PROBLEM 1

Data

SQUAREWAVE PULSE SIGNAL

$s_i(t) = A$ constant amplitude A in $0 < t < T_P = 10\mu s$

WHITE NOISE

$\sqrt{S_{bu}} = 20nV/\sqrt{Hz}$ unilateral ($\sqrt{S_{bb}} = 10nv/\sqrt{Hz}$ bilateral)

SWITCHED INTEGRATORS

Passive filter $R_1 = 10k\Omega$ C_1 *to be selected* $T_{F1} = R_1 C_1$

Active filter $R_2 = 100k\Omega$ $R_3 = 1k\Omega$ C_2 *to be selected* $T_{F2} = R_2 C_2$

REPETITIVE PULSE SIGNALS

in (B) constant repetition frequency f_R=*100 Hz*

in (C) variable repetition frequency f_R *from 100 Hz to 200Hz*

in (D) random arrival times with mean repetition rate 100 pulses/s

(A) Amplitude measurement of a single pulse

A1) Filter weighting function

With white noise the optimum filter weighting function is equal to the signal waveform, i.e. constant over the pulse duration T_P.

The weighting of a switched integrator filter in the interval 0<t< T_P is

$$\frac{1}{T_F}\exp(-t/T_F) \approx \frac{1}{T_F}(1 - t/T_F)$$ with approximation valid for $T_F >> T_P$

In order to limit to less than 1% the deviation from constant weight we must have T_{F1} and T_{F2} at least

$T_{F1} = T_{F2} = T_F = 100\ T_P = 1ms$

Therefore we select

C_1 = 100 nF e C_2 = 10 nF

In this condition the filter weighting function with good approximation has constant amplitude g

passive filter $$w_{g1}(t) = g_1 = \frac{1}{R_1 C_1} = \frac{1}{T_F}$$ in $0 < t < T_P = 10\mu s$

active filter $$w_{g2}(t) = g_2 = \frac{R_2}{R_3}\frac{1}{R_2 C_2} = \frac{R_2}{R_3}\frac{1}{T_F}$$ in $0 < t < T_P = 10\mu s$

At the filter output we have

Signal $$s_g = \int_0^{T_P} w_g(t) s_i(t)\, dt = \int_0^{T_P} Ag\, dt = AgT_P = Ag\frac{T_P}{T_F}$$

Noise $$\overline{n_g^2} = S_{bb}\cdot k_{ww}(0) = S_{bb}\cdot g^2 T_P$$ (bilateral spectral density $S_{bb} = S_{bu}/2$)

S/N $$\frac{S}{N}=\frac{s_g}{\sqrt{\overline{n_g^2}}}=A\frac{\sqrt{T_P}}{\sqrt{S_{bb}}}$$

A2) Minimum input pulse amplitude and corresponding output amplitude

The S/N is equal for the two filters, independent from the "gain" g. Therefore, also the minimum input pulse amplitude is equal

$$A_{\min,GI}=\frac{\sqrt{S_{bb}}}{\sqrt{T_P}}=\frac{\sqrt{S_{bu}}}{\sqrt{2T_P}}\approx 4,5\mu V$$

However, the two filters have very different amplitude of the output signal.

For the passive integrator the output amplitude is much lower than the input amplitude

$$s_{g1}=AT_Pg_1=A\frac{T_P}{T_F}=\frac{A}{100}<<A$$

For the active integrator, the output amplitude is equal to the input amplitude

$$s_{g2}=AT_Pg_2=A\frac{T_P}{T_F}\frac{R_2}{R_3}=s_{g1}\frac{R_2}{R_3}=100s_{g1}=A$$

A higher signal amplitude at the output of the filter is advantageous because it relaxes the requirements for the input noise and the amplification of the electronic circuits that follow the filter. These circuits have to amplify without adding significant noise the signal to be measured to bring it at proper level in the dynamic range of the measurement instrument (typically a ADC).

The two filters have output signal with different waveform. The output of the passive filter is a constant voltage (the capacitor is in "hold" state) well measurable with an ADC. The output of the Active filter is an exponentially decaying voltage, but it is still suitable for an ADC because the decay constant is very long (1 ms)

(B) Measurement of repetitive pulses with constant repetition rate

The measurement of the pulse amplitude can be improved by obtaining it as the average of the input pulses over a time interval not exceeding 10s. With repetition frequency $f_R = 100Hz$ we have $N_c=10\,f_R = 1000$ pulses in 10s. The synchronism signal available makes possible to synchronize the action of the filter (to close the switch) on each pulse.

