Sensors, Signals and Noise

COURSE OUTLINE

- Introduction
- Signals and Noise: 2) Types and Sources
- Filtering
- Sensors and associated electronics

Noise Sources and Types

- Shot Noise (or Shottky Noise) main features
- Shot Noise Mean, Mean Square and Power
- Shot Noise Power Spectrum and Autocorrelation Function
- Noise in diodes (Schottky Noise)
- Modeling any Noise with a Poisson Process
- Noise in Resistors (Johnson-Nyquist Noise)
- White Noise

and for those who want to gain a better insight

- APPENDIX 1: Autocorrelation Function of Shot Current
- APPENDIX 2: Diode Noise and conductance at zero bias
- APPENDIX 3: Conductor resistance and Noise

Shot Noise (or Schottky Noise) Main Features

Shot Noise (or Schottky Noise)

Real case: Shot current in a diode

- **Random** sequence of many **independent** pulses,
  i.e. «shots» due to single electrons that swiftly cross the junction depletion layer
- Pulses have rate *p*, charge q and very short duration T_h
  (shorter than transition times in the circuits)
- «Shot» current has mean value $\boxed{I = p \cdot q}$
- Shot current has fast fluctuations around the mean, called **shot noise**
  (or Schottky noise, after the name of the scientist who explained it)
  The technical literature reports that shot noise has constant spectral density

$$\boxed{S_{nu} = 2qI}$$ unilateral density in $0 < f < \infty$

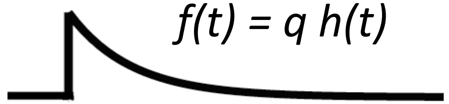
Let us see how this result can be inferred from the basic features of the shot process

Shot Current

Current in a reverse-biased p-n junction diode

random sequence of **independent** elementary pulses *f(t)*

(single carriers that fall down the potential barrier and cross the depletion layer)



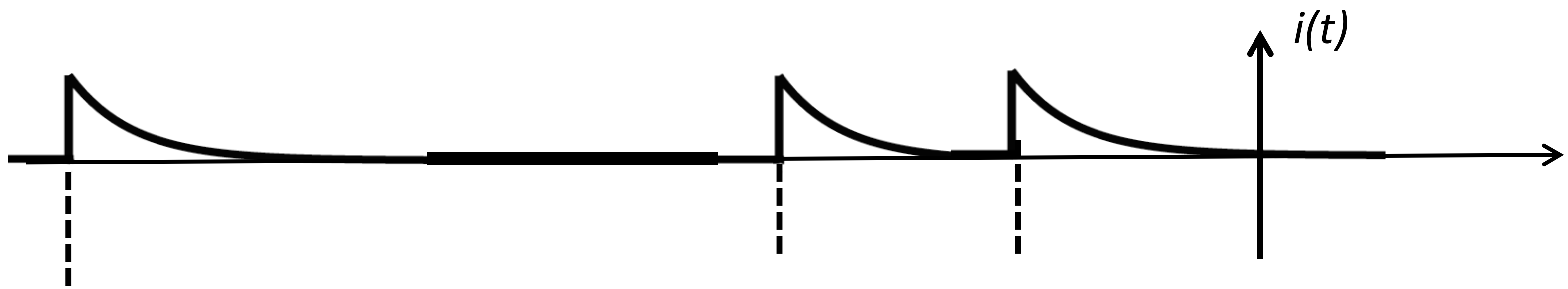
q pulse charge

h(t) normalized pulse shape: $\int_{-\infty}^{\infty} h(t)dt = 1$

POISSON STATISTICAL PROCESS

- The pulses are **independent statistical** events:
  the probability for a pulse to occur is independent of the occurrence of other pulses
- $p \cdot dt$ is the probability that a pulse starts in $t \leftrightarrow t+dt$
- We consider *p* constant (independent of t)

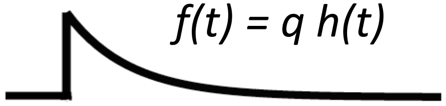
at time t the current *i* is the superposition of the contributions of pulses starting before t



