Sensors, Signals and Noise

COURSE OUTLINE

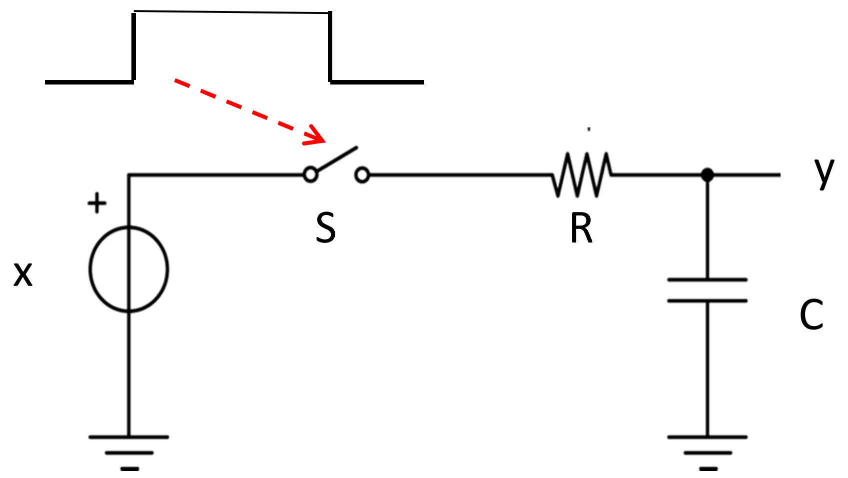
- Introduction
- Signals and Noise
- Filtering: LPF2 Switched-Parameter Filters
- Sensors and associated electronics

Switched-parameter filters

- Switched-parameter RC low-pass filters
- Sample and Hold S&H
- Gated Integrator GI

