PROBLEM 2

Data and notes

Applied Compression Force
- Rectangular pulse with constant amplitude F and duration $T_p = 5ms$

Piezoelectric force sensor
- A_q=10pC/N force-to-charge transduction factor
- C_L= 500pF total capacitance, sensor and circuit
- generator I_s represents the piezoelectric effect

Preamplifier
- R_{iA} high input resistance, to be considered infinite
- Wide Band-limit f_{pa}=50MHz
- $\sqrt{S_{v,u}} = 20nV/\sqrt{Hz}$ (unilateral) wide band
- $\sqrt{S_{i,u}} = 0,2\,pA/\sqrt{Hz}$ (unilateral) wide band
- 1/f noise components have not to be considered

Sinusoidal Electromagnetic Interference
- $f_d=20kHz$ frequency, known with uncertainty ±1%,
- Amplitude at preamplifier input $V_d \approx 100\mu V$

A) Optimum filtering

Noise is not white, therefore: optimum filter = noise whitening filter plus matched filter

Preamp Output voltage noise

$$S_T = S_v + \frac{S_i}{\omega^2 C_L^2} = S_v \frac{1+\omega^2 T_{nc}^2}{\omega^2 T_{nc}^2}$$

with $$T_{nc} = \frac{C_L\sqrt{S_v}}{\sqrt{S_i}} = 50\mu s$$

noise whitening filter = CR differentiator with time constant RC=T_{nc}

$$H_B = \frac{sT_{nc}}{1+sT_{nc}} \qquad \left|H_B(\omega)\right|^2 = \frac{\omega^2 T_{nc}^2}{1+\omega^2 T_{nc}^2}$$

"Whitened" noise $S_B = S_v$

Rectangular signal = positive step with amplitude V_F at t=0 plus negative step with amplitude V_F at t= T_P.
Signal after the whitening filter= positive exponential pulse at t=0 plus equal but negative exponential pulse a t= T_P .

$$v_{B2} = V_F \cdot \left\{ 1(t)\cdot\exp\left[-t/T_{nc}\right] - 1(t-T_P)\cdot\exp\left[-(t-T_P)/T_{nc}\right]\right\}$$

Optimum weighting function of the matched filter:

$$w_{m2} = 1(t)\cdot\frac{1}{T_{nc}}\exp\left[-t/T_{nc}\right] - 1(t-T_P)\cdot\frac{1}{T_{nc}}\exp\left[-(t-T_P)/T_{nc}\right]$$

There is practically no overlap of the two exponentials because

$$T_P \gg T_{nc}$$

Therefore, the measurement on the rectangular pulse can be directly obtained from the sum of two measurements on simple step pulses. With respect to the measurement on a single step pulse, the measure on the rectangular pulse has

1. Double signal amplitude
2. double mean square noise
3. S/N greater by a factor $\sqrt{2}$
4. Minimum measurable amplitude smaller by a factor $\sqrt{2}$

The single step signal at the output of the noise-whitening filter is

$$v_{B1} = V_F \cdot 1(t)\cdot\exp(-t/T_{nc})$$

its matched filter is

$$w_{m1} = \frac{1}{T_{nc}}\exp\left(-\frac{t}{T_{nc}}\right)$$

denoting by $S_{v,b}$ the bilateral noise density of the "whitened" noise we get for the single step

$$\left(\frac{S}{N}\right)_{op1} = \frac{\int_0^\infty v_{B1}(t)\,w_{m1}(t)\,dt}{\sqrt{S_{v,b}}\sqrt{\int_0^\infty w_{m1}^2(t)\,dt}} = \frac{V_F T_{nc}}{\sqrt{S_{v,b}}}\sqrt{\int_0^\infty w_{m1}^2(t)\,dt} = \frac{V_F\sqrt{T_{nc}}}{\sqrt{2S_{v,b}}} = \frac{V_F\sqrt{T_{nc}}}{\sqrt{S_{v,u}}}$$

The minimum amplitude at S/N=1 is

$$V_{F\min 1,op} = \frac{\sqrt{S_{vu}}}{\sqrt{T_{nc}}} \approx 2,8\mu V$$

and corresponds to

$$F_{\min 1,op} = \frac{V_{F\min 1,op}}{A_q/C_L} \approx 0,14\cdot 10^{-3} N = 140\mu N$$

Therefore, for the rectangular pulse we get

$$V_{F\min 2,op} = \frac{V_{F\min 1,op}}{\sqrt{2}} = \frac{\sqrt{S_{vu}}}{\sqrt{2T_{nc}}} \approx 2\mu V$$

which corresponds to

$$F_{\min 2,op} = \frac{F_{\min 1,op}}{\sqrt{2}} = \frac{V_{F\min 2,op}}{A_q/C_L} \approx 0,1\cdot 10^{-3} N = 100\mu N$$

B) Approximation of optimum filtering by a constant parameter filter

Also in this case the measurement on the rectangular pulse is obtained from the sum of two measurements on simple step pulses and we analyze first the measure of a single step pulse.