Shot Noise
Mean, Mean Square and Power

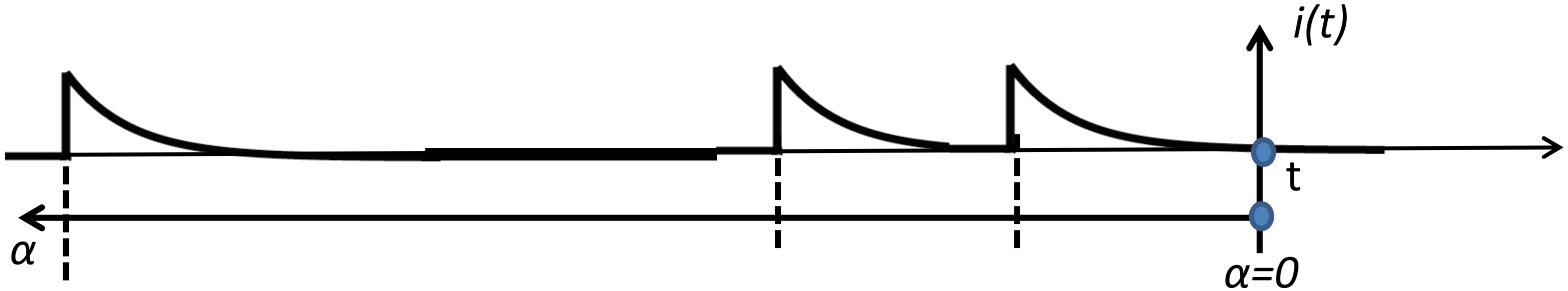
Shot Current: Mean Value

Shot current = **random** sequence of **independent** elementary pulses *f(t)*



q pulse charge

h(t) pulse shape (normalized $\int_{-\infty}^{\infty} h(t)dt = 1$)



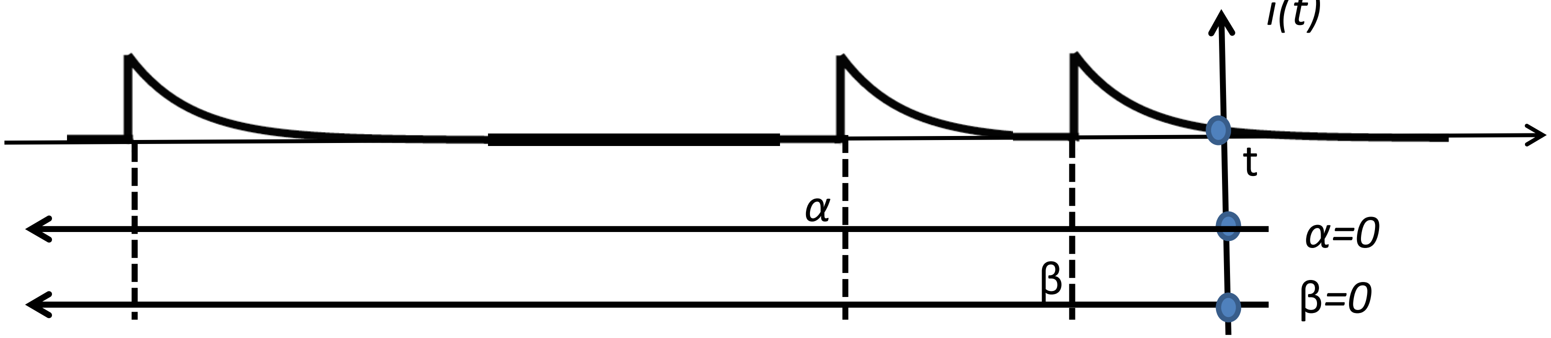
- A pulse which starts at time *α* contributes a current *q h(α)* at time *t*
- *pdα* probability that a pulse starts in *α* ⟷ *α+dα*

Mean current at time *t* : sum of the mean effects of all possible pulses

$$\overline{i(t)} = I = \int_0^{\infty} qh(\alpha) \cdot pd\alpha = pq \int_0^{\infty} h(\alpha)d\alpha = pq$$

Evident without computation: current = pulses at rate *p* each one carrying charge *q.*
However, with this approach to computation we can get also the **mean square** value

Shot Current: Mean Square



Let's consider a couple of pulses starting one at time α and the other at time β

- The contribution of the couple to the square $i^2(t)$ of the current at time t is
  $[qh(\alpha) + qh(\beta)]^2 = q^2 h^2(\alpha) + q^2 h^2(\beta) + qh(\alpha) \cdot q\, h(\beta) + q\, h(\beta) \cdot qh(\alpha)$
- i.e. a pulse gives two contributions, one of its own («square» terms $q^2 h^2(\alpha)$) plus one in collaboration with the other pulse («rectangular» terms $q\, h(\alpha) \cdot q\, h(\beta)$)
- The probability that a «square» contribution $q^2 h^2(\alpha)$ exists
  is **SIMPLY** the probability that the pulse in α exists, i.e. $pd\alpha$
- The probability that a «rectangular» contribution $qh(\alpha) \cdot qh(\beta)$ exists
  is the probability that the pulse in α exists **AND** the pulse in β exists, i.e. $pd\alpha \cdot pd\beta$
- The mean contribution given in total by the couple is therefore

$$di^2 = q^2 h^2(\alpha)pd\alpha + q^2 h^2(\beta)pd\beta + qh(\alpha)pd\alpha \cdot q\, h(\beta)pd\beta + q\, h(\beta)pd\beta \cdot qh(\alpha)pd\alpha$$

