PROBLEM 1

Specified data

Signal: rectangular pulses with duration T_p and amplitude A_p

$T_p = 10\mu s$

A_p variable, to be measured

An auxiliary signal synchronous with the pulse onset is available.
Measurements are carried out in runs of about 20 min, with interval of a few minutes between runs, available for auxiliary operations (baseline zero setting; calibrations; etc.)

Noise: white component $\sqrt{S_v} = 50\, nV/\sqrt{Hz}$ limited by a pole at f_a = 200 MHz

Where specified, also a *1/f* component with corner frequency f_c=100kHz

Hint: For better understanding this problem it is advisable to make reference to the course slides and in particular to the slides in the Section HPF2

A – Measurement with white noise only

A1) Without filtering

$$\sqrt{\overline{n_0^2}} = \sqrt{S_v \frac{\pi}{2} f_a} \approx 890\mu V$$

$$S/N = A_p \Big/ \sqrt{\overline{n_0^2}}$$

$$A_{p,\min} = \sqrt{\overline{n_0^2}} \approx 890\mu V$$

A2) With filtering by a LPF that produces strong reduction of the noise bandwidth and negligible reduction of the pulse amplitude.

It is considered negligible a reduction of the pulse amplitude by less than 1%.

We employ a RC single-pole LPF filter, with time constant $R_L C_L = T_L$. The output pulse waveform rises with exponential shape and reaches the maximum V_p at $t = T_p$

$$V_p = A_p \left(1 - e^{-\frac{T_p}{T_L}}\right)$$

For obtaining $V_p > 0{,}99 A_p$ it is necessary to have $T_p > 4{,}6 T_L$. Therefore, we select

$$T_L = T_p/5 = 2\mu s$$

The noise bandwidth is thus limited to $f_{Fn} = 1/4T_L = 125kHz$, hence

$$\sqrt{\overline{n_1^2}} = \sqrt{S_v}\sqrt{f_{Fn}} = \sqrt{S_v}\sqrt{1/4T_L} = \sqrt{S_v}\sqrt{5/4T_p} \approx 18\mu V$$

$$S/N = V_p \Big/ \sqrt{\overline{n_1^2}} \approx A_p \Big/ \sqrt{\overline{n_1^2}}$$

$$A_{p,\min} = \sqrt{\overline{n_1^2}} \approx 18\mu V$$

B) Measurements in presence also of the 1/f noise component with f_c= 100kHz (besides the white noise component) with high-pass filtering performed by CDS-FB

Filtering of the noise at high frequencies is still performed with the RC-LPF seen in (A).

The basic elementary approach for limiting the 1/f noise contribution at low frequency is to set at zero the amplifier baseline before each measurement run. Performing the zero-setting by measuring the baseline with a low-pass filter implements a CDS-FB that avoids the white-noise enhancement typical of a simple CDS. The duration of each run is about $T_q \approx 20$ min $\approx$ 1000s, therefore the CDS-FB establishes a high-pass cutoff at frequency about

$$f_q = 1/2\pi T_q < 1mHz$$

while at high frequency the 1/f noise is subject to a low-pass cutoff at frequency

$$f_L = 1/2\pi T_L$$

With the sharp-cutoff approximation, we evaluate that in this conditions the 1/f noise contribution

$$\sqrt{\overline{n_{f1}^2}} \approx \sqrt{S_v}\sqrt{f_c}\sqrt{\ln\left(\frac{f_L}{f_q}\right)} = \sqrt{S_v}\sqrt{f_c}\sqrt{\ln\left(\frac{T_q}{T_L}\right)} \approx 70\mu V$$

is much greater than the white noise contribution

C) Measurements in presence also of the 1/f noise component with f_c= 100kHz (besides the white noise component) with high-pass filtering performed by a constant-parameter CR filter

Let us consider to employ a CR differentiator with time constant $R_H C_H = T_H$, which establishes for the 1/f noise a high-pass cutoff at frequency

$$f_H = 1/2\pi T_H$$

However, this filter produces also a reduction of the pulse amplitude depending on the time constant T_H. The reduction can be evaluated by considering the step response of the CR filter

$$1(t)\, e^{-\frac{t}{T_H}}$$

and taking into account that the signal is still approximately a step, with a rise smoothed by the LPF that reaches its maximum at t=T_p. For keeping the amplitude loss below 1% it is necessary to have

$$e^{-\frac{T_p}{T_H}} \geq 1 - \frac{1}{100} \qquad \text{that is} \qquad T_H \geq 100 T_p$$

