PROBLEM 2

Data

LASER DIODE
Light emission at λ_1=800nm
Light power: available stationary (with constant diode current) or modulated to squarewave (by on-off modulation of the diode current at high frequency, up to 100kHz)

PIN SILICON PHOTODIODE

Optical absorption length	$L_a \approx 10\mu m$
Reflection coefficient	R=0,3
Surface layer thickness	w_n=*1 micron*
Depletion layer thickness	w_i=*20 micron*
Dark current	$I_{BD}=10^{-14}$ A

PMT PHOTOMULTIPLIER

Photocathode S20
Quantum detection efficiency $\eta_{DM} = 0,005 = 0,5\%$ at λ_1=800 nm

Dark-emission of cathode:	$n_{BM}=10^3$ elettroni/s
Gain	$G = 10^6$
Excess noise factor	F=1,5

CURRENT PREAMPLIFIER
S_{va} voltage noise referred to input is negligible because the source has high impedance
S_{ia}=0,01pA/Hz$^{1/2}$ white current noise referred to input (unilateral)
1/f noise component in S_{ia} with corner frequency f_c= *1kHz* , to be taken into account only in Sec. C

(A) EVALUATION OF NEP AND COMPARISON OF DETECTORS

Denoting by

- $I_{p,\min}$ minimum measurable photocurrent limited only by the detector dark current, with filtering bandwidth Δf=1Hz

$$I_{p,\min} = \sqrt{\overline{i_B^2}} = \sqrt{2qI_B\Delta f}$$ [in A; I_B dark current]

- η_D quantum detection efficiency

- S_D Radiant Sensitivity

$$S_D = \eta_D \frac{\lambda[\mu m]}{1,24}$$ [in A/W]

- The NEP is

$$NEP = \frac{I_{p,\min}}{S_D}$$ [in W]

<u>NEP of the PMT</u>

Dark emission produces at cathode $I_{BM} = n_{BM} q = 1,6 \cdot 10^{-16} A$ that gives shot noise

$$\sqrt{S_{iM}} = \sqrt{2qI_{BM}} = 7,1 \cdot 10^{-18} A / \sqrt{Hz}$$

with *Δf=1Hz* the rms noise is

$$\sqrt{\overline{i_{BM}^2}} = \sqrt{2qI_{BM}\Delta f} = 7,1 \cdot 10^{-18} A$$

The S20 cathode at λ_1=800 nm has quantum efficiency $\eta_{DM} = 0,005 = 0,5\%$ corresponding to radiant sensitivity

$$S_{DM} = 3,2 \cdot 10^{-3} A / W$$

Therefore $$(NEP)_M = \frac{\sqrt{\overline{i_{BM}^2}}}{S_{DM}} = 2,22^{-15} W$$

<u>NEP of the PIN</u>

Dark current $I_{BD} = 10^{-14} A$ produces shot noise

$$\sqrt{S_{iD}} = \sqrt{2qI_{BD}} = 5,6 \cdot 10^{-17} A / \sqrt{Hz}$$

with Δf=1Hz the rms noise is

$$\sqrt{\overline{i_{BD}^2}} = \sqrt{2qI_{BD}\Delta f} = 5,6 \cdot 10^{-17} A$$

The PIN quantum detection efficiency is η_{DD} is the product of the following probabilities for incident photons

1) NOT be reflected by the surface

2) NOT be absorbed in the input layer w_n

3) BE absorbed in the depletion layer w_i

$$\eta_{DD} = (1-R)\exp(-w_n / L_a)\left[1-\exp(-w_i / L_a)\right] \approx (1-0,3)0,905(1-0.135) = 0,55$$

that corresponds to Radiant Sensitivity

$$S_{DD} = \eta_{DD} \frac{\lambda_1}{1,24} = 0,355\ A / W$$

Therefore $(NEP)_D = \frac{\sqrt{\overline{i_{BD}^2}}}{S_{DD}} = 0,157^{-15} W$

<u>comparison of PMT and PIN based on NEP</u>

The PIN has better (smaller) NEP than the PMT because it has an advantage in terms of higher detection efficiency that overcomes the advantage of the PMT in terms of lower detector dark current.

$$\frac{(NEP)_M}{(NEP)_D} \approx 14,1$$

However, the comparison based on the NEP is not a fair comparison of the performance in operation. In fact, the detector is always connected to a front-end circuit (the preamplifier in our case) that contributes noise, whereas the NEP does not take into account the circuit noise. The circuit noise has significant effect in the actual measurement and cannot be neglected in the comparison.

