PROBLEM 2

Specified data

Pulse signal $x(t) = Pe^{(t-T_P)/T_D}$ in $0 \le t \le T_P$ and $x=0$ elsewhere

$T_P = 10\mu s$

$T_D = 10\mu s$

P pulse amplitude, to be measured

$\sqrt{S_{v,u}} = 20\, nV/\sqrt{Hz}$ unilateral

(A) – Filtering with gated integrator (GI)

The gate is open for the duration of the signal $T_G=T_P$

- Normalized GI weighting function $w_G = \dfrac{1}{T_P}$
- GI Output signal $u_G = \int_0^{T_P} x(t)\, w_G(t)\, dt = P\dfrac{T_D}{T_P}\left(1-e^{-T_P/T_D}\right)$
- GI weighting autocorrelation amplitude $k_{ww,G}(0) = \int_0^{T_P} w_G^2 dt = \dfrac{1}{T_P}$
- GI output noise $\overline{n_G^2} = S_{v,b}\, k_{ww,G}(0) = S_{v,u}\dfrac{1}{2T_P}$
- GI output S/N $\left(\dfrac{S}{N}\right)_G = P\dfrac{T_D}{T_P}\left(1-e^{-T_P/T_D}\right)\dfrac{\sqrt{2T_P}}{\sqrt{S_{v,u}}}$
- minimum pulse amplitude measurable with GI

$$P_{\min,G} = \frac{\sqrt{S_{v,u}}}{\sqrt{2T_P}}\frac{T_P}{T_D}\frac{1}{\left(1-e^{-T_P/T_D}\right)} = \frac{\sqrt{S_{v,u}}}{\sqrt{T_D}}\sqrt{\frac{T_P}{2T_D}}\frac{1}{\left(1-e^{-T_P/T_D}\right)} =$$

$$= \frac{\sqrt{S_{v,u}}}{\sqrt{T_D}}\frac{1}{\sqrt{2}\left(1-e^{-1}\right)} =$$

$$= 6{,}32\mu V \cdot 1{,}12 \approx 7{,}1\mu V$$

(B) – Optimum filtering and real circuits that can implement it

B1 - Optimum filtering

Pulse signal $x(t) = Pe^{(t-T_P)/T_D}$ in $0 \le t \le T_P$ and $x=0$ elsewhere

- Area of pulse signal $A = PT_D\left(1-e^{-T_P/T_D}\right)$

- Optimum weighting function: with white noise it is simply the matched function, that is, the pulse waveform normalized to unit area

$$w_o(t) = \frac{x(t)}{A} = \frac{1}{T_D\left(1-e^{-T_P/T_D}\right)} e^{(t-T_P)/T_D}$$

- Optimum weighting autocorrelation amplitude

$$k_{ww,o}(0) = \int_0^{T_P} w_o^2\, dt = \frac{1}{T_D^2\left(1-e^{-T_P/T_D}\right)^2}\int_0^{T_P} e^{2(t-T_P)/T_D}\, dt = \frac{1-e^{-2T_P/T_D}}{2T_D\left(1-e^{-T_P/T_D}\right)^2}$$

- Optimum filter output signal

$$u_o = \int_0^{T_P} x(t) w_o(t)\, dt = A\int_0^{T_P} w_o^2(t)\, dt = Ak_{ww,o}(0) = \frac{P}{2}\cdot\frac{1-e^{-2T_P/T_D}}{1-e^{-T_P/T_D}}$$

- Optimum filter output noise

$$\overline{n_o^2} = S_{v,b}\, k_{ww,o}(0) = \frac{S_{v,u}}{2} k_{ww,o}(0) = S_{v,u}\frac{1-e^{-2T_P/T_D}}{4T_D\left(1-e^{-T_P/T_D}\right)^2}$$

- Optimum filter output S/N

$$\left(\frac{S}{N}\right)_G = \frac{u_o}{\sqrt{\overline{n_o^2}}} = \frac{Ak_{ww,o}(0)}{\sqrt{\frac{S_{v,u}}{2}}\sqrt{k_{ww,o}(0)}} = \frac{A}{\sqrt{\frac{S_{v,u}}{2}}}\sqrt{k_{ww,o}(0)} =$$

$$= \frac{PT_D\left(1-e^{-T_P/T_D}\right)}{\sqrt{\frac{S_{v,u}}{2}}}\cdot\sqrt{\frac{1-e^{-2T_P/T_D}}{2T_D\left(1-e^{-T_P/T_D}\right)^2}} = \frac{PT_D}{\sqrt{\frac{S_{v,u}}{2}}}\sqrt{\frac{1-e^{-2T_P/T_D}}{2T_D}} =$$

