PROBLEM 2

Data summary

Plane Capacitive Sensors:

ε_o = 8,85 pF/m — dielectric constant (air)

A= 4 cm^2 — electrode area

l = 0,4 mm — electrode distance in quiescent condition

$$C_o = \varepsilon_o \frac{A}{\ell} = 8{,}85\,pF$$

V_C=20 mV — max voltage applied to the capacitor

Differential Preamplifier

High input resistance →∞

Wide band > 10MHz

$\sqrt{S_V}$ = 50nV Hz$^{-1/2}$ — voltage noise spectrum (unilateral) at the differential input

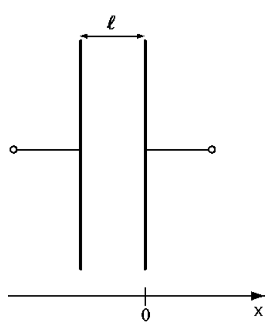
$\sqrt{S_i}$ = 0,1pA Hz$^{-1/2}$ — current noise spectrum (unilateral) at each input

$f_c \approx$ 10kHz — corner-frequency of 1/f noise component

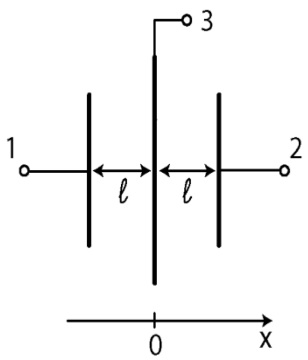
displacements to be measured:

- stationary displacement *x* (the value is constant over time intervals up to 10s)
- sinusoidal displacement *x* oscillating at frequency f_o with oscillation amplitude constant over time intervals up to 10s. The oscillation frequency f_o is in the interval from 40 to 100Hz, different from case to case; a reference signal with frequency and phase equal to the *x* oscillation is available.

(A) Sensor operating principle, circuit configuration and transduction gain



Simple capacitance sensor



Differential capacitance sensor

A1 Operating Principle

1) the variation of the capacitance due to the displacement is a known function

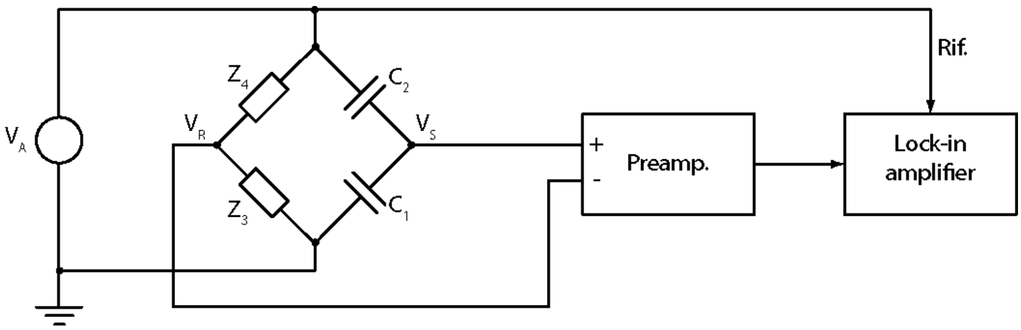
2) the measurement of the capacitance variation gives information about the displacement

with x=0 (quiescent condition) $\frac{1}{C} = \varepsilon_o \frac{\ell}{A}$

with displacement x $\frac{1}{C} = \varepsilon_o \frac{\ell + x}{A}$

A2 Circuit configuration

The variations of the sensor impedance can be measured in a Wheatstone bridge configuration



Sensor arm with simple capacitance sensor:

C_1= sensor

$C_2 = C_0$ constant capacitance.

Sensor arm with differential capacitance sensor:

C_1 capacitance connected between electrodes 1 and 3

C_2 capacitance connected between electrodes 2 and 3

Reference arm

two equal and constant impedances (e.g. small resistances $R_3 = R_4$ or capacitors $C_3=C_4=C_0$)

give the reference voltage $V_R = \frac{V_A}{2}$

The sensor has capacitive impedance, hence we employ a sinusoidal bridge voltage supply with frequency f_a and amplitude V_A :The sensor voltage is limited to V_C =20mV , hence we select amplitude $V_A = 2V_C$ = 40mV

A3 Transduction with simple capacitance sensor

With displacement x the output voltage of the sensor arm is

$$V_S = V_A \frac{Z_1}{Z_1 + Z_2} = V_A \frac{\frac{1}{C_1}}{\frac{1}{C_1} + \frac{1}{C_2}} = V_A \frac{\ell + x}{\ell + x + \ell} = \frac{V_A}{2} \frac{1 + \frac{x}{\ell}}{1 + \frac{x}{2\ell}}$$

