PROBLEM 2

Data and Notes

Optical Signal:

rectangular with duration T_P

Power P_S ; wavelength λ_i = 600 nm.

Optical background:

Power P_F specified in each case

PMT Photomultiplier:

photocathode S11 with quantum efficiency η_{DM} = 0,025 at λ_i = 600 nm

cathode dark current I_{bk} = 0,16 fA (corresponding to $n_{bk} = 10^3$ electrons/second)

multiplier with gain $G = 10^5$ and excess noise factor F= 2

PIN Silicon Photodiode:

quantum efficiency η_{DM} = 0,30 at λ_i = 600 nm

dark current I_b = 0,05 pA

Current Preamplifier with transimpedance configuration:

Bandpass limited by simple pole f_{pa} = 100 MHz

Current noise density (unilateral) referred to preamp input $S_{ip}^{1/2}$ = 0,05 pA/Hz$^{1/2}$

N.B: the voltage noise has negligible effect in this circuit configuration, therefore it is not specified.

Detector responsivity

The results computed in terms of electrical signals are translated to optical power by means of the

detector responsivity $$S_D = \eta \cdot \frac{\lambda q}{hc} = \eta \cdot \frac{\lambda[\mu m]}{1{,}24} \quad A/W$$

PMT at λ_i = 600 nm: η_{DM} = 0,025 hence S_{DM} = 0,012 A/W = 12mA/W

PIN at λ_i = 600 nm: η_{DP} = 0,30 hence S_{DP} = 0,145 A/W = 145mA/W

A) Measure of pulses with duration T_P = 1µs, without optical background P_F =0

A1) with photomoltiplier PMT

current noise spectral density

preamp input $S_{ip} = 25 \cdot 10^{-28}\, A^2/Hz$

dark current at PMT anode $S_{ibA} = 2qI_{bk} \cdot F \cdot G^2 = 51{,}2 \cdot 10^{-36} \cdot 2 \cdot 10^{10} = 1{,}024 \cdot 10^{-24}\, A^2/Hz$

since $S_{ip} << S_{ibA}$ the dark current noise is dominant $S_{iTA} = S_{ip} + S_{ibA} \approx S_{ibA} = 1{,}024 \cdot 10^{-24}\, A^2/Hz$

the total noise density referred to the photocathode is

$$S_{iTK} = \frac{S_{iTA}}{G^2} = \frac{S_{ibA}}{G^2} = 2qI_{bk} \cdot F = 1,024 \cdot 10^{-34} A^2 / Hz$$

This noise is white over a band much wider than the bandwidth of the signal.

Therefore, filtering by a Gated Integrator with duration equal to the signal T_G=T_P= 1μs is suitable and is a good approximation of the optimal filtering that can be employed in practice. The noise low-pass filtering bandwidth (unilateral) is

$$f_G = \frac{1}{2T_G} = \frac{1}{2T_P} = 500kHz$$

The noise referred to the cathode is

$$\sqrt{\overline{i_{nK}^2}} = \sqrt{S_{iTK} f_G} = \sqrt{\frac{2qI_{bk} \cdot F}{2T_P}} = 0,72 \cdot 10^{-14} A = 7,2 fA$$

So that the minimum detectable current at the cathode is

$$I_{PK\min} = \sqrt{\overline{i_{nK}^2}} = 7,2 fA$$

However, the filtering bandwidth is quite wide, so that the fluctuations of the photocurrent I_{pk} itself are quite high, whereas the dark current noise is very low. Therefore, it is necessary to check whether the fluctuations of the photocurrent I_{pk} set to the minimum detectable current a limit higher than that due to the dark current I_{bk}.