Shot Current: Mean Square and Noise Power

The mean square value $\overline{i^2(t)}$ of the current is given by the sum of the mean contributions of all possible single pulses and of all possible couples of pulses

$$\overline{i^2(t)} = \int_0^\infty q^2 h^2(\alpha) \cdot p d\alpha + \iint_0^\infty q h(\alpha)\, p d\alpha\, q h(\beta)\, p\, d\beta =$$

$$= pq^2 \int_0^\infty h^2(\alpha) d\alpha + pq \int_0^\infty h(\alpha) d\alpha \cdot pq \int_0^\infty h(\alpha) d\alpha =$$

$$= pq^2 \int_0^\infty h^2(\alpha) d\alpha + (pq)^2 = pq^2 \int_0^\infty h^2(\alpha) d\alpha + (\, \overline{i(t)}\,)^2 =$$

$$= pq^2 \int_0^\infty h^2(\alpha) d\alpha + I^2$$

The current **noise** is only the **fluctuation** of *i(t)* around the mean value *I*

$$n_i(t) = i(t) - \overline{i(t)} = i(t) - I$$

hence the noise power is the mean square **deviation** of i(t) :

$$\overline{n_i^2} = \overline{i^2(t)} - \left(\overline{i(t)} \right)^2 = pq^2 \int_0^\infty h^2(\alpha) d\alpha =$$

$$= qI \int_0^\infty h^2(\alpha) d\alpha$$

This is the **Campbell Theorem**

Shot Noise Power Spectrum
and
Autocorrelation Function

Shot Noise: Power Spectrum

$$\overline{n_i^2} = qI \int_0^{\infty} h^2(\alpha) d\alpha = qI \int_{-\infty}^{\infty} h^2(\alpha) d\alpha$$

NB: $-\infty$ *because* $h(\alpha) = 0$ *for* $\alpha<0$

- Let's denote by *H(f)* the Fourier transform of the elementary pulse shape *h(t)*

$$H(f) = F[h(t)]$$

- By the Parseval theorem, we see that the noise power is a sum of contributions of elementary components in frequency domain

$$\overline{n_i^2} = qI \int_{-\infty}^{\infty} h^2(\alpha) d\alpha = qI \int_{-\infty}^{\infty} |H(f)|^2 df$$

- The noise power computed from the power spectrum $S_n(f)$ is

$$\overline{n_i^2} = \int_{-\infty}^{\infty} S_n(f) df$$

- Hence the power spectrum is

$$S_n(f) = qI\,|H(f)|^2$$

Shot Noise: Autocorrelation Function

From the shot noise power spectrum

$$S_n(f) = qI|H(f)|^2$$

we obtain the shot noise autocorrelation

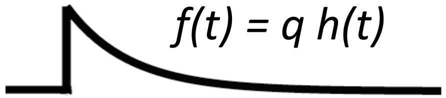
$$R_{nn}(\tau) = F^{-1}\left[S_n(f)\right] = qI \cdot F^{-1}\left[\left|H[f]\right|^2\right] = qI \cdot k_{hh}(\tau)$$

The shot noise power can be computed also in the time omain

$$n_i^2 = R_{nn}(0) = qI \cdot k_{hh}(0) = qI \cdot \int_0^\infty h^2(\alpha)\, d\alpha$$

Shot Noise: summary

Shot current *i(t)* : **random** sequence of **independent** elementary pulses *f(t)* with probability density *p* in time



q pulse charge

h(t) normalized pulse shape $\int_{-\infty}^{\infty} h(t)dt = 1$

Shot noise $n_i(t)$: **random fluctuations** of the current **around its mean value** *I = pq*

TIME DOMAIN

Autocorrelation

$$R_{nn}(\tau) = pq^2 \cdot k_{hh}(\tau) =$$

$$= qI \int_0^{\infty} h(\alpha)h(\alpha+\tau)\, d\alpha$$

Mean square value

$$\overline{n_i{}^2} = pq^2 \cdot k_{hh}(0) = qI \int_0^{\infty} h^2(\alpha)d\alpha$$

FREQUENCY DOMAIN

Power Spectrum

$$S_n(f) = pq^2 \cdot |H(f)|^2 =$$

$$= qI \cdot |H(f)|^2$$

Mean square value

$$\overline{n_i{}^2} = \int_{-\infty}^{\infty} S_n(f)df$$

Noise in Diodes

Diode Noise in **Reverse** Bias

In a p-n reverse-biased diode with mean current I_S we have:

- Elementary pulses: currents induced at the terminals by single carriers that fall down the potential barrier and cross the junction depletion layer
- Elementary pulse width T_h = transit time in the junction (from *a few ps* to *1ns* , since the p-n depletion layer ranges from 0,1 *μm* to *100 μm*)
- We can assume

$$h(t) \cong \delta(t) \qquad \text{and} \qquad |H(f)| \cong 1$$

with approximation valid for correlation times longer than T_h (i.e. down to ns) that is, for frequencies up to ≈ $1/T_h$ (i.e. up to GHz)

- With this approximation the noise spectrum is

$$S_{nB}(f) = qIS \cdot |H(f)|^2 \cong qI_S \qquad (S_{nB} \text{ bilateral density})$$

$$S_{nU}(f) = 2qI_S \cdot |H(f)|^2 \cong 2qI_S \qquad (S_{nU} \text{ unilateral density})$$

which is just the equation reported in the literature

- The corresponding noise autocorrelation is δ-like

$$R_{nn}(\tau) = qIS \cdot k_{hh}(\tau) \cong qIS \cdot \delta(t)$$

Diode Noise in **forward** bias

$$I = I_S\left(e^{\frac{qV}{kT}} - 1\right) = I_S\, e^{\frac{qV}{kT}} - I_S$$

The diode current is the result of opposite shot components with mean values:

a) $-I_S$ reverse current of minority carriers, which fall down the potential barrier