(B) <u>Minimum measurable power with white noise only</u>

B1) Filtering

The signal varies slowly over times of 0,1s or longer, hence its upper band-limit is roughly about 10Hz. A low-pass filter with band-limit about one decade higher $f_L \approx 100\ Hz$ reduces the noise and passes the signal.

Denoting by S_{iT} be the total white noise density that accompanies the current signal I_p

$$\left(\frac{S}{N}\right) = \frac{I_p}{\sqrt{\overline{n_{iT}^2}}} = \frac{I_p}{\sqrt{S_{iT} f_L}}$$

The minimum measurable current is

$$I_{p,\min} = \sqrt{\overline{n_{iT}^2}} = \sqrt{S_{iT} f_L}$$

The corresponding minimum optical power measurable is

$$P_{p,\min} = \frac{I_{p,\min}}{S_D} = \frac{\sqrt{S_{iT} f_L}}{S_D}$$

B2) Minimum measurable power with PIN

Total current noise power:

$$S_{iDT} = S_{iD} + S_{ia}$$

S_{iT} = PIN dark current noise S_{iD} + preamp current noise S_{ia} referred to the input.

$$\sqrt{S_{ia}} = 10^{-14}\, A/\sqrt{Hz} \gg \sqrt{S_{iD}} = 5,6\cdot 10^{-17}\, A/\sqrt{Hz}$$

S_{iD} is negligible $\qquad S_{iD} << S_{ia}$

Therefore $\qquad S_{iDT} \approx S_{ia}$

$$I_{pD,\min} = \sqrt{\overline{n_{iT}^2}} = \sqrt{S_{iDT} f_L} \approx \sqrt{S_{ia} f_L} = \sqrt{\overline{n_{ia}^2}} = 10^{-13} A = 100\, fA$$

$$P_{pD,\min} = \frac{I_{pD,\min}}{S_{DD}} = 2,82\cdot 10^{-13} W = 282\, fW$$

B3) Minimum measurable power with PMT

For evaluating the total current noise S_{iMT} the preamp noise is referred back to the photocathode, i.e. divided by the PMT gain square G^2 (and by the PMT excess noise factor F)

$$S_{iMT} = S_{iM} + \frac{S_{ia}}{FG^2}$$

The preamp current noise referred to the cathode with the high gain G=10^6

$$\frac{\sqrt{S_{ia}}}{G\sqrt{F}} \approx 8,1\cdot 10^{-21} A/\sqrt{Hz}$$

is negligible with respect to the detector noise

$$\sqrt{S_{iM}} = 7,1\cdot 10^{-18} A/\sqrt{Hz}$$

Therefore $\qquad S_{iMT} \approx S_{iM}$

and

$$I_{pM,\min} = \sqrt{\overline{n_{iT}^2}} = \sqrt{S_{iT} f_L F} \approx \sqrt{S_{iM} f_L F} = 87\cdot 10^{-18} A$$

$$P_{pM,\min} = \frac{I_{pM,\min}}{S_{DM}} = 27,2\cdot 10^{-15} W = 27,2\, fW$$

The measurable optical power with the PMT is much lower than that obtained with the PIN because the high internal gain G of the detector makes practically negligible the noise of the following preamplifier.

$$\frac{P_{pD,\min}}{P_{pM,\min}} \approx 10,4$$

We verify that the comparison based on the minimum measurable optical power is very different from the comparison based on the NEP, because the NEP does not take into account the noise of the circuit associated with the detector.

(C) <u>Minimum measurable power in presence of 1/f component in the preamp noise</u>

We must now take into account also a *1/f* component in the preamplifier current noise spectrum, that is we have

$$S_{iT} = S_{ia} + \frac{S_{ia} f_c}{f}$$

In the case of the PMT, this preamplifier noise spectrum is referred to the PMT cathode dividing it by the factor $G^2 \approx 10^{12}$. Therefore, also the 1/f component is strongly reduced and is small in comparison with the detector noise, so that the corner frequency in the total spectrum is very low and the effect of 1/f noise is negligible with respect to the detector noise.

