PROBLEM 1

Data summary

Pulse signal

V_P variable pulse amplitude
$T_P = 50$ µs pulse duration
$f_P = 1$ Hz pulse repetition frequency
$V_C \approx 10mV$ baseline level

Preamplifier
$S_V^{1/2} = 20nV/Hz^{1/2}$ white noise power density (unilateral)
$f_{pa} = 1\ MHz$ upper band-limit

in (b) and (c) the noise includes also:
$1/f$ noise component with corner frequency $f_c = 50$kHz

A) Filtering with constant-parameter filters

A1 - Filter for cancelling the baseline

High-Pass CR filter with time constant $T_A=R_A C_A$

Select value of $T_A >> T_P$, in order to have negligible reduction of the signal amplitude at the output of the differentiator. The factor of reduction is about $(1- T_P/T_A)$, therefore selecting $T_A=100T_P=5$ms the amplitude is reduced by less than 1%.

The high-pass band-limit f_i established by the CR filter is

$$f_i = \frac{1}{4T_A} \approx \frac{1}{400T_P} = 50\text{Hz}$$

A2 - Filter for reducing the wide-band noise

Low-Pass RC filter with time constant $T_B=R_B C_B$

Select value of T_B near to T_P, in order to have a low-pass filtering that approximates the optimum filtering

The low-pass band-limit f_S established by the RC filter is

$$f_S = \frac{1}{4T_B} \approx \frac{1}{4T_P} \approx 5kHz$$

A3 – Signal-to-Noise Ratio

Filtered noise:

The white noise is passed in the band with upper limit f_S =5kHz and lower limit f_i =50Hz and the effect of the lower limit is negligible because $f_i << f_S$

$$\sqrt{\overline{n_B^2}} = S_V^{1/2}\sqrt{f_S - f_i} \approx S_V^{1/2}\sqrt{f_S} = S_V^{1/2}\sqrt{1/4T_B}$$

Filtered signal:

$$V_B = V_P\left(1 - e^{-T_P/T_B}\right)$$

Signal to noise ratio

$$\frac{S}{N}=\frac{V_P}{S_V^{1/2}}\frac{1-e^{-T_P/T_B}}{\sqrt{\frac{1}{4T_B}}}=\frac{V_P}{S_V^{1/2}\sqrt{\frac{1}{2T_P}}}\frac{1-e^{-T_P/T_B}}{\sqrt{\frac{T_P}{2T_B}}}$$

The S/N value depends on the ratio $x=T_P/T_B$

$$\frac{S}{N}=\frac{V_P}{S_V^{1/2}\sqrt{\frac{1}{2T_P}}}\cdot\frac{1-e^{-x}}{\sqrt{2x}}$$

The function $\frac{1-e^{-x}}{\sqrt{2x}}$ reaches a maximum for $x=T_P/T_B$ =0,8 . The maximum is quite broad, the selection of T_B is not critical. Selecting $T_B \approx 0{,}8\ T_P$ =40 µs we get

$$\max\left(\frac{S}{N}\right)=\frac{V_P}{S_V^{1/2}\sqrt{\frac{1}{2T_P}}}\cdot 0{,}9$$

The minimum measurable amplitude with the RC filter in presence of white noise only thus is

$$V_{P,\min}=S_V^{1/2}\sqrt{\frac{1}{2T_P}}\cdot\frac{1}{0{,}9}=\frac{2\mu V}{0{,}9}\approx 2{,}2\mu V$$

<u>B) Filtering with time-variant filters</u>

B1 - Filter for cancelling the baseline

We can still employ the same filter as in (A1): high-pass CR filter with time constant $T_A=R_A\,C_A$ with $T_A=100T_P$= 5ms.

B2 - Filter for reducing the wide-band noise

A suitable low-pass filtering can be obtained with a Gated Integrator (GI). By selecting a duration of the gate $T_G=T_P$ we obtain a weighting function that approximates quite well the optimum weighting function, which in case of white noise is equal to the signal.

We include in the low-pass filtering stage a circuit that provides gain, in order to have filtering with unity gain, that is, with weighting function

$$w_G=\frac{1}{T_G}$$

The low-pass band-limit f_S established by the GI is

$$f_S=\frac{1}{2T_P}= 10\text{kHz}$$

Filtered noise:

$$\sqrt{\overline{n_U^2}}=S_V^{1/2}\sqrt{\frac{1}{2T_G}}=S_V^{1/2}\sqrt{\frac{1}{2T_P}}=2\mu V$$

Filtered signal

$$V_U=V_P$$

Signal to noise ratio

$$\frac{S}{N} = \frac{V_P}{S_V^{1/2}\sqrt{\dfrac{1}{2T_P}}} = \frac{V_P}{S_V^{1/2}\sqrt{\overline{n_U^2}}}$$

The minimum measurable amplitude thus is

$$V_{P,\min} = S_V^{1/2}\sqrt{\frac{1}{2T_P}} = \sqrt{\overline{n_U^2}} = 2\mu V$$

The result is better than that obtained in (A) because the GI is a better approximation of the optimum filtering.

