PROBLEM 2

Data

Strain gauges (SG):

R_S = 160 Ω resistance
G = 2,4 Gauge factor
P_{dmax}= 4 μW maximum power dissipation admitted in in each sensor

Temperature Coefficient $\alpha = \frac{\Delta R_S}{R_{S0}} = 4 \cdot 10^{-3}\ C^{-1}$

Temperature mismatch max $\Delta T_{max} \approx \pm 0{,}1°C$, variations over long time intervals >>10s

Differential Preamplifier:

Wide band f_p > 100MHz

$\sqrt{S_V}$ = 16 nV/Hz$^{1/2}$ (unilateral) wide band voltage noise referred to the preamp differential input; 1/f noise component with corner frequency f_{cv} = 100 kHz

$\sqrt{S_i}$ = 5 pA/Hz$^{1/2}$ (unilateral) wide band current noise referred to the preamp differential input; 1/f noise component with corner frequency f_{ci} = 100 kHz

Strain to be measured

In Sec.B: stationary strain with variation over long time intervals >10s ;

In Sec. C, D, E: sinusoidal oscillating strain of extension-contraction, caused by motor rotating at 12000rpm (revolutions per minute) i.e. with frequency f_m=200Hz

$$\varepsilon(t) = \varepsilon_S \cos(\omega_m t)$$

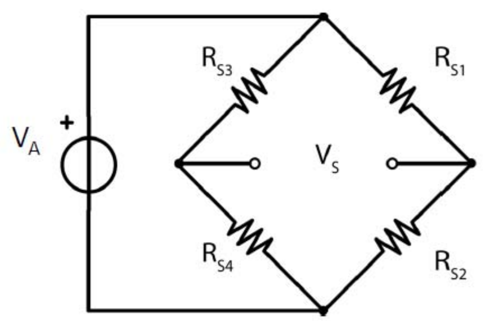
The amplitude of the oscillation slowly varies over time intervals >10s . A sinusoidal reference electrical signal is available with frequency and phase equal to the motor rotation

(A) Configuration, transduction gain, temperature mismatch effect and noise

- **Configuration**

  standard Wheatstone bridge configuration with four equal SG

  (R_{S1} , R_{S2} , R_{S3} , R_{S4}) and only one sensor subject to strain (R_{S2}).



- **Limit to the bridge supply voltage V_A**

  The DC voltage supply V_A is limited by the allowed power dissipation in the sensor

  $\frac{(V_A/2)^2}{R_S} \leq P_{d,\max}$ hence $V_A \leq 2\sqrt{P_{d,\max} R_S} = 50,5 mV$ we adopt $V_A = 50 mV$

- **Strain-to-voltage transduction factor**

  Small strain → small variations ΔR_S →small variations of bridge output ,

  $$V_S = \frac{V_A}{4} \cdot \frac{\Delta R_S}{R_{S0}}$$

  Strain $\varepsilon = \frac{\Delta l}{l}$ → resistance variation $\Delta R_S = G\varepsilon R_{S0}$ → output signal V_S

  $$V_S = \frac{V_A}{4} \cdot \frac{\Delta R_S}{R_{S0}} = \frac{V_A}{4} \cdot G\varepsilon = 30 mV \cdot \varepsilon$$

  Transduction factor

  $$\frac{dV_S}{d\varepsilon} = \frac{V_A}{4} \cdot G = \frac{30 mV}{strain} = \frac{30 nV}{\mu strain}$$

- **Effect of temperature mismatch of the sensors**

  Mismatch ΔT between active sensor (R_{S2}) and compensation sensor (R_{S1}) causes a difference $\Delta R_S = \alpha\, \Delta T \cdot R_{S0}$ between these resistances, i.e. a bridge unbalance. The stationary temperature difference ΔT thus simulates the presence of a stationary strain ε_T that produces this unbalance $\Delta R_S / R_{S0}$. This fake signal due to the mismatch ΔT is ruled by the temperature and is slowly varying, with frequency spectrum below 0,1 Hz

  $$\left(\frac{\Delta R_S}{R_{S0}}\right)_T = \alpha \cdot \Delta T$$

  and causes a bridge output voltage signal V_{ST}

  $$V_{ST} = \frac{V_A}{4} \cdot \left(\frac{\Delta R_S}{R_{S0}}\right)_T = \frac{V_A}{4} \cdot \alpha \cdot \Delta T$$