With the sharp-cutoff approximation we evaluate that in this conditions the 1/f noise contribution is

$$\sqrt{\overline{n_{fH}^2}} \approx \sqrt{S_v}\sqrt{f_c}\sqrt{\ln\left(\frac{f_L}{f_H}\right)} = \sqrt{S_v}\sqrt{f_c}\sqrt{\ln\left(\frac{T_H}{T_L}\right)}$$

By recalling that we have

$$T_L = T_p/5 = 2\mu s$$

and selecting

$$T_H = 100 T_p = 1ms$$

We get

$$\sqrt{\overline{n_{fH}^2}} \approx \sqrt{S_v}\sqrt{f_c}\sqrt{\ln\left(\frac{T_H}{T_L}\right)} = \sqrt{S_v}\sqrt{f_c}\sqrt{\ln(500)} \approx 39\mu V$$

The improvement is significant, but the 1/f contribution is still high and dominant over the white noise.

Furthermore, the action of the constant parameter differentiator on the pulse produces after each pulse a "tail" of opposite polarity with amplitude that decays exponentially with the time constant T_H. If the time interval between the pulses is not sufficient for the tail to decay at negligible level, the measurement of a pulse will be affected by a negative contribution due to the previous pulses.

In the case of pulse **repetition rate** $f_r = 100Hz$, the time interval between pulse $T_r = 1/f_r = 10ms$ is much greater than T_H and the CR filter can be correctly employed without interference between the pulses.

In the case of pulse **repetition rate** $f_r = 10kHz$**,** the time interval between pulse $T_r = 1/f_r = 100\mu s$ is smaller than T_H and the CR filter cannot be correctly employed. The baseline of the output waveform would have a negative shift that brings the DC component to zero, that is, a baseline shift of about $A_p\, T_p/T_r \approx 0{,}1\, A_p$, that is, about 10% of the pulse amplitude.

D) Measurements in presence also of the 1/f noise component with f_c= 100kHz (besides the white noise component) with high-pass filtering by a time-variant-parameter CR filter

A baseline restorer (BLR) is a $C_B R_B$ differentiator with resistance R_B controlled by a switch in series, which is open during the pulse (hence it is equivalent to infinite resistance). In the intervals free from pulses the BLR is a differentiator with time constant

$$R_B C_B = T_B$$

In the BLR the differentiator action is not applied to the pulses, but only to the noise. The BLR can thus be employed for avoiding the limitations and drawbacks of the constant-parameter CR differentiator.

The reader is referred to the Section HPF2 of the course slides for explanation and illustration of the action of the BLR and of the results obtained.

The BLR does not affect the pulse amplitude and establishes for the 1/f noise a high pass cutoff at frequency

$$f_B = 1/2\pi T_B$$

This cutoff frequency can be much lower than that allowed for a constant-parameter CR filter. However, it can be shown that it must not be very short, in order to avoid the noise enhancement effect typical of a simple CDS. Satisfactory results are obtained with time-constant somewhat longer than the pulse duration, that is,

$$T_B \approx 5T_p$$

With the sharp-cutoff approximation we evaluate that in this conditions the 1/f noise contribution is

$$\sqrt{\overline{n_{fB}^2}} \approx \sqrt{S_v}\sqrt{f_c}\sqrt{\ln\left(\frac{f_L}{f_B}\right)} = \sqrt{S_v}\sqrt{f_c}\sqrt{\ln\left(\frac{T_B}{T_L}\right)}$$

By recalling that we have

$$T_L = T_p/5 = 2\mu s$$

and selecting

$$T_B = 5T_p = 50\mu s$$

We get

$$\sqrt{\overline{n_{fB}^2}} \approx \sqrt{S_v}\sqrt{f_c}\sqrt{\ln\left(\frac{T_B}{T_L}\right)} = \sqrt{S_v}\sqrt{f_c}\sqrt{\ln(25)} \approx 28\mu V$$

The improvement with respect to the constant-parameter CR is remarkable. First, the output noise is significantly reduced. Second, all the problems met with the CR filter in case of measurements of signals in fast sequence are eliminated. In fact, no pulse tails are generated by the BLR because the differentiation is not applied to the pulse signals