PROBLEM 2

Specified data

DIODES IN THE THERMOMETRIC SENSOR

Diode D1: area A_1=100(μm)^2 ; default bias current I_1=100μA

Diode D2: area A_2=1000(μm)^2 ; default bias current I_2=100μA

INTEGRATED PREAMPLIFIER

Differential voltage amplifier with wide band, to be considered infinite

At each input:

- high input resistance (to be considered infinite)
- $\sqrt{S_{Va}} = 40\, nV/\sqrt{Hz}$ (unilateral) voltage noise referred to input
- $\sqrt{S_{ia}} = 0{,}1\, pA/\sqrt{Hz}$ (unilateral) current noise referred to input

In (C) and (D) the voltage noise has also a 1/f component with corner frequency f_c=200 kHz

A) Principle of the sensor operation and temperature-to-voltage transduction factor

Voltage and **current density** of p-n diodes are related by the Shockley equation

$$\mathrm{j} = \mathrm{j_s}\,(\mathrm{e}^{-\frac{qV}{kT}} - 1) \qquad \text{with} \quad j = \frac{I}{A} \qquad \text{with V=diode voltage; I=diode current; A=diode area}$$

With forward bias even at moderate level it is $\frac{\mathrm{j}}{\mathrm{j_s}} >> 1$

Therefore, with good approx. it is $\mathrm{j} = \mathrm{j_s}\,\mathrm{e}^{-\frac{qV}{kT}}$

and therefore $\mathrm{V} = \frac{kT}{q}\ln\left(\frac{\mathrm{j}}{\mathrm{j_s}}\right)$

This voltage dependence on T is neither well known nor well reproducible because of the dependence of saturation current density j_S on T. However, two diodes D_1 and D_2 with equal structure and equal material properties (“twin” diodes implemented in the same planar chip) have equal saturation current density j_S . Therefore, the difference V_m of the forward voltages of the two diodes does not depend on j_S and has a simple and reliable relation with temperature

$$\mathrm{V_m} = \mathrm{V_1} - \mathrm{V_2} = \frac{kT}{q}\ln\left(\frac{\mathrm{j_1}}{\mathrm{j_S}}\right) - \frac{kT}{q}\ln\left(\frac{\mathrm{j_2}}{\mathrm{j_S}}\right) = \frac{kT}{q}\ln\left(\frac{\mathrm{j_1}}{\mathrm{j_2}}\right)$$

Taking into account that $j_1 = \frac{I_1}{A_1}$ and $j_2 = \frac{I_2}{A_2}$

We get

$$\mathrm{V_m} = \mathrm{V_1} - \mathrm{V_2} = \frac{kT}{q}\ln\left(\frac{\mathrm{j_1}}{\mathrm{j_2}}\right) = \frac{kT}{q}\left[\ln\left(\frac{\mathrm{I_1}}{\mathrm{I_2}}\right) + \ln\left(\frac{\mathrm{A_2}}{\mathrm{A_1}}\right)\right]$$

with equal bias currents I_1=I_2

$$V_m = V_1 - V_2 = \frac{kT}{q}\ln\left(\frac{A_2}{A_1}\right)$$

In our case A_2/A_1=10 and we have

$$V_m = V_1 - V_2 = 2{,}3\frac{kT}{q}$$

That is, we have a linear transduction from temperature T to voltage difference between the diodes V_m=V_1-V_2 with conversion factor

$$\frac{\Delta V_m}{\Delta T} = 2{,}3\frac{k}{q} = 2{,}3\frac{25mV}{300K} = 191\frac{\mu V}{K} = 191\frac{nV}{mK}$$

B) <u>Error in the measured temperature due to the white noise</u>

The white noise component of the amplifier voltage noise is

$\sqrt{S_{va}} = 40\, nV/\sqrt{Hz}$.

The current noise components are converted to voltage by the resistance of the diode connected to the input

$$R_D = \frac{dV}{dI} = \frac{kT}{qI}$$ that is, with I_D=100μA R_D= 250 Ω

The contribution of the amplifier current noise is negligible $R_D\sqrt{S_{ia}} = 25\, pV/\sqrt{Hz}$.

The shot noise of the diode current

$$S_{iD} = 2qI$$

gives a negligible voltage noise S_{vD}

$$S_{vD}^{1/2} = R_D S_{iD}^{1/2} = \frac{kT}{q}\sqrt{\frac{2q}{I}}$$ that is, with I_D=100μA $S_{vD}^{1/2} = 1{,}4 nV/\sqrt{Hz}$

The voltage noise of the amplifier noise is clearly dominant: we evaluate the voltage noise taking into account a voltage noise generator $\sqrt{S_v} \approx \sqrt{S_{va}}$ at each input of the differential preamp.