Taking into account only the photocurrent fluctuations

$$\left(\frac{S}{N}\right)^2 = \frac{I_{pk}^2}{F 2qI_{pk} f_G}$$

the minimum measurabile current $I_{pk,min}$ corresponding to (S/N)=1 is

$$I_{pk,\min} = F 2qf_G = F \cdot \frac{q}{T_G} = 3,2 \cdot 10^{-13} A = 320 fA$$

We have thus verified that the limit $I_{pk,min}$ set by the photocurrent fluctuations is dominant, since it is much higher than the limit $I_{PK,min}$ set by the dark current fluctuations. The corresponding minimum detectable optical power is

$$P_{S\min} = \frac{I_{pk\min}}{S_{DM}} = \frac{320 \cdot 10^{-15}}{12 \cdot 10^{-3}} = 26,6 pW$$

A2) with PIN photodiode

current noise of the preamp: $S_{ip} = 25 \cdot 10^{-28} A^2 / Hz$

noise of the photodiode dark current $S_{iB} = 2qI_B = 1{,}6 \cdot 10^{-32} A^2 / Hz$

with the PIN the preamp noise is dominant $S_{iB} << S_{ip}$, so that the total noise density is

$$S_{iT} = S_{ip} + S_{iB} \approx S_{ip} = 25 \cdot 10^{-28} A^2 / Hz$$

Filtering with a GI with T_G= 1μs , noise bandwidth (unilateral) f_G = 1/2 T_G = 500kHz, we get

$$\sqrt{\overline{i_{nD}^2}} = \sqrt{S_{iT} f_G} = \sqrt{\frac{S_{ip}}{2T_P}} = 3{,}53 \cdot 10^{-11} A$$

the minimum measurable current is

$$I_{D\min} = \sqrt{\overline{i_{nD}^2}} = \sqrt{S_{iT} f_G} = \sqrt{\frac{S_{iT}}{2T_G}} = 3{,}53 \cdot 10^{-11} A = 35{,}3 pA$$

Let us check also in the case of PIN the role of the fluctuations of the photocurrent. The computation of S/N limited only by the photocurrent fluctuations in the PIN is like that made for the PMT, but without the excess noise due to the statistical multiplication of the PMT, that is, with F=1

$$\left(\frac{S}{N}\right)_p^2 = \frac{I_{Dp}^2}{2qI_{Dp} f_G}$$

The minimum measurable current $I_{Dp,min}$ limited by the photocurrent fluctuations thus is

$$I_{Dp\min} = 2qf_G = \frac{q}{T_G} = 16 \cdot 10^{-14} A = 160 fA$$

It is thus verified that the limit I_{Dpmin} set by the photocurrent fluctuations is much lower than that set by the preamplifier noise. The minimum measurable photocurrent thus is

$$I_{D\min} = \sqrt{\overline{i_{nD}^2}} = \sqrt{S_{iT} f_G} = 35{,}3 pA$$

which corresponds to a minimum measurable optical power

$$P_{S\min} = \frac{I_{D\min}}{S_{DP}} = \frac{35{,}3 \cdot 10^{-12}}{145 \cdot 10^{-3}} = 243 pW$$

The PIN sensitivity is limited by the noise of the preamplifier and therefore it is much lower than the PMT sensitivity. With the PMT the electronic circuit noise plays no role thanks to the high gain G of the electron multiplier. The gain fluctuations bring a moderate increase of the noise, given the moderate value F=2 of the excess noise factor.

B) <u>Measure of pulses with longer duration T_P , without optical background P_F =0</u>

B1) pulses with duration T_P = 10 μs (10 times longer)

Filtering by a Gated Integrator with duration equal to the signal T_G=T_P= 10μs, lowpass noise filtering band (unilateral)

$$f_G = \frac{1}{2T_G} = \frac{1}{2T_P} = 50kHz$$

With PMT

The increase of T_p by the factor 10 reduces the noise referred to the cathode by the square root of the factor $\sqrt{10}$

$$\sqrt{\overline{i_{nK}^2}} = \sqrt{S_{iTK} f_G} = \sqrt{\frac{2qI_{bk} \cdot F}{2T_P}} = 0,227 \cdot 10^{-14} A = 2,27\, fA$$

Therefore, the limit set to the minimum measurable current at the cathode by the dark-current fluctuations is reduced by the factor $\sqrt{10}$

$$I_{PK\min} = \sqrt{\overline{i_{nK}^2}} = 2,27\, fA$$

The limit set to the minimum measurable current at the cathode by the photocurrent fluctuations is reduced by the factor 10

$$I_{pk,\min} = F2qf_G = F \cdot \frac{q}{T_G} = 1,6 \cdot 10^{-13} A = 32\, fA$$