$$= \frac{P\sqrt{T_D}}{\sqrt{S_{v,u}}}\sqrt{1-e^{-2T_P/T_D}}$$

- minimum pulse amplitude measurable with the optimum filtering

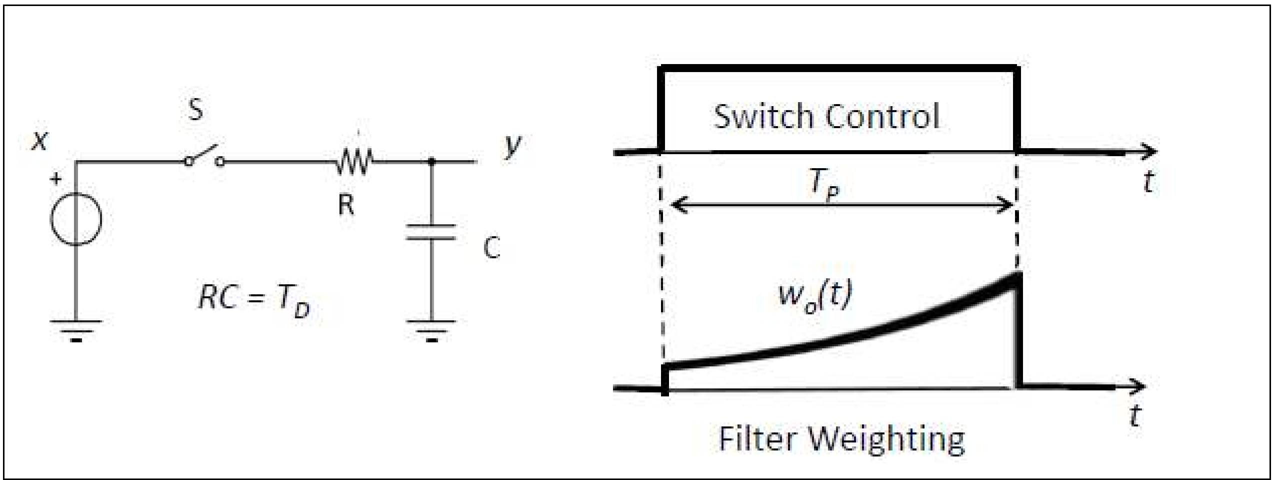
$$P_{\min,o} = \frac{\sqrt{S_{v,u}}}{\sqrt{T_D}}\frac{1}{\sqrt{1-e^{-2T_P/T_D}}} = \frac{\sqrt{S_{v,u}}}{\sqrt{T_D}}\frac{1}{\sqrt{1-e^{-2}}} = 6{,}32\mu V\cdot 1{,}075 \approx 6{,}8\mu V$$

- GI filtering compared to optimum filtering

$$\frac{P_{\min,G}}{P_{\min,o}} = \sqrt{\frac{T_P}{2T_D}}\frac{\sqrt{1-e^{-2T_P/T_D}}}{\left(1-e^{-T_P/T_D}\right)} \approx 1{,}04$$

B2 - Real circuits that can implement the optimum filtering

The optimum weighting function is obtained with a switched RC integrator like that employed for implementing a Gated Integrator, but with time constant RC modified and made equal to the parameter T_D of the pulse signal $RC = T_D$



(C) – Filter circuits that combine optimum weighting with exploitation of repetitive signals

C1 – Modified Boxcar Integrator (BI)

With repetitive signal with time interval T_R between consecutive pulses, i.e. with repetition frequency $f_R=1/T_R$, in order to exploit the information redundancy the switched integrator modified with RC=T_D can be employed as <u>Boxcar Integrator</u>. In Boxcar operation the measurement is obtained as an exponential average of measurements on single pulses, each one carried out with optimum weighting.

The attenuation ratio β of the weight from pulse to pulse is due to the capacitor discharge. In a Boxcar Integrator the discharge occurs only during the GATE-ON time interval. Therefore, for the Boxcar Integrator the exponential averaging ratio is

$$\beta = e^{-\frac{T_P}{T_D}} = e^{-1} \approx 0,368$$

The signal u_o is enhanced by the factor $\dfrac{1}{1-\beta}$ (sum of the signal terms decreasing exponentially with decrease ratio β)

The noise $\overline{n_o^2}$ is enhanced by the factor $\dfrac{1}{1-\beta^2}$ (sum of the quadratic noise terms decreasing exponentially with decrease ratio β^2).

Therefore, the improved SNR is

$$\left(\frac{S}{N}\right)_{AV} = \left(\frac{S}{N}\right)_o \frac{\sqrt{1-\beta^2}}{1-\beta} = \left(\frac{S}{N}\right)_o \sqrt{\frac{1+\beta}{1-\beta}} \approx 1,47\left(\frac{S}{N}\right)_o$$

The factor of improvement 1,47 thus obtained is significant, notwithstanding that the integration time constant employed is much shorter than the usual setting in a Boxcar Integrator and therefore the averaging is carried out over a moderate number of pulse signals. The minimum amplitude measurable is correspondingly reduced by the same factor 1,47.

C2 – Modified Ratemeter Integrator

The switched integrator modified with $RC=T_D$ can be a Ratemeter Integrator. In Ratemeter operation the measurement is obtained as exponential average of measurements on single pulses, each one carried out with optimum weighting.

The attenuation ratio ρ of the weight from pulse to pulse is due to the capacitor discharge. In a Ratemeter Integrator the discharge occurs during the entire time interval T_R between the pulses in sequence. Therefore, the RI exponential averaging ratio ρ is much smaller than the BI exponential averaging ratio β

$$\rho = e^{-\frac{T_P}{T_R}} = e^{-10} \approx 0$$

This means that with a Ratemeter Integrator circuit there is practically no averaging and therefore no SNR enhancement if the integrator time constant is set at the value required for achieving optimum filtering in the single measurements

$$\left(\frac{S}{N}\right)_{AV} = \left(\frac{S}{N}\right)_o \cdot \frac{\sqrt{1-\rho^2}}{1-\rho} \approx \left(\frac{S}{N}\right)_o \cdot 1$$