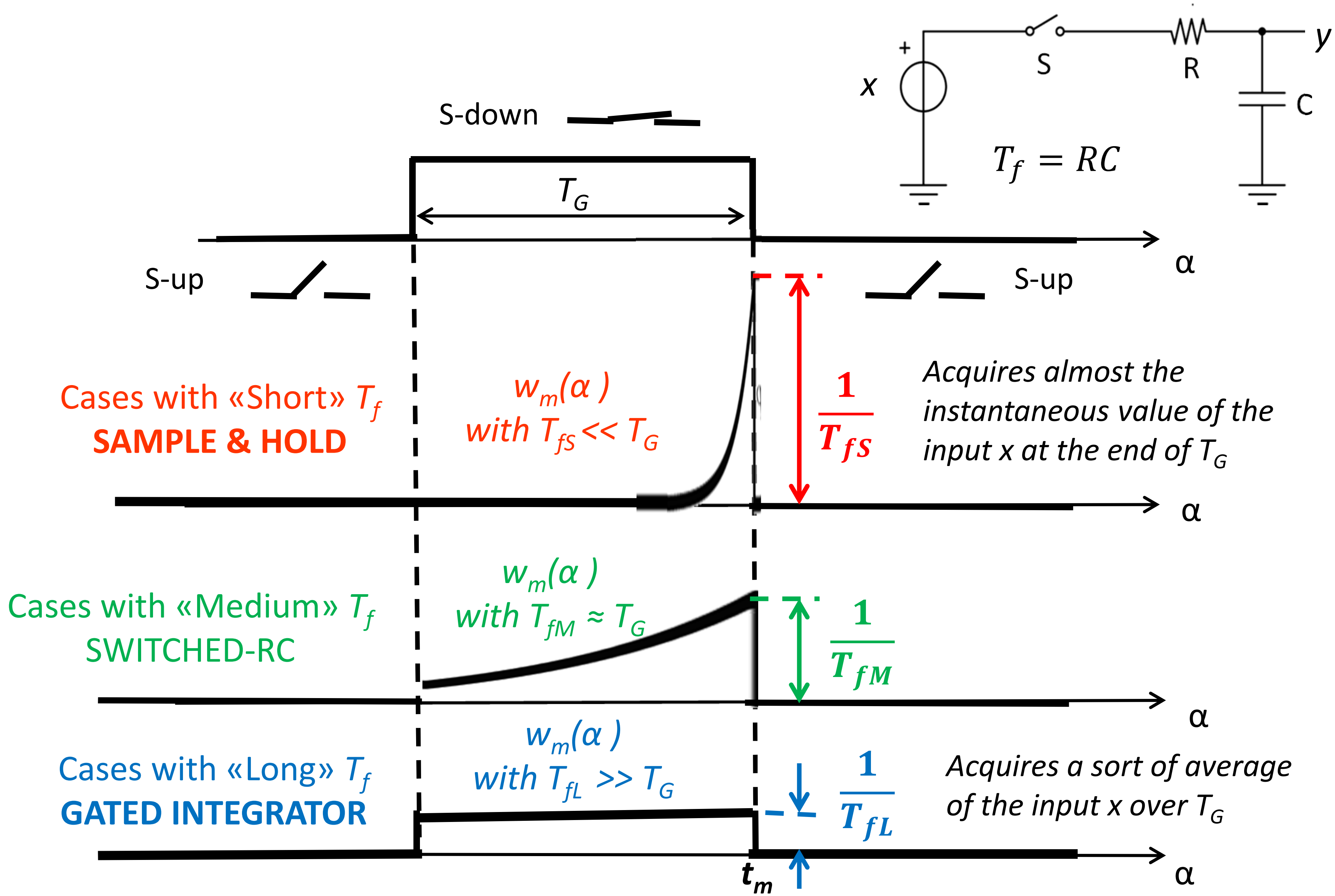
Switched-parameter RC low-pass filters

Switched-parameter RC low-pass filters



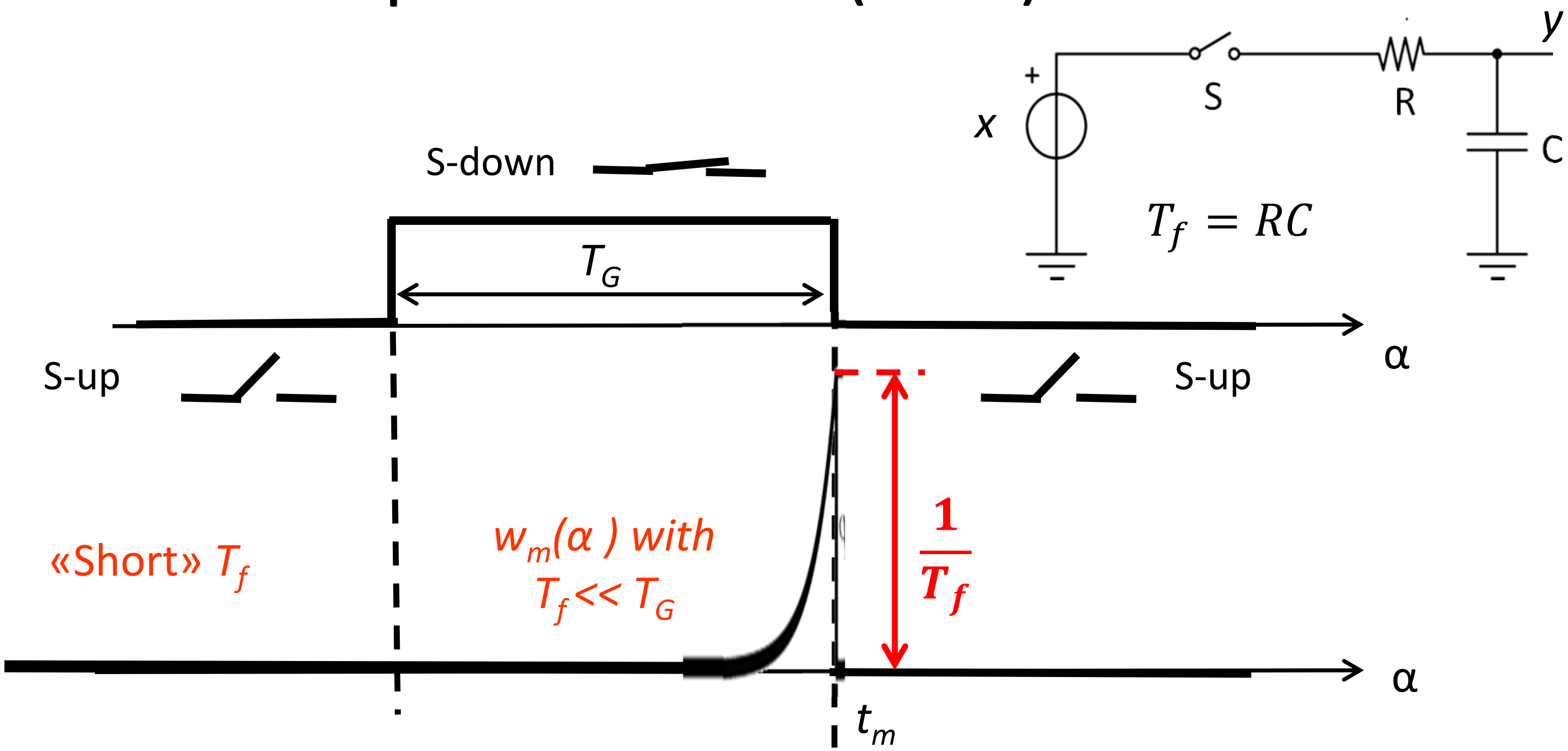
- State with *S* down (closed in short circuit): the circuit behaves like a constant-parameter RC integrator; current can flow in and out of C
- State with *S* up (open circuit): the circuit is in HOLD, no current can flow, the charge previously stored in C is maintained, the voltage on C stays constant.
- The state of *S* is controlled by a known command. The series resistance is switched from R (with S closed) to practically ∞ (with S open) or vice-versa.
- In the cases here considered:
  (a) the initial state is with *S* open and zero charge in C
  (b) the command closes *S* in synchronism with the signal to be acquired
  and re-opens S after the acquisition