Therefore, this limit is still much higher and dominant. The corresponding minimum detectable power is

$$P_{S\min} = \frac{I_{pk\min}}{S_{DM}} = \frac{16 \cdot 10^{-15}}{12 \cdot 10^{-3}} = 2,6\, pW$$

With PIN photodiode

The increase of T_p by the factor 10 reduces the noise referred to the cathode by a factor $\sqrt{10}$

$$\sqrt{\overline{i_{nD}^2}} = \sqrt{S_{iT} f_G} = \sqrt{\frac{S_{ip}}{2T_P}} = 1,12 \cdot 10^{-11} A$$

Therefore, it reduces by the same factor $\sqrt{10}$ the limit set to the minimum measurable current

$$I_{D\min} = \sqrt{\overline{i_{nD}^2}} = \sqrt{S_{iT} f_G} = \sqrt{\frac{S_{iT}}{2T_G}} = 1,12 \cdot 10^{-11} A = 11,2\, pA$$

The limit set by the photocurrent fluctuations to the minimum measurable current at the cathode is reduced by the factor 10

$$I'_{D\min} = 2qf_G = \frac{q}{2T_G} = 8 \cdot 10^{-15} A = 8 fA$$

Therefore, the limit due to the noise of the electronic circuit is even more dominant.

The corresponding minimum detectable power is

$$P_{S\min} = \frac{I_{D\min}}{S_{DP}} = \frac{11,2 \cdot 10^{-12}}{145 \cdot 10^{-3}} = 77\, pW$$

B2) pulses with duration T_P = 10 ms (10000 times longer)

Filtering by Gated Integrator with duration equal to the signal T_G=T_P= 10ms, lowpass noise filtering band (unilateral)

$$f_G = \frac{1}{2T_G} = \frac{1}{2T_P} = 50 Hz$$

With photomoltiplier PMT

The increase of the duration T_p by the factor 10000 reduces by a factor $\sqrt{10000} = 100$ the noise referred to the cathode

$$\sqrt{\overline{i_{nK}^2}} = \sqrt{S_{iTK} f_G} = \sqrt{\frac{2qI_{bk} \cdot F}{2T_P}} = 0,072 \cdot 10^{-15} A = 72 aA$$

Therefore, the limit set to the minimum cathode current detectable is also reduced by the factor 100

$$I_{PK\min} = \sqrt{\overline{i_{nK}^2}} = 72 aA$$

the limit set to the minimum detectable cathode current by the photocurrent fluctuations is reduced by the factor 10000

$$I_{pk\min} = F 2qf_G = F \cdot \frac{q}{2T_G} = 1,6 \cdot 10^{-17} A = 16 aA$$

We have thus verified that the limit $I_{pk,min}$ set by the photocurrent fluctuations in this case is lower than the limit set by the dark-current fluctuations. However, the ratio of the two limits is not very high, it is a factor 4,5. This implies that the photocurrent fluctuations will bring a slight increase of the detection limit with respect to the value I_{PKmin} computed with only the dark-current.

It is possible to have a more precise evaluation by writing the $(S/N)^2$ with both contributions of the fluctuation and computing the value of I_{PK} that gives $(S/N)^2 =1$, but the result will be only slightly higher than I_{PKmin} =72 aA and this more precise evaluation is not requested. The minimum detectable optical power corresponding to the limit I_{PKmin} =72 aA is

$$P_{S\min} = \frac{I_{PK\min}}{S_{DM}} = \frac{72\cdot 10^{-18}}{12\cdot 10^{-3}} = 6\,fW$$

As T_P is increased starting from T_P=1μs, we observe that the detection limit decreases first as $1/T_P$, i.e. in a first phase where the photocurrent fluctuations are dominant. At longer T_P the limit decreases progressively more slowly and when the dark-current fluctuations becomes dominant the detection limit decreases like $1/\sqrt{T_P}$.