B1 Passive Boxcar Integrator filter

The memory of the filter is discharged only during each pulse, it is in hold state in the interval between pulses. The circuit is a **BOXCAR INTEGRATOR (BI).** The decrease of weight from pulse to pulse is given by the ratio

$$r=\exp\left(-\frac{T_P}{T_{F1}}\right)\approx 1-\frac{T_P}{T_{F1}}$$

independent of the repetition frequency f_R. The exponential average must be limited to the pulses in 10s, that is, to $N_c=10\,f_R = 1000$ pulses.

For having negligible weight for pulses spaced by more than 10s we must have

$$r^{N_C}=\exp\left(-\frac{N_cT_p}{T_{F1}}\right)\le 1/100 \quad \text{that is} \quad \frac{N_cT_p}{T_{F1}}\ge 4,6 \quad \text{therefore} \quad T_{F1}\le\frac{N_cT_P}{4,6}=2{,}17ms$$

The value T_{F1} = 1ms previously considered for the GI is still suitable, but improved results can be obtained with a longer T_{F1}. We can take T_{F1} = 2ms, that is, C_1= 200nF.

Employed as GI this circuit with T_{F1} = 2ms would give the same S/N as in Sec. A with T_{F1} = 1ms, but output amplitude lower by a factor 2. However the circuit is here employed as BI, hence the output amplitude is equal to the input pulse amplitude A

$$s_{g1} = A\frac{T_P}{T_{F1}}\frac{1}{1-r} = A$$

In fact, with respect to the GI the BI has

output amplitude higher by the factor $\frac{1}{1-r} \approx \frac{T_{F1}}{T_P} = 200$

output noise higher by $\frac{1}{\sqrt{1-r^2}} \approx \sqrt{\frac{T_{F1}}{2T_P}} = 10$

S/N higher by $\frac{\sqrt{1-r^2}}{1-r} \approx \sqrt{\frac{2}{1-r}} = \sqrt{\frac{2T_{F1}}{T_P}} = 20$

In conclusion, with the BI the minimum measurable input pulse amplitude is improved to

$$A_{\min,BI} = A_{\min,GI}\frac{1-r}{\sqrt{1-r^2}} \approx \frac{A_{\min,GI}}{20} \approx 225nV$$

and the output amplitude is equal to the input amplitude

$$s_{g1} = A$$

This result is consistent with the fact that the DC Gain of the BI is unity

$$G_{BI} = 1$$

B2 Active Ratemeter Integrator filter

The memory of the filter is discharged all the time. The decrease of weight from pulse to pulse is given by the ratio

$$r = \exp\left(-\frac{T_R}{T_{F2}}\right) \approx 1 - \frac{T_R}{T_{F2}} = 1 - \frac{1}{f_R T_{F2}}$$

which depends on the repetition frequency $f_R = 1/T_R$. The circuit is a **RATEMETER INTEGRATOR (RI).**

In order to have negligible weight for the previous pulses beyond 10s we must have

$$\exp\left(-\frac{10s}{T_{F2}}\right) \le 1/100$$

Which implies $\frac{10s}{T_{F2}} \ge 4,6$ that is $T_{F2} \le \frac{10s}{4,6} = 2,17s$.

We select for the RI $T_{F2}=2s$ therefore $C_2=20\mu F$

This circuit employed as GI with T_{F2} = 2s would give the same S/N as the GI in Sec. A with T_{F1} = 1ms, but output amplitude lower by a factor 2000. However, the circuit is here employed as RI, so that with respect to the results that it would give employed as GI

The output signal is increased by the factor $\frac{1}{1-r} \approx \frac{T_{F2}}{T_R} = f_R T_{F2} = 200$

The output noise is increased by the factor $$\frac{1}{\sqrt{1-r^2}} \approx \sqrt{\frac{T_{F2}}{2T_R}} = \sqrt{\frac{f_R T_{F2}}{2}} = 10$$

The S/N is increased by the factor $$\frac{\sqrt{1-r^2}}{1-r} \approx \sqrt{\frac{2}{1-r}} = \sqrt{\frac{2T_{F2}}{T_R}} = \sqrt{2 f_R T_{F2}} = 20$$

The minimum measurable input amplitude is improved to the same level as with the BI

$$A_{\min,RI} = A_{\min,GI} \frac{1-r}{\sqrt{1-r^2}} \approx \frac{A_{\min,GI}}{20} \approx 225nV$$

