PROBLEM 2

Data summary

RTD Thermoresistance PT100

Reference value at 0°C (273 K) R_{T0} =100 Ω

temperature coefficient α = 3,9·10^{-3} /°C

Max dissipation P_d < 50μW

Si pn Junction Diode Sensors

Current-Voltage Characteristics $I = I_S\left[\exp\left(\frac{qV}{kT}\right)-1\right]$ with $I_s < 0{,}1\,pA = 100\,fA$

Max dissipation P_d< 50μW

Differential Preamplifier

Band-limit by a simple pole at f_{pa} = 1 MHz

Effective noise power density

$S_v^{1/2}$ = 80 nV/Hz$^{1/2}$ white (unilateral) and 1/f component with f_c = 20 kHz

$S_i^{1/2}$ = 1 pA/Hz$^{1/2}$ white (unilateral) and 1/f component with f_c = 20 kHz

(A) Sensor configuration for operation with DC voltage bias

A1) Thermoresistances

Wheatstone bridge with 4 equal resistances (1 RTD + 3 constant resistances) and DC voltage supply V_A .

Power dissipation in the RTD sensor

$$\left(\frac{V_A}{2}\right)^2\frac{1}{R_{T0}} < P_{d\max} = 50\mu\text{W} \qquad \text{hence} \qquad V_A < 2\sqrt{P_{d\max}R_{T0}} = 141mV$$

we select V_A= 140 mV.

Temperature variation ΔT produces resistance variation $\frac{\Delta R_T}{R_{T0}} = \alpha\,\Delta T$

hence voltage variation at bridge output

$$\Delta V_S = \frac{V_A}{4}\cdot\frac{\Delta R_T}{R_{T0}} = \frac{V_A}{4}\alpha\,\Delta T = 136{,}5\,\mu V\cdot\Delta T$$

The conversion factor is $\frac{dV_S}{dT} = 136{,}5\,\mu V/K$

A2) Si pn junction diode Sensors

For a silicon pn junction biased at forward current $I >> I_s \approx 100\, fA$

$$I = I_S \left[\exp\left(\frac{qV}{kT}\right) - 1 \right] \approx I_S \exp\left(\frac{qV}{kT}\right) \qquad \text{and} \qquad V = \frac{kT}{q} \ln\left(\frac{I}{I_S}\right)$$

A configuration with DC voltage bias employs two identical sensors submitted to the temperature T to be measured and biased at two different DC current levels I_1 e I_2. The difference V_D between the forward voltages of the two diodes is read with a differential preamplifier

$$V_D = \frac{kT}{q} \left[\ln\left(\frac{I_1}{I_S}\right) - \left(\frac{I_2}{I_S}\right) \right] = \frac{kT}{q} \ln\left(\frac{I_1}{I_2}\right)$$

For evaluating the power dissipated in a diode, the forward bias voltage can be considered with good approximation V= 600 mV.

$$I \cdot V \approx I \cdot 600mV < P_{d\max} = 50\mu\text{W} \qquad \text{hence} \qquad I < \frac{P_{d\max}}{0,6\,V} = 83\,\mu A$$

Let us select $I_1 = 80\ \mu A$ and $I_2 = 8\ \mu A$ that is $\frac{I_1}{I_2} = 10$.

We get then

$$V_D = \frac{kT}{q} \ln\left(\frac{I_1}{I_2}\right) = 2,3\frac{kT}{q}$$ that is, at room temperature T=300k we have $V_D \approx 57,5$ mV

The conversion factor is

$$\frac{dV_D}{dT} = \frac{k}{q} \ln\left(\frac{I_1}{I_2}\right) = \frac{V_D}{T} = 192\ \mu V/K$$

(B) <u>Noise and sensitivity limits in measurements with sensors operating with DC bias</u>

The voltage noise S_V of the preamplifier is the dominant noise contribution.

The contribution of the current noise S_i of the preamplifier is negligible, because it is converted in voltage noise by the low resistance R_S of the signal source. In the case of thermoresistances, the R_S is the sensor resistance $R_T \approx 100\Omega$. In the case of Silicon pn diodes the R_S is the resistance of the diode in forward bias

$$R_{D1} = \frac{dV_1}{dI_1} = \frac{kT}{qI_1} = 312\,\Omega \qquad\qquad R_{D2} = \frac{dV_2}{dI_2} = \frac{kT}{qI_2} = 3{,}12\ k\Omega$$

therefore, in all cases $R_S S_i^{1/2} < 10\, nV/\sqrt{Hz} << S_v = 80\, nV/\sqrt{Hz}$.

The intrinsic noise of the sensors (pn junctions or thermoresistances) can be directly verified to be negligible.