$b)$ $I_S\, e^{\frac{qV}{kT}}$ forward current of majority carriers,which jump over the potential barrier

- The mean current is the difference of the components
- The independent current fluctuations are quadratically added in the spectrum

$$S_{nU}(f) = 2qI_S e^{\frac{qV}{kT}} + 2qI_S = 2q(I + I_S) + 2qI_S = 2qI + 4qI_S$$

- In forward bias it is $I \gg I_S$ and the spectrum is

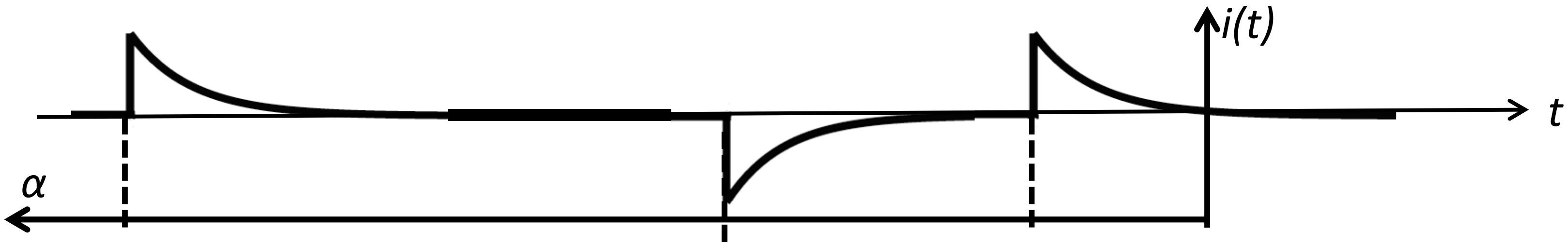
$$\boxed{S_{nU}(f) \approx 2qI}$$

- At zero bias it is I=0 and the spectrum is

$$\boxed{S_{nU}(f) \approx 4qI_S}$$

Modeling any Noise with a Poisson Process

Poisson process with zero mean value



For a shot current superposition of **independent positive and negative** random pulses (q and –q) with **equal pulse shape and equal probability density** ($p_+ = p_-$) the **mean current is nil** (equal positive and negative current)

$$\overline{i(t)} = I_+ - I_- = p_+ q - p_- q = 0$$

The equation for shot noise is thus directly obtained for the current

$$\overline{n_i^2} = \overline{i^2(t)} = (p_+ + p_-) q^2 \int_0^\infty h^2(\alpha)\, d\alpha = q(I_+ + I_-) \int_0^\infty h^2(\alpha)\, d\alpha$$

$$S_{nB}(f) = q(I_+ + I_-)\left|H(f)\right|^2$$ (NB: bilateral spectral density)

CONCLUSION :
whichever noise can be modeled by a Poisson process
with proper pulse shape *h(t)* and zero mean value

Noise in Resistors
(Johnson-Nyquist noise)

Resistor thermal noise (Johnson-Nyquist noise)

- The voltage V between the terminals of a conductor with resistance R shows random fluctuations that do not depend on the current *I*
- The technical literature reports that this noise has voltage spectral density S_{vU} constant up to very high frequency >> 1GHz: denoting by R the resistance and by T the absolute temperature it is

$$S_{vU}(f) = 2kTR \quad \text{(bilateral)}$$

- This noise can be described also in terms of current in the conductor terminals: denoting by $G = 1/R$ the conductance, the current spectral density is

$$S_{iU}(f) = 2kTG \quad \text{(bilateral)}$$

- This noise is known as Johnson-Nyquist noise, after the name of the scientists that first studied and explained it.
- It is generated by the agitation of the charge carriers (electrons) in the conductor in thermal equilibrium at temperature T
- We will outline how this noise can be interpreted in terms of a Poisson pulse process with zero mean value.

Resistor thermal noise (Johnson-Nyquist noise)

With *V = 0* i.e. no voltage applied to the terminals

in the conductor there is **NO electric field *E*** and the situation is in thermal equilibrium:

- A huge population of electrons ($\approx 10^{22} el/cm^3$ in the volume of the conductor) is in random thermal agitation with frequent collisions
- In each collision the velocity of an electron changes randomly, i.e. the velocity after the collision is statistically independent of the velocity before
- An elementary pulse is the current induced on the terminal electrode by an electron traveling between a collision and the next one
- The + or - pulse sign correspond to electron moving away or towards the electrode
- The pulse duration is the time interval t_C between two collisions, which is very short $t_c \approx 10^{-14} s$
- The individual velocities of electrons are very high, but the drift velocity v_D (mean value over the population) is zero. That is, there is no current (no charge transport)

Resistor thermal noise (Johnson-Nyquist noise)