The output amplitude of the RI is remarkably higher than that obtained with the GI, but lower than that obtained with the BI

$$s_{RI} = s_{g2} \frac{1}{1-r} \approx A \frac{T_P}{T_{F2}} \frac{R_2}{R_3} \frac{T_{F2}}{T_R} = A \frac{R_2}{R_3} \frac{T_P}{T_R} = A \cdot \frac{1}{10}$$

This result is consistent with the fact that the DC gain of the RI is

$$G_{RI} = \frac{R_2}{R_3} \frac{T_P}{T_R} = 0{,}1$$

(C) <u>Measurement of repetitive pulses with variable repetition rate</u>

C1 Passive Boxcar Integrator filter

The filter as defined in Sec. B1 is well suitable for measuring the pulse amplitude also in cases where the repetition frequency is not controllable and varies in the range from $f_R = 100\ Hz$ to $f_R = 200\ Hz$. In fact:

1) The output signal amplitude and the S/N do NOT DEPEND on the repetition frequency f_R
2) The condition of having negligible weight for the previous pulses beyond 10s is still fulfilled. If the repetition frequency is increased above 100Hz the weight given to the previous pulses decreases more rapidly and becomes lower than 1/100 at time distance shorter than 10s

C2 Active Ratemeter Integrator filter

In cases where the pulse repetition frequency is variable, the RI is unsuitable for carrying out measurements of the pulse amplitude, because the output signal amplitude depends also on the repetition frequency f_R and not only on the input pulse amplitude A. If the amplitude is constant but the frequency varies the output signal will show a variation that the user may misinterpret as due to a variation of the amplitude.

(D) <u>Measurement of repetitive pulses with random repetition rate</u>

Let us consider a measurement of the amplitude of pulses generated in a detector that receives ionizing radiations from a source (for instance gamma rays from a Co^{60} source in a hospital laboratory).

The arrivals of the radiations at the detector time are independent and random; they are ruled by the Poisson statistics. The time intervals between the pulses are not constant, but statistical. The number of pulses in a time interval is a statistical variable ruled by Poisson statistics (mean square number = mean number; etc). The intensity of the radioactive source is characterized by the mean number of pulses per second, currently called mean repetition rate (or simply repetition rate). In our case the repetition rate is $m_R = 10\ c/s$.

The pulses have equal amplitude, so they can be measured by measuring an average of the pulse amplitudes in sequence. An auxiliary electrical signal is available that signals the arrival of each pulse, so it is possible to employ circuits with a gate, such as Boxcar Integrators or Ratemeter integrators.

D1 Passive Boxcar Integrator filter

The performance of the BI does not depend on the idle interval between the pulses in sequence: in this idle interval the circuit is in "hold" state, its memory is frozen, that is, no charge is brought in the capacitor and no charge is taken from it. The averaging weighting of the BI is carried out on a given number of pulses set by the filter parameters, independently from the time interval where they occur. It is irrelevant how long the idle time between two pulses is; it is irrelevant whether it is constant or variable. The output signal will result from the exponential average with a ratio *r* not dependent on the repetition interval

$$r = \exp\left(-\frac{T_P}{T_{F1}}\right) \approx 1 - \frac{T_P}{T_{F1}}$$

Therefore, the result obtained with the BI working on a statistical sequence of pulses is equal to the result obtained working on a periodical sequence (of course with equal pulses in the two sequences).

We conclude that the BI is perfectly suitable to the purpose also in case of statistical sequence of pulse signal

D2 Active Ratemeter Integrator filter

The performance of the RI does depend on the idle interval between the pulses in sequence, because the weighting of the filter is done over a given time interval set by the filter parameters. The fact that the input pulses are statistical introduces further fluctuations. The number of pulses covered by the averaging (that is, pulses occurring over the duration of the weighting function) is a statistical variable that introduces its fluctuations in the result. Let's make reference to the RI in Sec. B2, with time constant $T_{F2} = 2s$. We can consider the duration of the weighting to be $5T_{F2} = 10s$ and we have a random pulse rate $m_R = 100\ c/s$. Therefore

mean number of pulses weighted $\quad N_w = 5T_{f2} m_r = 1000$

mean square fluctuation of the number $\quad \sqrt{\sigma_N^2} = \sqrt{N_w} \approx 32$

additional relative fluctuation introduced $\quad \frac{\sqrt{\sigma_N^2}}{N_w} \approx 0,03 = 3\%$

Further fluctuations should be considered (e.g. fluctuations in the weights given to the pulses caused by the random position in time of the pulses) but even considering only the number of pulses weighted we see that with the RI is not well suitable for measurements on statistical pulse sequences, because it is subject to additional fluctuations that are avoided employing a BI