Which by denoting $\delta = \frac{x}{\ell} < 1$ can be written

$$V_S = \frac{V_A}{2} \frac{1+\delta}{1+\frac{\delta}{2}}$$

The output of the bridge thus is

$$V_U = V_S - V_R = \frac{V_A}{2}\left(\frac{1+\delta}{1+\frac{\delta}{2}} - 1\right) = \frac{V_A}{4}\delta \cdot \frac{1}{1+\frac{\delta}{2}} \approx \frac{V_A}{4}\delta$$

With respect to the ideal linear transduction

$$V_U = \frac{V_A}{4}\delta$$

the factor of deviation from linear is

$$\frac{1}{1+\frac{\delta}{2}} \approx 1 - \frac{\delta}{2}$$

For keeping the deviation lower than 1% , the measurement range must be limited to 0< δ< 0,02, that is, to displacements

$$x_{lin} < 0,02\,\ell = 8\mu m$$

The transduction from displacement x to output voltage in the linear rangethus is

$$V_U = V_S - V_R \approx \frac{V_A}{4}\delta = \frac{V_A}{4}\frac{x}{\ell} =$$

with the maximum allowed supply voltage of the bridge V_A = 40mV (i.e. twice the maximum allowed voltage 20mV on the sensor) the transduction factor is

$$G_S = \frac{V_U}{x} \cong \frac{V_A}{4\ell} = 25\frac{\mu V}{\mu m}$$

A4 Transduction with differential capacitance sensor

With displacement x the sensor capacitances are

$$\frac{1}{C_1}=\frac{\ell+x}{\varepsilon_o A} \qquad \text{e} \qquad \frac{1}{C_2}=\frac{\ell-x}{\varepsilon_o A}$$

the output voltage of the sensor arm is

$$V_S=V_A\frac{Z_1}{Z_1+Z_2}=V_A\frac{\dfrac{1}{C_1}}{\dfrac{1}{C_1}+\dfrac{1}{C_2}}=V_A\frac{\ell+x}{\ell+x+\ell-x}=\frac{V_A}{2}\left(1+\frac{x}{\ell}\right)=\frac{V_A}{2}\left(1+\delta\right)$$

The output of the bridge thus is

$$V_U=V_S-V_R=\frac{V_A}{2}\delta$$

The transduction is linear over all the dynamic range $\delta<1$ without approximation.

$$V_U=V_S-V_R=\frac{V_A}{2}\delta=\frac{V_A}{2}\frac{x}{\ell}$$

With the maximum allowed supply voltage V_A = 40mV the transduction constant is

$$G_D=\frac{V_U}{x}=\frac{V_A}{2\ell}=50\frac{\mu V}{\mu m}$$

with measurement dynamic

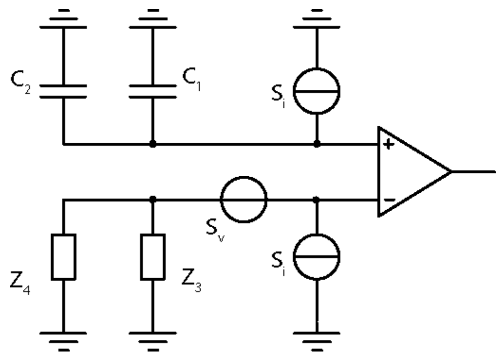
$$|x|<\ell=400\mu m$$

The differential capacitance sensor has the following advantages over the simple capacitance sensor

- wider dynamics of the x measurement
- higher output voltage for the same displacement x

(B) <u>Filtering for measuring stationary displacement</u>

The equivalent circuit of the sensor bridge connected to the differential preamplifier is



The output signal is sinusoidal at the bridge supply frequency f_a with narrow bandwidth $\Delta f_U \approx 0{,}1\text{Hz}$ (since the amplitude V_A varies over time intervals >10s). A suitable filtering is a narrow-band filter centered on f_a , with bandwidth greater than the signal bandwidth, and frequency f_a selected where the noise spectral density is lower.