With PIN photodiode

The increase of T_p by the factor 10000 reduces the noise by the factor $\sqrt{10000} = 100$

$$\sqrt{\overline{i_{nD}^2}} = \sqrt{S_{iT} f_G} = \sqrt{\frac{S_{ip}}{2T_P}} = 3,53\cdot 10^{-13} A = 354\,fA$$

The limit to the minimum detectable current is reduced by the same factor 100

$$I_{D\min} = \sqrt{\overline{i_{nD}^2}} = \sqrt{S_{iT} f_G} = \sqrt{\frac{S_{iT}}{2T_G}} = 3,53\cdot 10^{-13} A = 354\,fA$$

This limit coresponds to a minimum detectable optical power

$$P_{S\min} = \frac{I_{D\min}}{S_{DP}} = \frac{353\cdot 10^{-15}}{145\cdot 10^{-3}} = 2,43\,pW$$

It is not necessary to check the limit set by the fluctuations of the photocurrent, which is reduced by the factor 10000 and is even more negligible than in the previous case..

C) <u>Measure of pulses with T_P = 10ms in presence of optical background P_F</u>

The photodetector receives a steady background light with power P_F , which has wavelength equal to the signal and therefore is not attenuated by the optical filters. It produces a further constant current I_F that is added to the dark-current I_b , bringing further shot noise. We have to compute the level of power P_F that makes the mean square value of the shot noise double of that in the previous case with negligible background.

With photomultiplier PMT

We have seen in the case without optical background that dark-current noise density at the PMT cathode is $S_{ibK} = 2qI_{bk}$ and produces at the anode a current noise density $S_{ibA} = 2qI_{bk} \cdot F \cdot G^2$ which is dominant over that of the preamp $S_{ibA} >> S_{ip}$. The total noise at the preamp input was therefore $S_{iTA} = S_{ibA} + S_{ip} \approx S_{ibA} = 2qI_{bk} \cdot F \cdot G^2$ and the corresponding noise density referred to the cathode was

$$S_{iTK} = \frac{S_{iTA}}{G^2} = \frac{S_{ibA}}{G^2} = 2qI_{bk} \cdot F = 1,024 \cdot 10^{-34} A^2 / Hz$$

The steady current I_{Fk} generated at the cathode by the optical background is simply added to the dark current, giving an increased steady total shot current

$$I_{bk} + I_{Fk}$$

The corresponding increased spectral density is $2q(I_{bk} + I_{Fk})$ makes even more negligible the preamp noise. The total noise density referred to the cathode is now

$$S_{iTK} = \frac{S_{iTA}}{G^2} = 2q(I_{bk} + I_{Fk}) \cdot F$$

The noise with GI filtering andreferred to the cathode is

$$\overline{i_{nK}^2} = S_{iTK} f_G = \frac{2q(I_{bk} + I_{Fk})F}{2T_P}$$

The noise doubles with respet to the previous case when the background current is equal to thedark current, that is

$$I_{Fkm} = I_{bK} = 0,16 fA$$

The background power corresponding to this level is

$$P_{Fm} = \frac{I_{FKm}}{S_{DM}} = \frac{0,16 \cdot 10^{-15}}{12 \cdot 10^{-3}} = 13,3 fW$$

In these conditions the mean square noise is increased by a factor 2 and therefore the minimum measurable optical power is increased by a factor $\sqrt{2}$

$$P_{S\min} = \sqrt{2} \cdot 6 fW = 8,5 fW$$

With PIN photodiode

Also in the case of the PIN the steady current I_{Fp} generated by the optical background is simply added to the dark current I_b and the total shot current is

$$I_b + I_{Fp}$$

with a corresponding noise spectral density $2q(I_b + I_{Fp})$. However, with a PIN the dark-current shot noise is negligible with respect to the current noise of the preamplifier S_{ip}. In order to have a total current noise a factor 2 greater than the case without optical background, the shot noise due to the background has to be equal to the preamplifier current noise

$$2qI_{Fpm} = S_{ip} = 25 \cdot 10^{-28}\, A^2 / Hz$$

This corresponds to have a background current

$$I_{Fpm} = \frac{S_{ip}}{2q} = 7,8nA$$

The background optical power that generates such a current is

$$P_{F\mathrm{m}} = \frac{I_{Fp\mathrm{m}}}{S_{DP}} = \frac{7,8 \cdot 10^{-9}}{0,145} = 53,8nW$$

In presence of such a background the mean square value of the noise is increased by a factor 2 and therefore the minimum measurable optical power is increased by a factor $\sqrt{2}$

$$P_{S\min} = \sqrt{2} \cdot 2,43pW = 3,4pW$$