrac{qI_{Ek}}{T_F}F}}$$

The minimum (corresponding to S/N=1) measurable amplitude $I_{p,\min B}$ limited by the stationary background noise is

$$\boxed{I_{p,\min B} = \sqrt{q\frac{I_{Ek}}{T_F}F} \approx 255 \cdot 10^{-15} A = 255\, fA}$$

which corresponds to a minimum photoelectron emission rate

$$\boxed{n_{p,\min B} = \frac{I_{p,\min B}}{q} = \sqrt{\frac{I_{Ek}}{qT_F}F} \approx 1{,}6 \cdot 10^{6}\, el/s}$$

B3 – Minimum amplitude limited by the photocurrent noise

Let us now consider the minimum measurable pulse amplitude limited only by the statistical fluctuations of the photocurrent, neglecting the stationary background noise. We get

$$\frac{S}{N} = \frac{I_p}{\sqrt{2qI_p\dfrac{1}{2T_F}F}} = \sqrt{\frac{I_p T_F}{qF}}$$

The minimum measurable amplitude limited by the photocurrent fluctuations thus is

$$I_{p,\min P} = \frac{qF}{T_F} \approx 32 \cdot 10^{-15} A = 32 fA$$

and corresponds to

$$n_{p,\min P} = \frac{I_{p,\min P}}{q} = \frac{F}{T_F} \approx 2 \cdot 10^5 \; el/s$$

B4 – Actual limit for measurements with PMT in daylight

We can conclude that in these measurements the minimum measurable amplitude is limited by the stationary background noise, while the limit due to the photocurrent noise is irrelevant because it is much lower.

The minimum optical pulse power measurable in daylight operation is therefore

$$\boxed{P_{\min} = \frac{I_{p,\min B}}{S_R} = 10,6 \cdot 10^{-12} = 10,6 pW}$$

corresponding to a minimum detected photon rate

$$\boxed{n_{f,\min B} = \frac{n_{p,\min B}}{\eta} \approx 3,11 \cdot 10^7 \; ph/s}$$

(C) Measurements with PMT in the night

C1 – Minimum amplitude limited by the stationary background noise

In measurements performed in the night, the environment-light current is negligible and the dark current noise density S_{iDk} is dominant over the noise of the electronic circuitry. Therefore, the total stationary background shot noise density S_{iB} in nighttime is practically equal to S_{iDk}

$$\sqrt{S_{iB}} = \sqrt{S_{iDk} + S_{iUk}} \approx S_{iDk}$$

Employing the same gated integrator we get in this case

$$\frac{S}{N} = \frac{I_p}{\sqrt{F S_{iDk} f_F}} = \frac{I_p}{\sqrt{F 2q I_{iDk} \frac{1}{2T_F}}} = \frac{I_p}{\sqrt{\frac{q I_{iDk} F}{T_F}}}$$

The minimum measurable amplitude $I_{p,\min B}$ limited by the stationary background noise is now

$$I_{p,\min B} = \sqrt{q \frac{I_{Dk}}{T_F} F} \approx 5,7 \cdot 10^{-15} A = 5,7 \, fA$$

C2 – Actual limit for measurements in nighttime

The limit due to the stationary background noise is definitely lower than the minimum measurable amplitude limited by the photocurrent fluctuations, which therefore is the actual limit for the measurement in nighttime

$$\boxed{I_{p,\min P} = \frac{qF}{T_F} \approx 32 \cdot 10^{-15} A = 32 fA}$$

The minimum optical pulse power measurable in nighttime is therefore

$$\boxed{P_{\min} = \frac{I_{p,\min P}}{S_R} = 1,32 \cdot 10^{-12} W = 1,32 pW}$$

that corresponds to a minimum detected photon rate

$$\boxed{n_{f,\min B} = \frac{n_{p,\min B}}{\eta} \approx 4 \cdot 10^{6}\ ph/s}$$

(D) Minimum signal measurable in daylight with the PIN photodiode

D1 – Stationary Background Noise

The detector does not have internal gain and all the noise component are evaluated referred to the photodiode output (i.e. the preamplifier input).

