PROBLEM 1

Data and Notes

Applied force F

first case: step pulse

second case: rectangular pulse with duration T_p = 5ms

Piezoelectric force sensor

A_q=10pC/N force-to-charge conversion

C_L= 500pF total capacitance of sensor and connected circuit

Preamplifier

Input resistance R_{iA} very high > 500 MΩ , to be treated as ∞

f_{pa}=20MHz band limit

$\sqrt{S_{v,u}} = 20nV/\sqrt{Hz}$ wide-band (unilateral)

$\sqrt{S_{i,u}} = 0,2\,pA/\sqrt{Hz}$ wide-band (unilateral);

where it is stated, take into account in S_i also a *1/f* component with corner frequency f_c=1KHz

(A) Optimum Filtering

Noise is not white, therefore: optimum filter = noise-whitening filter followed by matched filter.

Voltage noise at the preamp output

$$S_T = S_v + \frac{S_i}{\omega^2 C_L^2} = S_v \frac{1+\omega^2 T_{nc}^2}{\omega^2 T_{nc}^2}$$

with $$T_{nc} = \frac{C_L\sqrt{S_v}}{\sqrt{S_i}} = 50\mu s$$

Noise-whitening filter = CR differentiator with time constant RC=T_{nc}

$$H_B = \frac{sT_{nc}}{1+sT_{nc}} \qquad \left|H_B(\omega)\right|^2 = \frac{\omega^2 T_{nc}^2}{1+\omega^2 T_{nc}^2}$$

"whitened" noise $S_B = S_v$

Case 1: step applied force

A step force F applied to the sensor generates in the capacitor a piezoelectric charge Q = A_q F and therefore a step voltage signal

$$V_F = \frac{A_q}{C_L} \cdot F$$

The force-to-voltage conversion factor thus is

$$\frac{A_q}{C_L} = 20mV / Newton$$

The output signal of the whitening filter is

$$v_{B1} = V_F \cdot 1(t) \cdot \exp(-t / T_{nc})$$

The matched filter weighting function has the same shape as this signal

$$w_{m1} = \frac{1}{T_{nc}} \exp\left(-\frac{t}{T_{nc}}\right)$$

Denoting by $S_{v,b}$ the bilateral density and by $S_{v,u}$ the unilateral density of the white noise at the input of the matched filter, the S/N is

$$\left(\frac{S}{N}\right)_{op1} = \frac{\int_0^\infty v_{B1}(t)\, w_{m1}(t)\, dt}{\sqrt{S_{v,b}}\sqrt{\int_0^\infty w_{m1}^2(t)\, dt}} = \frac{V_F T_{nc}}{\sqrt{S_{v,b}}}\sqrt{\int_0^\infty w_{m1}^2(t)\, dt} = \frac{V_F\sqrt{T_{nc}}}{\sqrt{2S_{v,b}}} = \frac{V_F\sqrt{T_{nc}}}{\sqrt{S_{v,u}}}$$

The minimum measurable amplitude of the voltage signal, which corresponds to S/N=1, is

$$V_{F\min 1,op} = \frac{\sqrt{S_{vu}}}{\sqrt{T_{nc}}} \approx 2{,}8\mu V$$

The corresponding minimum force is

$$F_{\min 1,op} = \frac{V_{F\min 1,op}}{A_q / C_L} \approx 0{,}14 \cdot 10^{-3} N = 140\mu N$$

<u>Case 2: applied force rectangular with duration T_P</u>

The whitening filter and its output noise are the same of case 1.

The rectangular signal is the sum of a positive step with amplitude V_F aplied at t=0 and a negative step with equal amplitude V_F aplied at a t= T_P . Therefore, the signal at the output of the whitening filter is composed by two equal exponential pulses, a positive one a t=0 and a negative one at t= T_P .

$$v_{B2} = V_F \cdot \left\{1(t) \cdot \exp\left[-t / T_{nc}\right] - 1(t - T_P) \cdot \exp\left[-(t - T_P) / T_{nc}\right]\right\}$$

Therefore, in this case the optimum weighting function is

$$w_{m2} = 1(t)\cdot\frac{1}{T_{nc}}\exp\left[-t/T_{nc}\right] - 1(t-T_P)\cdot\frac{1}{T_{nc}}\exp\left[-(t-T_P)/T_{nc}\right]$$

It is worth noting that there is negligible superposition of the two exponential pulses because $T_P \gg T_{nc}$

We see therefore that in the case 2 the measure can be obtained by subtracting from the measure of the first exponential pulse (as made in case 1) the measure of the second exponential pulse (negative pulse). We note that

1. The signal amplitude doubles
2. The mean square noise doubles
3. the S/N is increased by the factor $\sqrt{2}$ and the minimum amplitude is reduced by this factor

This result can be obtained also without considering the composition of two measures, but just by computing the S/N with the correct functions v_{B2} and w_{m2} and taking into account that $T_P \gg T_{nc}$.