The noise contribution from the reference arm (the arm with impedances Z_3 and Z_4) can be made negligible with respect to that of the sensor arm by using low impedances Z_3 and Z_4 (for instance two resistances R_3 and R_4 with small value e.g. 100Ω). The noise spectrum at the differential preamplifier input thus is

$$S_n = S_V + S_i \left|Z_S\right|^2 = S_V + S_i \frac{1}{\left(2\pi f\right)^2 \left(C_1 + C_2\right)^2} \approx S_V + S_i \frac{1}{\left(2\pi f\right)^2 \left(2C_o\right)^2}$$

The contribution of the white voltage noise S_i is constant at any frequency and the contribution of the white current noise S_i decreases as the frequency increases. They are equal at frequency

$$f_{nc} = \frac{1}{4\pi C_o} \frac{\sqrt{S_i}}{\sqrt{S_V}} \approx 18kHz$$

Selecting $f_a > 5f_{nc}$ the contribution of S_i at the signal frequency is f_a is negligible with respect to that of S_V. Also the 1/f noise component contribution is negligible at f_a , given the low noise corner frequency $f_c \approx 10\text{kHz}$. The S/N can thus be evaluated taking into account only the white voltage noise, that is, by considering

$$S_n\left(f_a\right) \approx S_V$$

Let us select

$$f_a = 200kHz$$

A lock-in-amplifier (LIA) can be employed for obtaining the required narrow-band filtering, with reference signal obtained from the Wheatstone bridge supply and with low-pass filter adjusted to pass the signal variations in time, that is, with low-pass band-limit f_L that establishes a pass-band $\Delta f_L = 2 f_L$ somewhat wider than that of the signal

$$\Delta f_L = 2 f_L \approx 10 \Delta f_U \approx 1 Hz$$

Only the noise components that have frequency and phase equal to the reference signal contribute to the LIA output. We obtain

$$\left(\frac{S}{N}\right) = \frac{V_U}{\sqrt{2 S_n\left(f_a\right) f_L}}$$

and the minimum measurable signal is

$$V_{U\min} = \sqrt{2 S_n\left(f_a\right) f_L} \approx \sqrt{2 S_V f_L} \approx 70\text{nV}$$

The correspong minimum measurable displacement is

$$x_{\min} = \frac{V_{U\min}}{G_D} = 1,4\ nm$$

(C) <u>Filtering for measuring oscillating displacement</u>

The considerations about noise made in Sec.B for selecting the frequency f_a of the bridge supply are still valid in this case. The oscillating displacement is still a narrowband signal, but centered at frequency f_o . The output signal of the bridge is therefore a double line in frequency at frequencies

$$f_a \pm f_o$$

Employing the LIA with reference signal given by the bridge supply, the lines are shifted back to the base band at frequency $\pm f_o$. Setting the low-pass filter of the LIA to a bandlimit $f_L \approx 0{,}5$ Hz is no more suitable; for passing the signal it is necessary to employ a higher bandlimit $f_{L1} > 10 f_o$. Therefore we employ now

$$f_{L1} = 10 kHz$$

However, this increases the output noise and the minimum measurable signal by the factor

$$\sqrt{f_{L1} / f_L} \approx 100$$

We note also that the LIA output is now a sinusoidal signal, not a constant signal as in Sec. A.

However, this output signal is still a narrow-band signal with bandwidth $\Delta f_U \approx 0{,}1 Hz$ (oscillation amplitude varies in time intervals > 10s) and we can exploit this with a further narrowband filtering.

Since the oscillation frequency has uncontrolled variations in time, we will not use a constant parameter filter, but a second Lock-in-amplifier (called here LIA2) that employs as reference the auxiliary signal with frequency and phase equal to the oscillation of the displacement. The low-pass filter in LIA2 can be equal to that employed in Sec. B since it has now to select a narrow bandwidth centered at $f=0$.

In summary, we can summarize the operations as follows

a) The white noise S_V at the LIA1 input centered at frequency f_a is brought to the LIA1 output shifted down in frequency, centered at $f=0$ and limited by the band f_{L1} defined by the low-pass filter in LIA1

b) this spectral density at the LIA1 output is doubled $2S_V$ (this is called the "spectrum folding" effect)

c) the signal is brought to the LIA1 output at frequency $\pm f_o$ with its correct amplitude

Therefore LIA2 receives at its input

- the oscillating signal at frequency f_o with amplitude V_U
- a white noise spectrum with density $2S_V$ limited in band up to f_{L1}

For measuring the oscillating signal amplitude we can employ in LIA2 the low-pass filter with bandlimit $f_L \approx 0,5 Hz$ as discussed in Sec.B and obtain

$$\left(\frac{S}{N}\right) = \frac{V_U}{\sqrt{4S_V f_L}}$$

The minimum measurable signal amplitude thus is

$$V_{U\min} = \sqrt{4S_V f_L} \approx 100\, nV$$

and the corresponding minimum measurable oscillating displacement is

$$x_{\min} = \frac{V_{U\min}}{G_D} = 2\; nm$$