  Strain ε causes a difference $\Delta R_S = G\varepsilon \cdot R_{S0}$ and therefore a bridge unbalance

  $$\left(\frac{\Delta R_S}{R_{S0}}\right)_S = G\varepsilon$$

  and a bridge output voltage signal V_{SS}

$$V_{SS} = \frac{V_A}{4} \cdot G\varepsilon$$

Therefore, the temperature mismatch generates strain errors up to

$$\varepsilon_{Tm} = \pm \frac{\alpha \cdot \Delta T_{\max}}{G} = \pm 1,66 \cdot 10^{-4} \, strain = \pm 166 \; \mu strain$$

- **Noise referred to the preamp input**

  The equivalent source resistance between the output terminals of the Wheatstone bridge is equal to the resistance R_S of one SG.

  The voltage noise due to the source resistance is

  $\sqrt{4kT \cdot R_S} \approx 1,6 \, nV/\sqrt{Hz}$ without appreciable 1/f noise component

  The voltage noise due to the preamp current noise is

  $R_S \sqrt{S_i} = 0,8 nV/\sqrt{Hz}$ with 1/f corner frequency f_{cv} = 20 kHz

  The preamp voltage noise is

  $\sqrt{S_V} = 16 nV/\sqrt{Hz}$ with 1/f corner frequency f_{cv} = 20 kHz

  Therefore, S_v is dominant in the total noise $S_n(f)$ and the other contributions are negligible

(B) <u>Measurement of stationary strain carried out with DC voltage supply V_A</u>

- **<u>Effect of the noise in the measurement of a stationary strain</u>**

  Since the signal is slowly varying, the noise can be filtered at high frequency by a simple low-pass filter (for instance with band-limit $f_h \approx 1Hz$). At low frequency a high-pass band-limit f_l can be enforced (typically at $f_l \approx 1mHz$) by resetting to zero the amplifier baseline before each measurement run (for instance every 15 minutes). The preamp voltage noise is dominant, other components can be neglected in comparison; the contribution of the 1/f noise component is dominant and can be evaluated with the sharp-bandlimit approximation

$$\sqrt{\overline{v_{n,f}^2}} \approx \sqrt{S_V f_{cv}} \sqrt{\ln\left(\frac{f_h}{f_l}\right)} \approx 5.865 \text{ µV}$$

  The corresponding error in the measurement of strain is very high

$$\sqrt{\overline{\varepsilon_c^2}} = \frac{\sqrt{\overline{v_{n,c}^2}}}{\frac{dV_S}{d\varepsilon}} \approx 195 \mu strain$$

- **Effect of the temperature mismatch in measurement of a stationary strain**

  See in Sec. A the analysis and evaluation, which shows that there is a stationary error

  $$\varepsilon_{Tm} = \pm \frac{\alpha \cdot \Delta T_{\max}}{G} = \pm 1,66 \cdot 10^{-4}\, strain = \pm 166\ \mu strain$$

(C) Measurement of stationary strain carried out with AC supply voltage

- **Effect of the noise**

  We consider applied to the bridge a sinusoidal voltage with amplitude V_A at frequency f_A=200kHz.

  At the output of the bridge both the stationary signals considered in Sec. A (the useful one generated by the stationary strain and the fake one generated by the stationary temperature mismatch) are now modulated and shifted at frequency f_A. The noise is unmodified (not modulated), therefore the modulated signal is brought out of the spectral region where 1/f noise is dominant. We can then exploit a Lock-in Amplifier (LIA1) (employing a reference signal obtained from the bridge supply) for filtering the output of the preamplifier, bringing back to the base band (about f=0) the modulated signal accompanied by the noise found at the modulation frequency f_A =200kHz, that is, practically only the wide-band noise without 1/f noise. The LIA1 low-pass filter (last stage of the LIA1) must extract the low-frequency signal from this wide-band noise, therefore it can have a narrow band-limit $f_L \approx 1Hz$. The wide-band voltage noise S_V of the preamplifier is dominant and sets to the minimum measurable signal the limit

  $$\sqrt{\overline{v_{n,w}^2}} \approx \sqrt{2 S_V f_L} \approx S_V^{1/2} \sqrt{2 f_L} \approx 23\text{nV}$$

  that is a limit

  $$\sqrt{\overline{\varepsilon_{n,w}^2}} \approx 0,8 \mu strain$$

- **Effect of the temperature mismatch**

  This temperature mismatch causes a fake stationary signal (fake unbalance of the bridge) which is processed just like the true signal due to the strain. It is modulated and demodulated without being filtered; the error generated is not reduced, it is still the same.