With voltage *V* applied to the terminals

An electric field E is established in the resistor and current flows, but in **ohmic regime** (i.e. with **moderate value V**) the perturbation of the situation is very moderate

- The **mean value** of the velocity of the electrons (v_D drift velocity) is no more zero, but it is still negligible with respect to the thermal random velocity.
- The drift velocity $v_D = \mu E$ is proportional to the electric field E (with μ electron mobility) leading to the macroscopic **ohmic behaviour *V = R I***
- As concerns the statistical fluctuations, **the noise is the same as with V = 0** because the thermal equilibrium can be considered unperturbed

Resistor thermal noise (Johnson-Nyquist noise)

A simple approximate analysis of the random motion of the electrons shows that p (pulse rate) and q (pulse charge) of the Poisson process are related to the conductance G and to the thermal energy kT by the equation

$$S_i = pq^2 = 2kT\,G$$

This conclusion is in fact the equation of the Johnson noise currently reported in the technical literature.

For those who wish to gain a better insight, the approximate analysis is reported in Appendix 3.

Essentially, the conclusion relies on the following facts:

- the probability density p , the mobility μ (and therefore the conductance G) and the elementary pulse charge q are all related to the time interval t_c between collisions
- the pulse charge q depends also on the thermal agitation velocity, hence on kT

Resistor thermal noise (Johnson-Nyquist noise)

In conclusion, for Johnson noise

$$R_{ii}(\tau) = pq^2 \cdot k_{hh}(\tau) = 2kTG \cdot k_{hh}(\tau)$$

$$S_{iB}(f) = pq^2 \cdot |H(f)|^2 = 2kTG \cdot |H(f)|^2$$

The elementary pulse width T_h is the mean interval between the carrier collisions, that is $T_h \approx 10^{-14} s$. Therefore, the approximation

$$h(t) \cong \delta(t)$$

$$|H(f)| \cong 1$$

is valid for Johnson noise as long as one deals with correlation at time intervals longer than T_h (i.e. ≥ *0,01 ps)* or bandwidth lower than $1/T_h$ (i.e. ≤ *100 THz*)

In such conditions

$$S_{iB}(f) \cong 2kTG$$

with S_{iB} bilateral density. The corresponding unilateral density is

$$S_{iU}(f) = 2S_{iB}(f) \cong 4kTG$$

or, in terms of voltage noise

$$S_{vU}(f) = R^2 S_{vU}(f) \cong 4kTR$$

White Noise

White Noise (stationary)

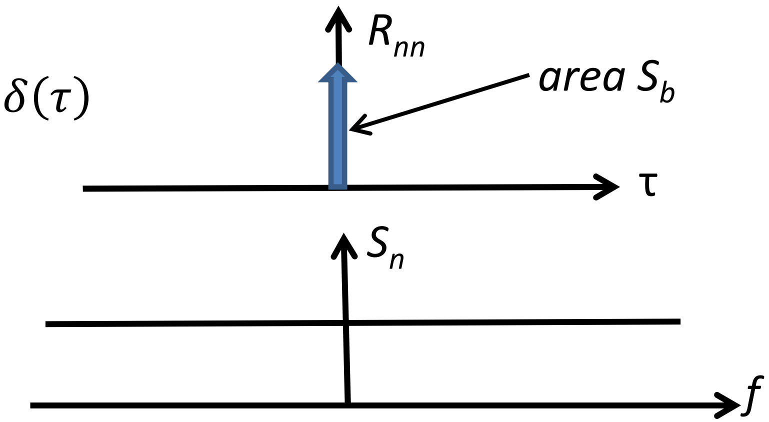
The **IDEAL «white» noise** is a concept extrapolated from Johnson noise and shot noise and is defined by the **essential** characteristic feature:

no autocorrelation at any time distance τ, no matter how small

$$R_{nn}(\tau) = S_b \cdot \delta(\tau)$$

and therefore constant spectrum

$$S_n(f) = S_b$$



In reality such a noise does not exist: it would have divergent power $\overline{n^2} \to \infty$

A **REAL «white»** noise has

- **Very small width** of autocorrelation, shorter than the minimum time interval of interest in the actual case and therefore approximated to zero
- **Very wide band** with constant spectral density S_b , wider than the maximum frequency of interest in the actual case and therefore approximated to infinite

White Noise (non-stationary)

Also in non-stationary cases the IDEAL «white» noise *n(t)*
is defined by the **essential** characteristic feature:

no correlation at any finite time distance *τ*, no matter how small,

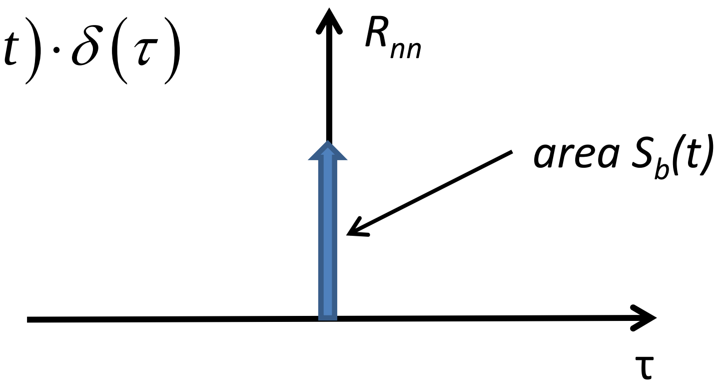
but the noise intensity is no more constant, it varies with time *t*