In conclusion, the total noise to be taken into account is given by the voltage noise of the preamplifier.

$S_T(f) \approx S_v(f)$

B1) Measurement without low-pass filtering and with CDS

The measurement is carried out with correlated double sampling CDS, setting to zero the amplifier baseline in the available interval every T_i = 15-20 min ≈ 1000s .

The noise is subject to

high-pass filtering with limit $f_i = 1/T_i \approx 0,001 Hz$ due to the CDS

low-pass filtering with limit $f_S = \pi f_{pa}/2 \approx 1,5 MHz$ due to the preamp band-limit

The contributions of white noise and 1/f noise are

$$\sqrt{\overline{n_B^2}} = S_v^{1/2}\sqrt{2}\sqrt{f_S - f_i} \approx \sqrt{2}\, S_v^{1/2} f_S^{1/2} = 138 \mu V$$

$$\sqrt{\overline{n_f^2}} = \sqrt{2}\, S_v^{1/2} f_C^{1/2} \sqrt{\ln\left(\frac{f_S}{f_i}\right)} \approx 74 \mu V$$

total noise

$$\sqrt{\overline{n_T^2}} = \sqrt{\overline{n_B^2} + \overline{n_f^2}} = 156 \mu V = \Delta V_{S\min}$$

In this condition the white noise is dominant and the minimum measurable voltage variation $\Delta V_{S,min}$ corresponds to a temperature variation higher than the specification:

$$\Delta T_{\min} = \frac{\Delta V_{S\min}}{\frac{dV_S}{dT}} \approx 1140 mK \qquad \text{with the RTD}$$

$$\Delta T_{\min} = \frac{\Delta V_{D\min}}{\frac{dV_D}{dT}} \approx 810 mK \qquad \text{with the diode sensor}$$

B2) Measurement with low-pass filtering and CDS

A low-pass filter is employed at the preamplifier output for reducing the white noise contribution and CDS as outlined in Sec.B1 is carried out on the filter output for reducing the 1/f noise contribution. Temperature variations over time intervals down to 0,1 s must be measured, hence the low-pass filter must pass frequency components up to a few 10 Hz; therefore, we employ a low-pass with band-limit $f_S = 100\ Hz$. We get

$$\sqrt{\overline{n_B^2}} = S_v^{1/2}\sqrt{2}\sqrt{f_S - f_i} \approx \sqrt{2}\, S_v^{1/2} f_S^{1/2} \approx 1,1 \mu V$$

$$\sqrt{\overline{n_f^2}} = \sqrt{2}\, S_v^{1/2} f_C^{1/2} \sqrt{\ln\left(\frac{f_S}{f_i}\right)} \approx 54,2 \mu V$$

the total noise is

$$\sqrt{\overline{n_T^2}} = \sqrt{\overline{n_B^2} + \overline{n_f^2}} \approx \sqrt{\overline{n_f^2}} \approx 54,2\mu V = \Delta V_{S\min}$$

In this condition the 1/f noise is dominant and the minimum measurable voltage variation $\Delta V_{S,min}$ corresponds to a temperature variation which is reduced with respect to Sec. B1, but still higher than the specification:

$$\Delta T_{\min} = \frac{\Delta V_{S\min}}{\dfrac{dV_S}{dT}} \approx 400mK \qquad \text{with the RTD}$$

$$\Delta T_{\min} = \frac{\Delta V_{D\min}}{\dfrac{dV_D}{dT}} \approx 282mK \qquad \text{with the diode sensor}$$

We note that the noise is so high because with DC bias the signal to be measured is in a low frequency region, where the 1/f noise density is very high, much higher than the white noise density.

(C) <u>Measurement with sensor under time-varying bias (modulated or commuted voltage)</u>

Employing a periodic time varying bias with fundamental frequency f_m higher than the 1/f corner frequency f_c , the signal can be shifted to f_m before entering in the preamplifier, i.e., before the addition of 1/f noise. It is thus possible to employ a narrow-band filtering centered at f_m for collecting the signal accompanied by a reduced noise.

The two sensor have different features and will be dealt with separately.

C1) Thermoresistances RTD

The sensor is a linear resistor and we can simply employ a sinusoidal voltage supply with frequency f_m =100kHz and amplitude V_A equal to the DC supply $V_A = 140mV$, thus maintaining the same conversion factor (it would also be possible to employ an amplitude V_A higher by a factor $\sqrt{2}$, i.e. $V_A = 200mV$ without exceeding the dissipation limit).