- In conclusion, with respect to the measurement with DC supply:

  the error due to the noise is greatly reduced

  but the error due to temperature mismatch is not reduced

(D) Measurement of oscillating strain carried out with AC supply voltage

The signal to be measured is the fundamental component of the strain oscillation, at the revolution frequency of the motor

$$f_m = 12000rpm = 200Hz$$

An auxiliary signal with same frequency and phase as the signal is available (obtained by monitoring the rotation of the motor).

The signal amplitude V_S varies slowly, over time intervals >10s,

$v(t) \approx V_S \cos(\omega_m t)$ with slowly varying V_S

In the frequency domain, the signal is a line at frequency f_m, where the 1/f noise is very high. However with the bridge AC voltage supply at f_A =200kHz also the oscillating strain signal is modulated up and brought at $f_A \pm f_m$. We exploit then the Lock-in Amplifier (LIA1) with the reference signal obtained from the bridge supply for filtering the output of the preamplifier, and bring back to the base band (about f=0) the modulated signal accompanied by the noise found about the modulation frequency f_A =200kHz (practically only wide-band noise without 1/f noise). It must be noted, however, that the LIA1 low-pass filter (last stage of the LIA1) must now have a wider band-limit because it must pass the oscillating strain signal at f_m =200Hz. Therefore, the band-limit f_{L1} must be greater than the strain oscillation frequency f_m, typically $f_{L1} \approx$2kHz.

The S/N obtained can be computed as the ratio of:
(1) the amplitude V_S of the signal brought back in the baseband at f_m (by the frequency-domain convolution with the reference signal)
(2) the input spectral density S_n around $f_m \pm f_A$ brought back in the baseband (by the frequency-domain convolution with the spectrum of the reference signal) and filtered by the low-pass band-limit f_L of the LIA1 low-pass filter. We note that the convolution doubles the spectral density $2S_n$; this effect is commonly denoted "spectrum folding"

$$\left(\frac{S}{N}\right)_m = \frac{V_S}{\sqrt{2S_n(f_m + f_A) f_{L1}}} \qquad (S_n \text{ unilateral density})$$

At the LIA1 output we have an oscillating signal; the minimum measurable amplitude is limited by the noise at

$$V_{S\min} = \sqrt{2S_n(f_m + f_A)}\sqrt{f_{L1}} \approx \sqrt{2S_V}\sqrt{f_{L1}} = 1010nV$$

that is, the minimum strain measurable is

$$\sqrt{\overline{\varepsilon_n^2}} = \frac{\sqrt{\overline{v_n^2}}}{\frac{dV_S}{d\varepsilon}} \approx 34\mu strain$$

Effect of the temperature mismatch

This temperature mismatch causes a fake stationary signal (fake unbalance of the bridge) which is processed just like a true signal due to stationary strain. It is modulated and demodulated without being filtered out; the error due to the temperature mismatch is not reduced.

(E) Improving the measurement of oscillating strain by further filtering

At the LIA1 output, the fake signal due to the temperature mismatch and the signal due to oscillating strain can be clearly distinguished. The oscillating strain has narrow spectrum centered at f_m=200Hz, while the fake signal has a low-frequency spectrum in the range below 0,1Hz. The error due to temperature mismatch can be avoided (or at least strongly reduced) by applying to the LIA1 output a further filter that eliminates the fake signal by rejecting the low frequency components below 1Hz. By suitably selecting this filter, it is also possible to reduce the noise and the corresponding minimum measurable amplitude.

Such efficient filtering can be obtained with a second Lock-in Amplifier (LIA2) that employs the reference signal of the oscillation and operates on the output of the first Lock-in Amplifier (LIA1). The LIA2 eliminates the fake signal due to temperature mismatch and the associated error and sets a very narrow filtering bandwidth centered on the oscillation frequency, which strongly reduces the noise passed. For the low-pass filter in the LIA2, we select a band-limit f_{L2} just greater than the signal bandwidth $\Delta f_S < 1Hz$.

$f_{L2} = 1Hz$

We must take into account, however, that the LIA2 receives at the input the noise spectrum doubled by the spectrum folding in LIA1 and therefore we get

$$V_{S\min,L} = \sqrt{2}\sqrt{2S_V}\sqrt{f_{L2}} \approx 32nV$$

that is $$\varepsilon_{\min,L} = \frac{V_{S\min,L}}{\frac{dV_S}{d\varepsilon}} = 1{,}1\mu strain$$