that is

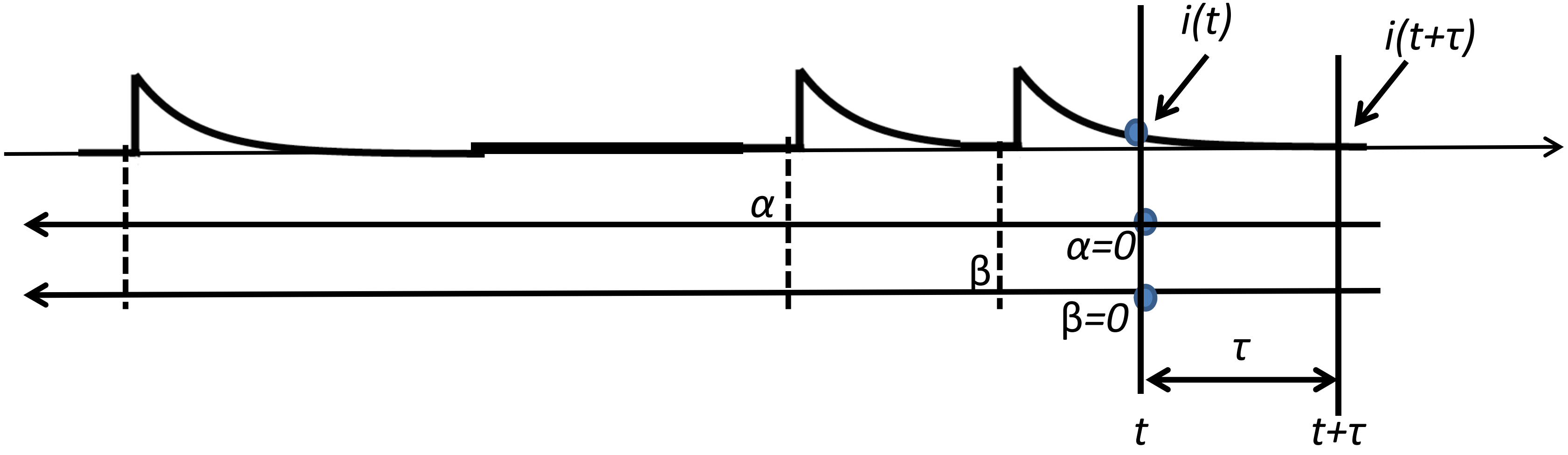
the autocorrelation function is δ-like,

but has time-dependent area $S_b(t)$

$$R_{nn}\left(t, t+\tau\right) = S_b\left(t\right) \cdot \delta\left(\tau\right)$$



APPENDIX 1: Autocorrelation Function of Shot Current



$$R_{ii}(\tau) = \overline{i(t)i(t+\tau)}$$

- *i(t)* is a sum of contributions $q\,h(\alpha) + q\,h(\beta) + \dots\dots$
- *i(t+τ)* is a sum of contributions $q\,h(\alpha+\tau) + q\,h(\beta+\tau) + \dots\dots$
- $i(t) \cdot i(t+\tau)$) is a sum of «square» contributions due to **single** pulses $qh(\alpha) \cdot qh(\alpha+\tau)$ plus a sum of «rectangular» contributions due to **couple** of pulses $q\,h(\alpha) \cdot q\,h(\beta+\tau)$
- The probability that a «square» contribution $q\,h(\alpha) \cdot qh(\alpha+\tau)$ exists is **just** the probability that a pulse in α exists, i.e. $pd\alpha$
- The probability that a «rectangular» contribution $qh(\alpha) \cdot q\,h(\beta+\tau)$ exists is the probability that the pulse in α exists **and** the pulse in β exists, i.e. $pd\alpha \cdot pd\beta$

APPENDIX 1: Autocorrelation Function of Shot Current

The ensemble average of $i(t)i(t+\tau)$ is the sum of the mean contributions of all possible single pulses and of all possible couples of pulses

$$R_{ii}(\tau) = \overline{i(t)i(t+\tau)} =$$

$$= \int_{-\infty}^{\infty} q^2 h(\alpha)h(\alpha+\tau)\, p d\alpha + \iint_{-\infty}^{\infty} qh(\alpha)\, p d\alpha \cdot qh(\beta+\tau)\, p\, d\beta =$$