(B) Approximation of the matched filter with a low-pass filter with constant parameters

Case 1: step applied force

The weighting function of the matched filter shows that it is a low-pass filter. Therefore, also the filter to be employed as approximation of the matched filter must be a low-pass filter. We note that in the frequency domain the matched filter has module of the weighting function equal to that of a simple RC low-pass filter with $RC=T_{nc}$. Therefore, with such a filter the filtered white noise is equal to that of the matched filter.

$$\overline{n_U^2} = S_{v,u}\frac{1}{4T_{nc}} \approx 1,4\mu V$$

However, the result obtained for the filtered signal is different and obviously less favourable. The output signal of the whitening filter is

in time domain $v_{B1} = V_F \cdot 1(t)\cdot\exp(-t/T_{nc})$ in Laplace domain $V_{B1} = V_F T_{nc}\dfrac{1}{1+sT_{nc}}$

The action of the RC low-pass filter with $RC=T_{nc}$ is obtained by means of

δ response in time $h_L = \dfrac{1}{T_{nc}}\exp\left(-\dfrac{t}{T_{nc}}\right)$ transfer function in Laplace domain $H_L = \dfrac{1}{1+sT_{nc}}$

The output of the RC low-pass filter is

in time $v_{U1} = V_F \cdot 1(t)\cdot\dfrac{t}{T_{nc}}\exp(-t/T_{nc})$ in Laplace domain $V_{F1} = V_F T_{nc}\dfrac{1}{(1+sT_{nc})^2}$

With constant parameter filters the pulse measurement is obtained by measuring the peak amplitude of the output signal in time. In the present case the maximum of the signal is found at $t=T_{nc}$

$$s_{U1} = \frac{V_F}{e} \approx 0,37 \cdot V_F$$

We have thus

$$\left(\frac{S}{N}\right)_{L1} = \frac{V_F\sqrt{T_{nc}}}{\sqrt{S_{v,u}}} \cdot \frac{2}{e} = \frac{2}{e} \cdot \left(\frac{S}{N}\right)_{op1} \approx 0,74\left(\frac{S}{N}\right)_{op1} \approx \frac{1}{1,36}\left(\frac{S}{N}\right)_{op1}$$

$$V_{F\min 1,L} = \frac{e}{2}\frac{\sqrt{S_{vu}}}{\sqrt{T_{nc}}} \approx 3,8\mu V$$

<u>Case 2: applied force rectangular with duration T_P</u>

We can employ also in this case the same procedure as in (A) for computing the optimum result. The measurement is obtained by summing (algebrically) the measurement of the (positive) step signal at t=0 and the measurement of the (negative) step signal signal at $t= T_P$.

It is therefore concluded that also employing as approximate matched filter a simple constant-parameter low-pass filter, with a rectangular pulse the S/N and the minimum measurable amplitude are improved by a factor $\sqrt{2} \approx 1,41$ with respect to the case of step pulse

(C) <u>Measurement in presence of 1/f component in the current noise</u>

We take now into account also a 1/f component with corner frequency f_c in the noise current

$$S_f = S_i \frac{f_c}{f}$$

The corresponding noise component added to the output of the whitening filter is a *1/f* component filtered by the capacitance C_s and by the high-pass CR whitening filter.

$$S_i \frac{\omega_c}{\omega} \cdot \frac{1}{\omega^2 C_L^2} \cdot \frac{\omega^2 T_{nc}^2}{1+\omega^2 T_{nc}^2} = S_v \frac{\omega_c}{\omega} \cdot \frac{1}{1+\omega^2 T_{nc}^2}$$

It is therefore a *1/f* noise component filtered only by a low-pass filter (a simple pole with time constant T_{nc}), without any high-pass filtering.

<u>Case 1: step applied force</u>

The filter employed as an approximation of the matched filter(see in B) is a simple <u>low-pass</u> filter with band limit f_{nc} (equal to that of the matched filter).

We know (seen in B) that the white noise component gives a contribution $\sqrt{\overline{n_U^2}}$ to the filtered noise.

$$\overline{n_U^2} = S_{v,u} \frac{1}{4T_{nc}} \approx 1,4\mu V$$

As concerns the 1/f noise, the low-pass filter does not provide any high-pass filtering action that limits the 1/f contribution $\sqrt{\overline{n_f^2}}$, which is thus dominant with respect to the white noise $\sqrt{\overline{n_U^2}}$.