<u>C) Filtering in case of 1/f noise</u>

The contribution of the filtered *1/f* noise $\overline{n_{1/f}^2}$ can be well evaluated with the sharp cutoff approximation (since the ratio of the upper band-limit f_S to the lower band-limit f_i is high)

$$\overline{n_{1/f}^2} \approx S_V f_C \ln\left(\frac{f_S}{f_i}\right) \approx S_V f_S \frac{f_C}{f_S} \ln\left(\frac{f_S}{f_i}\right) \approx \overline{n_U^2}\frac{f_C}{f_S}\ln\left(\frac{f_S}{f_i}\right)$$

The total noise thus is

$$\overline{n_T^2} = \overline{n_U^2} + \overline{n_{1/f}^2} \approx \overline{n_U^2}\left[1 + \frac{f_C}{f_S}\ln\left(\frac{f_S}{f_i}\right)\right]$$

Therefore, the worsening factor brought by the 1/f noise is

$$\sqrt{\frac{\overline{n_T^2}}{\overline{n_U^2}}} = \sqrt{1 + \frac{f_C}{f_S}\ln\left(\frac{f_S}{f_i}\right)}$$

with $f_S = 10kHz$ $\quad f_i = 50Hz$ $\quad f_C = 50KHz$ we get

$$\sqrt{\frac{\overline{n_T^2}}{\overline{n_U^2}}} \approx \sqrt{27{,}5} = 5{,}24$$

That is

$$\sqrt{\overline{n_T^2}} \approx 5{,}24 \cdot 2\mu V \approx 10{,}48\mu V$$

The signal to noise ratio is

$$\frac{S}{N} = \frac{V_P}{\sqrt{\overline{n_U^2}}} \cdot \frac{1}{5{,}24}$$

The minimum measurable amplitude is much higher than that obtained with white noise only.

$$V_{P,\min} = S_V^{1/2}\sqrt{\frac{1}{2T_P}} \cdot 5{,}24 = \sqrt{\overline{n_U^2}} \cdot 5{,}24 = 10{,}48\mu V$$

<u>D) Filtering for reducing the 1/f noise contribution</u>

For reducing the contribution of the 1/f noise, the high-pass band-limit f_i must be increased, bringing it nearer to the upper band-limit f_s. However, the increase of f_i must be obtained without reducing significantly the amplitude of the filtered signal.

Therefore, it is not advisable to obtain an increased f_i simply by making shorter the time constant T_A of the CR filter, because with T_A not much longer than the pulse duration T_P the factor of reduction of the signal ($1-T_P/T_A$) would be considerable.

A better approach is to employ a correlated double filtering. The GI considered in (C) can be employed and the measurement can be obtained as the difference of two GI acquisitions. The first acquisition is with gate synchronized to the signal and the second with a delay T_R equal to the pulse duration T_P or longer, for instance $T_R \approx T_P$. This approach establishes for the *1/f* noise a filtering passband with lower limit f_i (high-pass limit) set by the interval T_R between the two measurements

$$f_i = \frac{1}{2\pi T_R} \approx \frac{1}{2\pi T_P} \approx 3{,}18 kHz$$

and higher limit f_S (low-pass limit) set by the GI

$$f_S = \frac{1}{2T_P} \approx 10 kHz$$

Furthermore, it must be taken into account that the double measurement increases roughly by a factor 2 the noise power collected in the passband of the filter,.

Therefore, the worsening factor brought by the 1/f noise in the measurement with correlated double filtering by GI can be approximately evaluated as

$$\sqrt{\frac{\overline{n_T^2}}{\overline{n_U^2}}} = \sqrt{2}\sqrt{1 + \frac{f_C}{f_S}\ln\left(\frac{f_S}{f_i}\right)}$$

With $f_S=10kHz$ $f_i=3{,}18\ kHz$ $f_C=50KHz$ we get

$$\sqrt{\frac{\overline{n_T^2}}{\overline{n_U^2}}} \approx \sqrt{13{,}45} = 3{,}7$$

That is

$$\sqrt{\overline{n_T^2}} \approx 3{,}7 \cdot 2\mu V \approx 7{,}4\mu V$$

The signal to noise ratio is

$$\frac{S}{N} = \frac{V_P}{\sqrt{\overline{n_U^2}}} \cdot \frac{1}{3{,}7}$$

and the minimum measurable amplitude is

$$V_{P,\min} = S_V^{1/2}\sqrt{\frac{1}{2T_P}} \cdot 3{,}7 = \sqrt{\overline{n_U^2}} \cdot 3{,}7 = 7{,}4\mu V$$