(NB: the integrals are extended to $-\infty$ *because* $h(\theta) \equiv 0$ *for negative argument* ϑ*)*

$$R_{ii}(\tau) = pq^2 \int_{-\infty}^{\infty} h(\alpha)h(\alpha+\tau)\, d\alpha + pq \int_{-\infty}^{\infty} h(\alpha)d\alpha \cdot pq \int_{-\infty}^{\infty} h(\beta+\tau)d\beta =$$

$$= pq^2 \int_0^{\infty} h(\alpha)h(\alpha+\tau)\, d\alpha + (pq)^2 =$$

$$= pq^2 \int_0^{\infty} h(\alpha)h(\alpha+\tau)\, d\alpha + (\,\overline{i(t)}\,)^2 =$$

$$= qI \int_0^{\infty} h(\alpha)h(\alpha+\tau)\, d\alpha + I^2$$

APPENDIX 1: Autocorrelation Function of Shot Current

The current shot noise is

$$n_i(t) = i(t) - \overline{i(t)} = i(t) - I$$

hence the autocorrelation of the shot noise is

$$R_{nn}(\tau) = R_{ii}(\tau) - \left(\overline{i(t)}\right)^2 = R_{ii}(\tau) - I^2 =$$

$$= qI \int_0^\infty h(\alpha)h(\alpha+\tau)\, d\alpha$$

That is

$$R_{nn}(\tau) = qI\, k_{hh}(\tau)$$

Shot Noise: Power Spectrum

From the definition

$$S_n(f) = F[R_{nn}(\tau)]$$

we obtain

$$S_n(f) = qI|H(f)|^2$$

which is consistent

$$\overline{n_i{}^2} = R_{nn}(0) = \int_{-\infty}^{\infty} S_n(f) df$$

APPENDIX 2: Diode Noise and conductance in forward bias

- In the processing of **small signals** (i.e. small voltage deviations from the bias voltage) the diode is equivalent to a **resistor with conductance G**

$$G = \frac{dI}{dV} = \frac{q}{kT} I_S e^{\frac{qV}{kT}} = \frac{q}{kT}\left(I + I_S\right) = \frac{q}{kT} I + \frac{q}{kT} I_S$$

but <u>in forward bias</u> it is $I \gg I_S$, therefore

$$\boxed{G = \frac{q}{kT} I}$$

- The noise of an ordinary resistor with conductance G is

$$S_{GU}(f) = 4kTG$$

but a diode with equivalent G has lower noise

$$S_{nU}(f) \approx 2qI = 2kT\,\frac{qI}{kT} = 2kTG < 4kTG$$

- In circuit design the resistance of a forward-biased diode can be employed for small signals as a resistor with lower noise (i.e. a so-called **«cold resistance»**)

APPENDIX 2: Diode Noise and conductance at zero bias

At zero bias, i.e. with *V = 0 and I = 0*

for small signals around zero bias, a diode is equal to **a resistor with conductance**