The signal has significant frequency components up to about 1Hz (variations over time longer than 1s) and the noise is wide-band. Therefore, we can filter the noise with a low-pass filter with noise band-limit

$f_S \approx 10\ Hz$

We get a voltage noise

$$\sqrt{\overline{v_{nb}^2}} = \sqrt{2}\sqrt{S_v}\sqrt{f_S} \approx 178\text{nV}$$

which corresponds to an error in the measured temperature

$$\varepsilon_m = \frac{\sqrt{\overline{v_{nb}^2}}}{\frac{dV_m}{dT}} \approx 0,0009\text{ K} = 0,9\text{m K}$$

C) <u>Error in the measured temperature due to the 1/f noise</u>

The contribution of the 1/f noise is limited at high frequency by the filter considered in (B) and at low frequency by basic elementary approach of setting the LPF-baseline to zero in the interval between the measurement runs, that is, by Correlated Double Sampling with Filtered baseline (CDS-FB). The time interval is about ≈1000s, hence the cutoff frequency of the high-pass filter is f_i ≈ 0,001 Hz. We can evaluate the 1/f noise contribution with the approximation of sharp-cutoff , taking into account the contributions of the two inputs

$$\sqrt{\overline{v_{nf}^2}} \approx \sqrt{S_v}\cdot\sqrt{f_c}\sqrt{\ln\left(\frac{f_S}{f_i}\right)} \approx 18\mu\text{V}\cdot\sqrt{\ln(10000)} = 54,6\mu\text{V}$$

This corresponds to a temperature error due to the 1/f noise that largely exceeds the required level

$$\varepsilon_m = \frac{\sqrt{\overline{v_{nf}^2}}}{\frac{dV_m}{dT}} \approx 0,286\text{ K} = 286\text{mK}$$

D) <u>Modified approach for reducing the error due to the 1/f noise</u>

D1- Modulation of the electrical signal

The 1/f noise sources are in the preamplifier, therefore an efficient approach for reducing the effect of the 1/f noise is to modulate the signal before it reaches the preamplifier and thus bring it in a frequency range where 1/f noise is negligible.

In the case considered it is not possible to modulate the signal by contacting the diode terminals with switches. However, it is possible to control the current generators that produce the current in the diodes, and thereby to modulate the diode currents and obtain a suitably modulated differential voltage signal.

It is not advisable to use a sinusoidal modulated current component. The strongly nonlinear current-voltage equation of the diode would produce a non-sinusoidal voltage waveform, not well suitable to our purpose. It is instead advisable to employ a squarewave current commutation as described in the following lines.

a) in the first half of the period the situation is as previously described: the two currents are equal $I_1=I_2=100\mu A$, the current densities are different by a factor 10 and the voltage difference is

$$V_m = V_1 - V_2 = \frac{kT}{q}\ln\left(\frac{A_2}{A_1}\right) = 2,3\frac{kT}{q}$$

b) in the other half period, the current in D1 is commuted to $I_1=10\mu A$, so that the current densities are now equal $j_1 = I_1/A_1 = I_2/A_2 = j_2$ and the voltage difference is zero

$$V_{m0} = \frac{kT}{q}\ln\left(\frac{j_1}{j_2}\right) = 0$$

We have thus a squarewave differential voltage signal $v_m(t)$ with peak-to-peak amplitude V_m , that is, oscillation amplitude $A=V_m/2$.

By driving the commutation of the current generator with a squarewave at frequency f_m quite higher than the 1/f noise-corner frequency, we can bring the voltage signal in a frequency range where the 1/f noise is negligible and only the white noise must be taken into account. For instance, we can employ f_m =2MHz , i.e. f_m 10 times higher than f_c.

D2- Measurement of the modulated signal

An efficient way of measuring the amplitude $A=V_m/2$ is to use a Lock-in amplifier (LIA) employing as reference the same squarewave signal r(t) that drives the current modulation. Denoting by B the amplitude of the reference signal, the output signal V_L of the LIA is

$$V_L = B \cdot A = B \cdot \frac{V_m}{2}$$

The considerations made in (B) about the selection of the band-limit of the LPF filter are still valid now for the selection of the LPF in the LIA, hence we adopt

$f_S \approx 10\ Hz$

Since the modulated signal and the reference of the LIA have the same waveform, the S/N is optimized and equal to the ratio of the full signal power (in-phase with the reference) to one half of the white noise power in the noise band of the LIA's LPF $\Delta f_n = 2f_S$ (reduction given by the selection in phase of the noise components)

The output signal power is

$$P = A^2 = \frac{V_m^2}{4}$$

The output noise power the in the band Δf_n is

$$\sqrt{\overline{n_{nL}^2}} = \sqrt{2S_{vb}(f_m)\Delta f_n} = \sqrt{S_{vu}(f_m)\Delta f_n} = \sqrt{S_{vu}(f_m)2f_S}$$

where S_{vb} denotes the <u>bilateral</u> noise density and $S_{vu} = S_v \approx S_{va}$ the unilateral noise density.

Hence the S/N

$$\left(\frac{S}{N}\right) = \frac{A}{\sqrt{2S_{vb}(f_m)\Delta f_n}} = \frac{A}{\sqrt{S_{vu}(f_m)2f_S}}$$

The minimum measurable amplitude of the squarewave signal thus is

$$A_{\min} = \frac{V_{m,\min}}{2} = \sqrt{S_{vu}(f_m)2f_S}$$

The minimum signal for the temperature measurement is

$$V_{m,\min} = 2A_{\min} = 2\sqrt{S_{vu}(f_m)2f_S} \approx 2\sqrt{S_v(f_m)}\sqrt{2f_S} \approx 358nV$$

The corresponding error in temperature fulfills the requirement

$$\varepsilon_m = \frac{\sqrt{\overline{v_{nL}^2}}}{\frac{dV_m}{dT}} \approx 0,0019\text{ K} = 1,9\text{mK}$$

The basic reasons that make possible such a remarkable improvement are the capability of shifting the signal at a frequency well above the range of the dominant 1/f noise and the capability of the LIA of filtering the high frequency signal with very narrow bandwidth.