The weighting function denotes that the matched filter it is a low-pass filter, therefore also its approximation must be a low-pass filter. A single-pole low-pass RC filter with RC=T_{nc} has weighting function in frequency with module equal to the optimum weighting function. Therefore, this RC filter will filter the white noise just like the optimum filter

However, the action on the signal will not be the optimum.

The signal corresponding to a single step at the whitening filter output is

Time domain $v_{B1} = V_F \cdot 1(t)\cdot \exp(-t/T_{nc})$ Laplace $V_{B1} = V_F T_{nc} \frac{1}{1+sT_{nc}}$

The low-pass filter has

δ-response $h_L = \frac{1}{T_{nc}} \exp\left(-\frac{t}{T_{nc}}\right)$ Laplace $H_L = \frac{1}{1+sT_{nc}}$

The low-pass filter output is

$v_{U1} = V_F \cdot 1(t)\cdot \frac{t}{T_{nc}} \exp(-t/T_{nc})$ Laplace $V_{F1} = V_F T_{nc} \frac{1}{(1+sT_{nc})^2}$

The measure is taken at the maximum of the output pulse, which occurs at t=T_{nc} and is

$$s_{U1} = \frac{V_F}{e} \approx 0,37\cdot V_F$$

Therefore

$$\left(\frac{S}{N}\right)_{L1} = \frac{V_F\sqrt{T_{nc}}}{\sqrt{S_{v,u}}}\cdot\frac{2}{e} = \frac{2}{e}\cdot\left(\frac{S}{N}\right)_{op1} \approx 0,74\left(\frac{S}{N}\right)_{op1} \approx \frac{1}{1,36}\left(\frac{S}{N}\right)_{op1}$$

$$V_{F\min 1,L} = \frac{e}{2}\frac{\sqrt{S_{vu}}}{\sqrt{T_{nc}}} \approx 3,8\mu V$$

Therefore, with the rectangular pulse we get a minimum amplitude

$$V_{F\min 2,L} = \frac{V_{F\min 1,L}}{\sqrt{2}} = \frac{e}{2}\frac{\sqrt{S_{vu}}}{\sqrt{2T_{nc}}} \approx 2,7\,\mu V$$

which is worse than the optimum by the factor 1,36 pointed out in the S/N. The minimum measurable force is higher by the same factor

$$F_{\min 2,L} = \frac{F_{\min 1,L}}{\sqrt{2}} = \frac{V_{F\min 2,L}}{A_q / C_L} \approx 0,136\cdot 10^{-3}\,N = 136\,\mu N$$

C) Approximation of the optimum filtering by a switched-parameter filter

Also in this case the measurement on the rectangular pulse is obtained from the sum of two measurements on simple step pulses, we analyze first the measure of a single step pulse and scale then the result to the case of rectangular pulse.

As approximation of the optimum filtering we employ a simple Gated Integrator (GI) and select the gate duration T_G for maximizing the S/N. Denoting by w_G the GI weighting function we get at GI output

signal $$V_G = \int_0^{\infty} v_B(\alpha) w_G(\alpha)\, d\alpha$$

noise $$\sqrt{\overline{v_{n,G}^2}} = \int_{-\infty}^{\infty} S_{V,bil}\delta(\tau)\, k_{ww,G}(\tau)\, d\tau = S_{V,bil}\, k_{ww,G}(0) = \frac{S_V}{2}\, k_{ww,G}(0)$$

Let us consider a normalized GI

$$w_G(\alpha) = \frac{1}{T_G} \qquad \text{for } 0 < \alpha < T_G$$

With a single step signal we have

signal $$V_{G1} = \frac{Q}{C_L}\cdot\frac{1-e^{-T_G/T_{nc}}}{\dfrac{T_G}{T_{nc}}}$$

noise $$\sqrt{\overline{v_{n,G1}^2}} = \frac{S_{v,u}^{1/2}}{\sqrt{2T_G}} = \frac{S_{v,u}^{1/2}}{\sqrt{2T_{nc}}}\sqrt{\frac{T_{nc}}{T_G}}$$

hence $$\left(\frac{S}{N}\right)_{G1} = \frac{Q}{C_s}\cdot\frac{1}{\dfrac{S_{v,u}^{1/2}}{\sqrt{T_{nc}}}}\cdot\sqrt{2}\cdot\frac{1-e^{-T_G/T_{nc}}}{\sqrt{\dfrac{T_G}{T_{nc}}}}$$

that is, by denoting $x = \frac{T_G}{T_{nc}}$ we get

$$\left(\frac{S}{N}\right)_{G1} = \frac{Q}{C_s} \cdot \frac{1}{\frac{S_{v,u}^{1/2}}{\sqrt{T_{nc}}}} \cdot \sqrt{2} \cdot \frac{1-e^{-x}}{\sqrt{x}} = \left(\frac{S}{N}\right)_{op1} \cdot \sqrt{2} \cdot \frac{1-e^{-x}}{\sqrt{x}}$$

The function $g(x) = \frac{1-e^{-x}}{\sqrt{x}}$ with x≈1,25 attains its maximum $g_{max} \approx 0{,}638$ and gives a factor $\sqrt{2} g_{max} = 0{,}9$. We can conclude that with $T_G \approx 1{,}25\ T_{nc} \approx 62{,}5\ \mu s$ we get a result not much inferior to the optimum filter, namely just 10% lower.

$$\left(\frac{S}{N}\right)_{G1} = \frac{Q}{C_s} \cdot \frac{1}{\frac{S_{vu}^{1/2}}{\sqrt{T_{nc}}}} \cdot 0{,}9 \cong \left(\frac{S}{N}\right)_{op1} \cdot 0{,}9$$

We note that the selection of T_G is not critical, because the maximum of g(x) is very wide: from x=1 to x=1,6 the function g(x) has small variation, just a few %.