$$G = \frac{q}{kT} I_S$$

the diode noise spectrum is

$$S_{nU}(f) = 4qI_S$$

which can also be expressed as

$$S_{nU}(f) = 4qI_S = 4kT\frac{q}{kT}I_S = 4kTG$$

Therefore, in a diode employed as a resistor for small signal at zero bias

the noise is equal to that of an ordinary resistor with equal conductance G

$$S_{GU}(f) = 4kTG$$

Appendix 3: Conductor Resistance and Noise

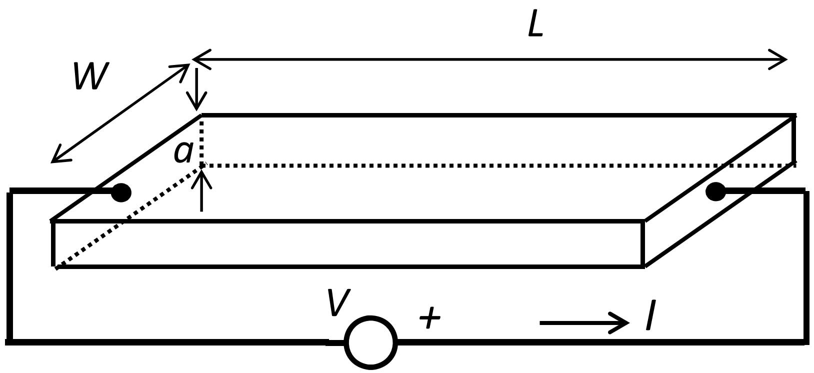
$E = V/L$ *electric field*

$I = G\,V$; *G conductance*

$j = \sigma E$; σ *conductivity*

$A = W\,a$ *cross section*

$$G = \sigma \frac{A}{L}$$

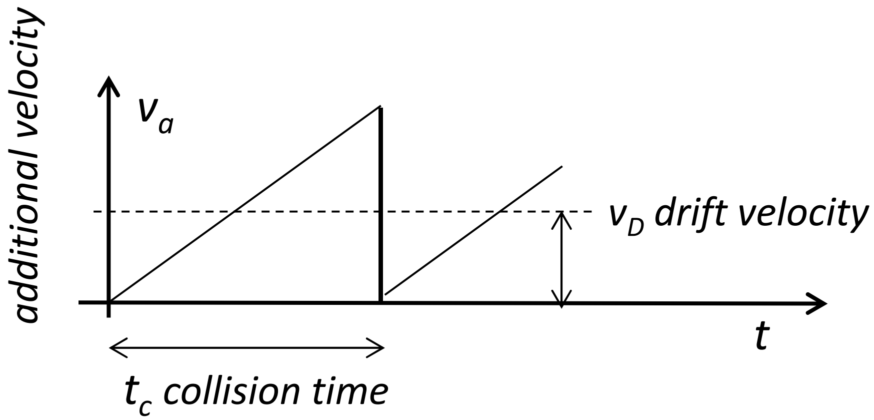


In Ohmic regime (moderate *E*) the carrier thermal random agitation is same as with *E=0* but the <u>mean velocity</u> is no more zero: the action of *E* coherently adds to every carrier a small component v_D (drift velocity) parallel to *E* . Every carrier is accelerated, it gains energy, then transfers it to the lattice by colliding; it is accelerated again etc.

<u>Simplified model</u>:

- The time t_C between collisions is assumed constant
- All the additional energy gained is assumed lost in the collision

Appendix 3: Conductor Resistance and Noise



$$v_{a,\max} = at_c = \frac{qE}{m} t_c$$ maximum additional velocity

$$v_D = \frac{v_{a,\max}}{2} = \frac{qE}{2m} t_c$$ Drift velocity = mean additional velocity

$$v_D = \mu E$$ μ Carrier Mobility

$$\boxed{\mu = \frac{v_D}{E} = \frac{q}{2m} t_c}$$

μ Carrier Mobility is proportional to collision time t_c

Appendix 3: Conductor Resistance and Noise

Carrier density $n = \frac{N}{AL}$

Conduction current density $j = nqv_D = nq\mu E = n\frac{q^2}{2m}t_c E$

Conductivity $\sigma = \frac{j}{E} = nq\mu = n\frac{q^2}{2m}t_c$

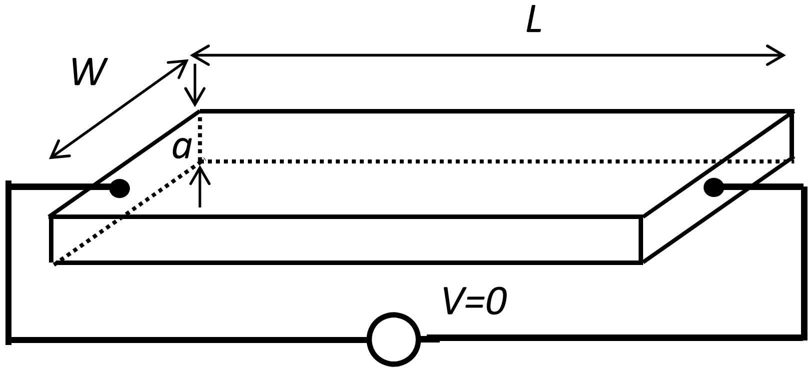
Conductance $G = \sigma\frac{A}{L} = \frac{1}{2}\frac{nA}{L}\frac{q^2}{m}t_c = \frac{1}{2}\frac{N}{L^2}\frac{q^2}{m}\ t_c$

NB: the conductance G is proportional to the mean collision time t_c

Appendix 3: Conductor Resistance and Noise

N free carriers in conductor G

$n = N/AL$ carrier density



Carriers in thermal agitation randomly collide with lattice and generate noise

t_c time between collisions

v_T longitudinal component of the thermal velocity (parallel to field E in conduction)

Simplified model:

- v_T has constant module, but the directon is randomly switched in each collision
- Between two collisions, a carrier generates an elementary pulse with current i_e

Appendix 3: Conductor Resistance and Noise

- The elementary pulse can be computed with the Shockley-Ramo theorem:

  square pulse with duration t_c , constant current $i_e = qv_T \frac{1}{L}$ and charge

  $$Q = i_e t_c = qv_T \frac{t_c}{L}$$

- Rate of generation of elementary pulses by a single carrier = collision rate = $1/t_c$

  hence the total generation rate by the N carriers in the conductor is

  $$\boxed{p = \frac{N}{t_c}}$$

- The current generated by the thermal agitation of carriers has zero mean value (positive and negative pulses) and shot noise with bilateral spectral density

  $$S_{iB} = pQ^2 = \frac{N}{t_c}\left(qv_T \frac{t_c}{L}\right)^2 = N \frac{q^2}{L^2} t_c v_T^2$$

NB : S_i is proportional to the mean collision time t_c as the conductance G is

Appendix 3: Conductor Resistance and Noise

The bilateral spectrum is $S_{iB} = N \frac{q^2}{L^2} t_c v_T^2$

Recalling that $G = \frac{1}{2} \frac{N}{L^2} \frac{q^2}{m} t_c$ we get $S_{iB} = 2G \cdot m v_T^2$

Because of the equipartition of the thermal energy on the degrees of freedom

it is $\frac{1}{2} m v_T^2 = \frac{1}{2} kT$

which finally leads to

$$\boxed{S_{i,B} = 2kTG}$$ bilateral spectral density

$$\boxed{S_{i,U} = 2S_{i,B} = 4kTG}$$ unilateral spectral density