The noise components due to the dark-current I_{Dd} and to the environment-daylight current I_{Ed} are

$I_{Dd}= 10^{-12}$ A $\qquad \sqrt{S_{iDd}} = \sqrt{2qI_{Dd}} \approx 5,7 \cdot 10^{-16}\ A/\sqrt{Hz}$

$I_{Ed}=2\ 10^{-12}$ A $\qquad \sqrt{S_{iEk}} = \sqrt{2qI_{Ek}} \approx 8 \cdot 10^{-16}\ A/\sqrt{Hz}$

The components of the electronic circuitry noise are:

$$\sqrt{S_{iA}} = 1\, pA/\sqrt{Hz}\ , \qquad \sqrt{S_{vA}}/R_L = 1\, pA/\sqrt{Hz}\ , \qquad \sqrt{4kT/R_L} = 4\, pA/\sqrt{Hz}$$

Therefore, the total electronic circuitry noise is

$$\sqrt{S_{iU}} = \sqrt{S_{iA} + S_{vA}/R_L^2 + 4kT/R_L} \approx 4,2 \cdot 10^{-12}\ A/\sqrt{Hz} = 4,2\, pA/\sqrt{Hz}$$

The electronic circuit noise is clearly dominant among the stationary background current noise components

$$\sqrt{S_{iT}} = \sqrt{S_{iU} + S_{iD} + S_{Ed}} \approx \sqrt{S_{iU}} = 4,2 \cdot 10^{-12}\ A/\sqrt{Hz} = 4,2\, pA/\sqrt{Hz}$$

D2 – Minimum amplitude limited by the stationary background noise

By considering only the stationary background noise and neglecting the noise due to the fluctuations of the photocurrent, we get

$$\frac{S}{N} = \frac{I_p}{\sqrt{S_{iT} f_F}} = \frac{I_p}{\sqrt{\frac{S_{iT}}{2T_F}}}$$

The minimum (corresponding to S/N=1) measurable amplitude $I_{p,minBd}$ limited by the stationary background noise is

$$\boxed{I_{p,\min Bd} = \sqrt{S_{iT} f_F} = \sqrt{\frac{S_{iT}}{2T_F}} \approx 0,94 \cdot 10^{-9}\, A = 940\, pA}$$

which corresponds to a minimum rate of photoelectrons

$$\boxed{n_{p,\min Bd} = \frac{I_{p,\min Bd}}{q} = \sqrt{\frac{I_{Ek}}{qT_F}} \approx 5,9 \cdot 10^{9}\, el/s}$$

D3 – Minimum amplitude limited by the photocurrent noise

It has still the value computed in Sec. B3, therefore it is very much lower and it is totally irrelevant.

D4 – Comparison of PMT and PIN results

The minimum optical pulse power measurable in daylight operation with the PIN diode is therefore

$$\boxed{P_{\min.d} = \frac{I_{p,\min Bd}}{S_{Rd}} = 3,26 \cdot 10^{-9} = 3,26 nW}$$

corresponding to a minimum detected photon rate

$$\boxed{n_{f,\min Bd} = \frac{n_{p,\min Bd}}{\eta_d} \approx 9,83 \cdot 10^{9}\, ph/s}$$

We see that for the PIN the minimum detector photocurrent $I_{p,minBd}$ is higher than that for the PMT $I_{p,minB}$ by a very high factor

$$\frac{I_{p,\min Bd}}{I_{p,\min B}} = \frac{940 pA}{255 fA} \approx 3686$$

However, the ratio of minimum detected optical power for the PIN and for the PMT is much less high, because the PIN has a much higher photon detection efficiency that at least in part counterbalances the lower sensitivity of the PIN due to the lack of detector gain

$$\frac{P_{p,\min Bd}}{P_{p,\min B}} = \frac{3,26 nW}{10,6 pW} \approx 307$$