For the rectangular pulse we have then

signal $$V_{G2} = \frac{Q}{C_L} \cdot 2 \frac{1-e^{-T_G/T_{nc}}}{\frac{T_G}{T_{nc}}}$$

noise $$\sqrt{\overline{v_{n,G2}^2}} = \frac{S_{vu}^{1/2}}{\sqrt{T_G}} = \frac{S_{vu}^{1/2}}{\sqrt{T_{nc}}} \sqrt{\frac{T_{nc}}{T_G}}$$

hence $$\left(\frac{S}{N}\right)_{G2} = \frac{Q}{C_L} \cdot \frac{1}{\frac{S_{vu}^{1/2}}{\sqrt{T_{nc}}}} \cdot 2 \cdot \frac{1-e^{-T_G/T_{nc}}}{\sqrt{\frac{T_G}{T_{nc}}}} = \left(\frac{S}{N}\right)_{Mr} \sqrt{2} \cdot \frac{1-e^{-T_G/T_{nc}}}{\sqrt{\frac{T_G}{T_{nc}}}}$$

and selecting x≈1,25 we get

$$\left(\frac{S}{N}\right)_{G2} \cong \left(\frac{S}{N}\right)_{op2} \cdot 0{,}9$$

and obtain a minimum measurable force

$$F_{g,2\min} = \frac{F_{op2,\min}}{0{,}9} = 111 \mu N$$

D) Filtering the electromagnetic interference

The prospect of attenuating the electromagnetic interference signal by employing the filters already considered with minor modifications does not appear promising, because the interference frequency f_d =20kHz is fairly near to the band-limits of these filters and is therefore just moderately attenuated. In fact:

1) The whitening filter is a single-pole high-pass filter with the pole at frequency $f_h = 1/2\pi T_{nc} \approx 3kHz$

2) The low-pass filter with RC=T_{nc} , employed as approximation of the matched filter, has the same pole frequency $f_h = 1/2\pi T_{nc} \approx 3kHz$.

3) The gated integrator, employed with T_G=62,5μs as approximation of the matched filter, has action comparable to a low-pass RC filter with RC= T_G/2≈31μs , that is, with pole at frequency $f_l = 1/2\pi RC \approx 5kHz$.

However, the GI weighting function in frequency has also a sequence of local zeros at frequencies f_z multiple of 1/T_G, which can be well exploited for strongly attenuating the narrow-band disturbance. It is sufficient to make f_z =1/T_G or a multiple of it coincident with the frequency f_d=20kHz of the interfering signal. This can be obtained by selecting T_G = 50μs and we know that this will cause very little degradation of the noise filtering.

We have to consider, however, that nil interference transmission is achieved only with perfect coincidence of the GI-zero-frequency f_z with the interference frequency f_d. Perfect coincidence cannot be guaranteed since the interference frequency is known with limited precision; we have to consider the effect of deviations Δf up to 1% between interference frequency f_d and GI-zero-frequency f_z.

Since the relative deviation Δf is small, that is, $\Delta f << f_z$ we can evaluate the transmission with first-order approximation

$$W_G = \frac{\sin(\pi f T_G)}{\pi f T_G} \qquad \text{hence} \qquad \frac{dW_G}{df} = \frac{\cos(\pi f T_G)}{f} - \frac{\sin(\pi f T_G)}{f^2}$$

At $f=f_z$ it is $\sin(\pi f_z T_G) = 1 \qquad \cos(\pi f_z T_G) = 1 \qquad$ hence $\left|\frac{dW_G}{df}\right|_{f=f_z} = \frac{1}{f_z}$

We can conclude that the residual transmission is simply given by the relative deviation in frequency of the interference signal with respect to the frequency of zero transmission.

$$\Delta W_G \approx \left|\frac{dW_G}{df}\right|_{f=f_z} \cdot \Delta f_d = \frac{\Delta f_d}{f_z}$$

In our case we have deviations up to $\Delta f_d / f_z \approx \pm 0,01$ and therefore $\Delta W_G \approx \pm\ 0,01$ which causes a residual transmission of the disturbance

$$V_d \cdot \Delta W_G \approx \pm 1\mu V$$

In comparison with the noise at the GI output

$$\sqrt{n_G^2} = \frac{\sqrt{S_v}}{\sqrt{2T_G}} \approx 2\mu V \qquad \text{with } T_G = 50\mu s$$

the residual disturbance is therefore acceptable