On the contrary, working with the PIN the preamplifier noise is dominant and the effect of the 1/f component must be thoroughly taken into account in the evaluation of the minimum measurable amplitude

C1) Measurement with constant light power and initial baseline reset

The introduction of a reset to zero of the baseline of the preamp at the start of a measurement cycle of about 15 min complements the low-pass filtering with a high-pass filtering with frequency cutoff roughly about $f_i \approx 10^{-3}\ Hz$. The upper and lower band-limits are widely spaced; hence the 1/f noise can be evaluated with the sharp band-limit approximation

$$\overline{n_f^2} = S_{ia} f_c \ln\left(\frac{f_L}{f_i}\right) \approx \overline{n_{ia}^2} \frac{f_c}{f_L} \ln\left(\frac{f_L}{f_i}\right)$$

with $\sqrt{\overline{n_{ia}^2}} \approx \sqrt{S_{ia} f_L} = 10^{-13} A = 100\, fA$

therefore

$$\overline{n_f^2} \approx \overline{n_{ia}^2} \frac{f_c}{f_L} \ln\left(\frac{f_L}{f_i}\right) = \overline{n_{ia}^2} \cdot 115$$

In these conditions the noise is much higher than with the white noise contribution only

$$\sqrt{\overline{n_f^2}} = \sqrt{\overline{n_{ia}^2}} \sqrt{115} \approx 10,7 \sqrt{\overline{n_{ia}^2}} = 1,07 \cdot 10^{-12} A = 1,07\, pA$$

The minimum measurable optical power is correspondingly higher than that with white noise only

$$P_{pD,\min f} \approx \frac{\sqrt{\overline{n_f^2}}}{S_{DD}} = 3{,}34 \cdot 10^{-12} W = 3pW$$

(D) Measurement with modulated light power

By modulating on and off at frequency f_m the current in the Laser diode we obtain on-off squarewave modulation of the emitted light P_m at frequency f_m . By selecting a modulation frequency f_m quite higher than the noise corner frequency $f_m > f_c$ the signal is brought out of the spectral region where 1/f noise is higher than the white noise S_a. For instance, we can select $f_m = 40kHz$ or higher.

A squarewave reference signal (with frequency and phase equal to the light signal) can be obtained from the modulating current of the laser diode and employed as reference signal for a lock-in amplifier (LIA) that processes the photodetector signal and noise.

The signal is modulated on-off from I_p to zero, hence it is the sum of

a) a constant signal $I_p/2$ and

b) a squarewave signal with equal amplitude $I_p/2$.

The squarewave signal is efficiently recovered with a LIA that employs the squarewave reference. The lowpass filter of the LIA must have bandwidth higher than that of the slowly varying optical power: therefore we can use a bandwidth $f_L=100Hz$ equal to the low-pass filter employed in Sec.B.

The LIA selects in frequency and phase the components of signal and noise. In frequency it selects the components within bandwidth $2f_L$ centered on the components of the reference (fundamental f_m and odd harmonics). In phase it selects the components with phase equal to the reference. Since signal and reference have the same waveform, the S/N is the ratio of

the full power of the squarewave signal $\left(I_p/2\right)^2$

half of the noise power in the selection bandwidth $S_{iT}\left(f_m\right) f_L \approx S_{ia} f_L$

$$\frac{S}{N} = \frac{I_p}{2\sqrt{S_{ia} f_L}}$$

The minimum measurable amplitude of the modulated current is therefore

$$I_{pm,\min} = 2\sqrt{S_{ia} f_L} = 2\sqrt{\overline{n_{ia}^2}} = 200 \cdot 10^{-15} A = 200 fA$$

and the corresponding minimum measurable amplitude of the modulated optical power is

$$P_{pm,\min} = \frac{I_{pm,\min}}{S_{DD}} = 563 \cdot 10^{-15} W = 563\, fW$$

which is just slightly higher than the value computed taking into account the white noise only.