For limiting $\sqrt{\overline{n_f^2}}$ it is thus necessary to introduce after the low-pass filter a suitable high-pass filter that introduces a band limit f_i at low frequency. At high frequency the band limit f_s is given by the pole of the low-pass filter with time constant T_{nc}., therefore $f_s \sim 1/T_{nc}$

Since it is $f_s >> f_i$, we can use the sharp band-cutoff approximation for evaluating the filtered 1/f noise

$$\sqrt{\overline{n_f^2}} = \sqrt{S_v}\sqrt{f_c} \cdot \sqrt{\ln\left(\frac{f_s}{f_i}\right)} \approx 0,63\mu V \cdot \sqrt{\ln\left(\frac{f_s}{f_i}\right)}$$

Let us consider some different types of high pass filters that can be employed for limiting the 1/f noise.

The simplest filtering approach is to manually set to zero the baseline at the start of a cycle of measurements.
If the cycle has a duration T_m , the lower band-limit is $f_i \sim 1/T_m$.
The upper band-limit is set by the low-pass filter pole $f_s \sim 1/T_{nc}$

For a measurement cycle of $T_m \approx 15\min = 1000s$ we can count on a high-pass cutoff with

$T_m \approx 15\min = 1000s$ $\qquad f_i = 1mHz$

We have thus $\frac{f_s}{f_i} = \frac{T_m}{T_{nc}} = 2 \cdot 10^7$

Therefore it is $\sqrt{\overline{n_f^2}} \approx 0,63\mu V \cdot 4,1 = 2,6\mu V$ almost double of the white noise

A better result can be obtained by employing a constant parameter CR high-pass filter. The differentiation time constant T_D must be long with respect to the duration of the signal, in order to avoid a reduction of the signal amplitude. For instance, we can use

$T_D = 100T_{nc} = 5ms$ thus obtaining $\frac{f_s}{f_i} = \frac{T_D}{T_{nc}} = 100$

and therefore $\sqrt{\overline{n_f^2}} \approx 0,63\mu V \cdot 2,14 = 1,35\mu V$ almost equal to the white noise

This result is quite satisfactory, but it is possible to obtain a further improvement by employing a Baseline Restorer BLR. Since the BLR is a switched-parameter filter, it is possible to employ a shorter differentiation time constant T_B without reducing the signal amplitude .
For instance we may employ

$T_B = 10T_{nc} = 500\mu s$ which gives $\frac{f_s}{f_i} = \frac{T_B}{T_{nc}} = 10$

and therefore $\sqrt{\overline{n_f^2}} \approx 0,63\mu V \cdot 1,41 = 0,9\mu V$ lower than the white noise contribution

Case 2: applied force rectangular with duration T_P

The measurement is obtained as difference of two subsequent measurements, spaced by a time interval T_P . It is a Correlated Double Filterig CDF, which adds to the low-pass filtering (with time constant T_{nc}) a high-pass filtering, with lower band-limit equivalent to that of a constant-parameter CR filter with differentiation time constant equal to the interval T_P . In our case it is

$T_P = 5ms$ therefore $$\frac{f_s}{f_i} = \frac{T_P}{T_{nc}} = 100$$

A low frequency cutoff is produced by this CDF filtering without needing to introduce any other filtering. However, it must be noted that the CDF doubles the mean square noise in the band defined by the limits f_s and f_i

$$\sqrt{\overline{n_f^2}} \approx \sqrt{2} \cdot \sqrt{S_v}\sqrt{f_c} \cdot \sqrt{\ln\left(\frac{f_s}{f_i}\right)} = \sqrt{2} \cdot 0,63\mu V \cdot 2,14 = 1,9\mu V$$

The CDF also doubles the mean square noise due to the white noise in the filtering band

$$\sqrt{\overline{n_{U2}^2}} \approx \sqrt{2}\sqrt{S_{v,u}}\frac{1}{\sqrt{4T_{nc}}} \cdot \approx 2\mu V$$

We see that the two contributions of the 1/f noise and of the white noise in this case are almost equal.

We can conclude that in the measurement of the rectangular signal it is not necessary to introduce a further high-pass filter for limiting the contribution of the 1/f noise. In fact, the filtering that gives an optimized (or approximately optimized) measurement with white noise inherently includes a high-pass filtering that provides adequate reduction of the 1/f noise contribution.