A lock-in amplifier (LIA) with reference obtained from the voltage supply (which points out the frequency and phase of the signal) can be employed for filtering the preamp output with a narrow frequency band. The LIA demodulates the signal bringing it back to the base band (i.e. around f=0) and filters it with its internal low-pass filter. For the reasons explained in Sec. B2, the band-limit of this filter is set at f_S *=100 Hz.*

The S/N obtained at LIA output is

$$\frac{S}{N} = \frac{V_S}{\sqrt{2 S_v \, f_S}} = \frac{V_S}{S_v^{1/2} \sqrt{2 \, f_S}}$$

as it can be shown in various ways (employing the LIA weighting function; analyizing the power transfer of the various frequency components of the input and taking into account the selection in frequency and phase by the LIA; etc.)

The minimum measurable amplitude variation (S/N=1) is

$$\Delta V_{S\min} = S_v^{1/2}\sqrt{2\, f_S} = 1{,}13 \mu V$$

and the corresponding minmum measurable temperature variation is adequate to the specification

$$\Delta T_{\min} = \frac{\Delta V_{S\min}}{\frac{dV_S}{dT}} \approx 8 mK$$

C2) Silicon pn junction Sensors

A sinusoidal current supply is not well suitable in this case. The pn junction has strongly nonlinear characteristics, the modulated voltage waveform is strongly distorted with respect to the sinusoid of the current and it is not simple to obtain the temperature value from the measured voltage signal amplitude.

It is more suitable to employ bias current modulated at squarewave and obtain a squarewave voltage signal with peak-to-peak amplitude $V_D = V_1 - V_2$, with V_1 and V_2 corresponding to levels I_1 and I_2 in the diode, as shown in Sec.A2. It is then possible to measure directly the peak-to-peak amplitude $V_D = V_1 - V_2$, or to measure the fundamental component of the squarewave, which is $\frac{4}{\pi}\frac{V_D}{2}$.

Various current commutation schemes can be employed for producing the squarewave modulation. In the scheme with two identical diodes seen in Sec.A2, the current can be commuted between I_1 and I_2 in a diode and kept constant at I_2 in the other diode. In an alternative scheme that employs just one diode, the current in the diode can be commuted between I_1 and I_2 .

We can employ a sinusoidal voltage signal generator at f_m =100kHz and with a suitable switching circuit obtain from it a square-wave signal for driving the current commutation in the diode.

We can then employ the sinusoidal signal as reference for a LIA that has the same settings as in Sec.C1 and processes signal and noise coming from the preamp. In this case the LIA recovers only the fundamental component at frequency f_m , which has amplitude

$$\frac{4}{\pi}\frac{V_D}{2} = \frac{2V_D}{\pi}$$

We get then

$$\frac{S}{N} = \frac{\frac{2V_D}{\pi}}{\sqrt{2 S_v\, f_S}} = \frac{V_D\sqrt{2}}{S_v^{1/2}\pi\sqrt{f_S}}$$

and minimum measurable amplitude

$$\Delta V_{D\min} = S_v^{1/2} \frac{\pi}{\sqrt{2}} \sqrt{f_S} = 1{,}8 \mu V$$

which corresponds to a minimum measurable temperature variation adequate to the specification

$$\Delta T_{\min} = \frac{\Delta V_{D\min}}{\dfrac{dV_D}{dT}} \approx 9mK$$

If we instead employ the squarewave as reference for the LIA, then reference and signal have the same waveform, the LIA exploits all the power of the signal and gives a better S/N

$$\frac{S}{N} = \frac{V_D}{2\sqrt{S_v \; f_S}} = \frac{V_D}{S_v^{1/2} 2\sqrt{f_S}}$$

with improved minimum measurable amplitude

$$\Delta V_{D\min} = S_v^{1/2} 2 \sqrt{f_S} = 1{,}6 \mu V$$

and correspondingly improved minimum measurable temperature variation

$$\Delta T_{\min} = \frac{\Delta V_{D\min}}{\dfrac{dV_D}{dT}} \approx 8{,}3mK$$

(D) <u>Conclusions, summary and comment</u>

It is required to measure small temperature variations (down to 10mK) occurring over fairly short time intervals (down to 0,1 s). The analysis of the problem shows that

- For attaining the required goal, the contribution of the 1/f noise must be drastically limited.
- This result cannot be obtained by schemes employing stationary bias of the sensor, because the signal produced is in the spectral region where 1/f noise is dominant.
- Satisfactory results are obtained by employing a time varying bias of the sensor (modulated or commuted) which produces signals in a high-frequency spectral region where the 1/f noise is negligible.
- In order to extract efficiently the signal from noise a narrow-band filtering must be employed, with bandpass just wider than that of the signal, that is, such that the variations in time of the temperature are faithfully transmitted
- The required narrow-band filtering can be obtained simply and efficiently with a LIA.