Switched-parameter RC low-pass filters



Sample and Hold S&H

Sample and Hold (S&H)



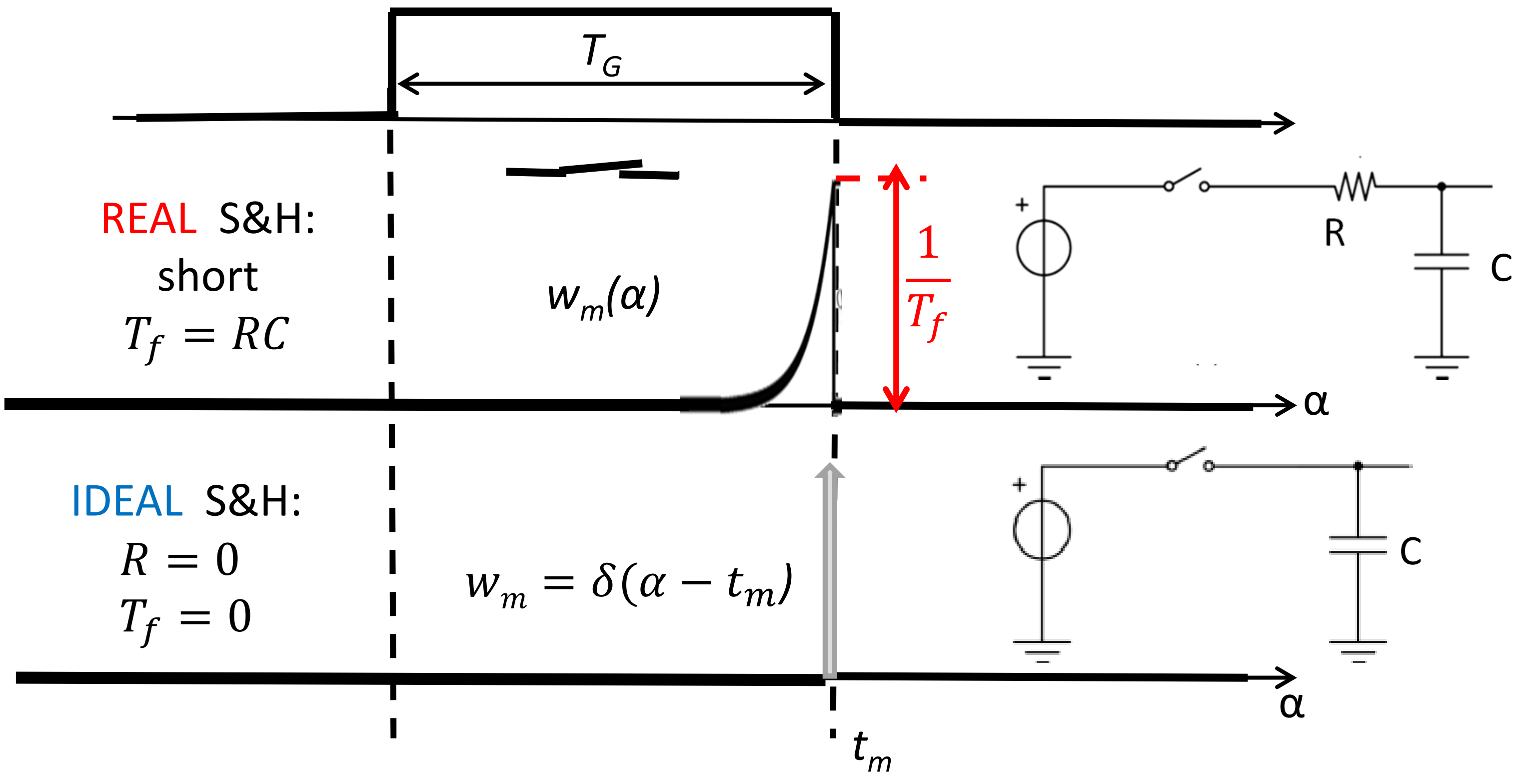
The S&H has **unity DC gain** (C is fully charged at the input voltage within T_G)

$$W_m(0) = \int_0^\infty w_m(\alpha)d\alpha = 1$$

The S&H has very mild filtering action, equivalent to that of a constant-parameter RC integrator with equal time constant T_{fS} . With wide-band input noise S_b (bilateral)

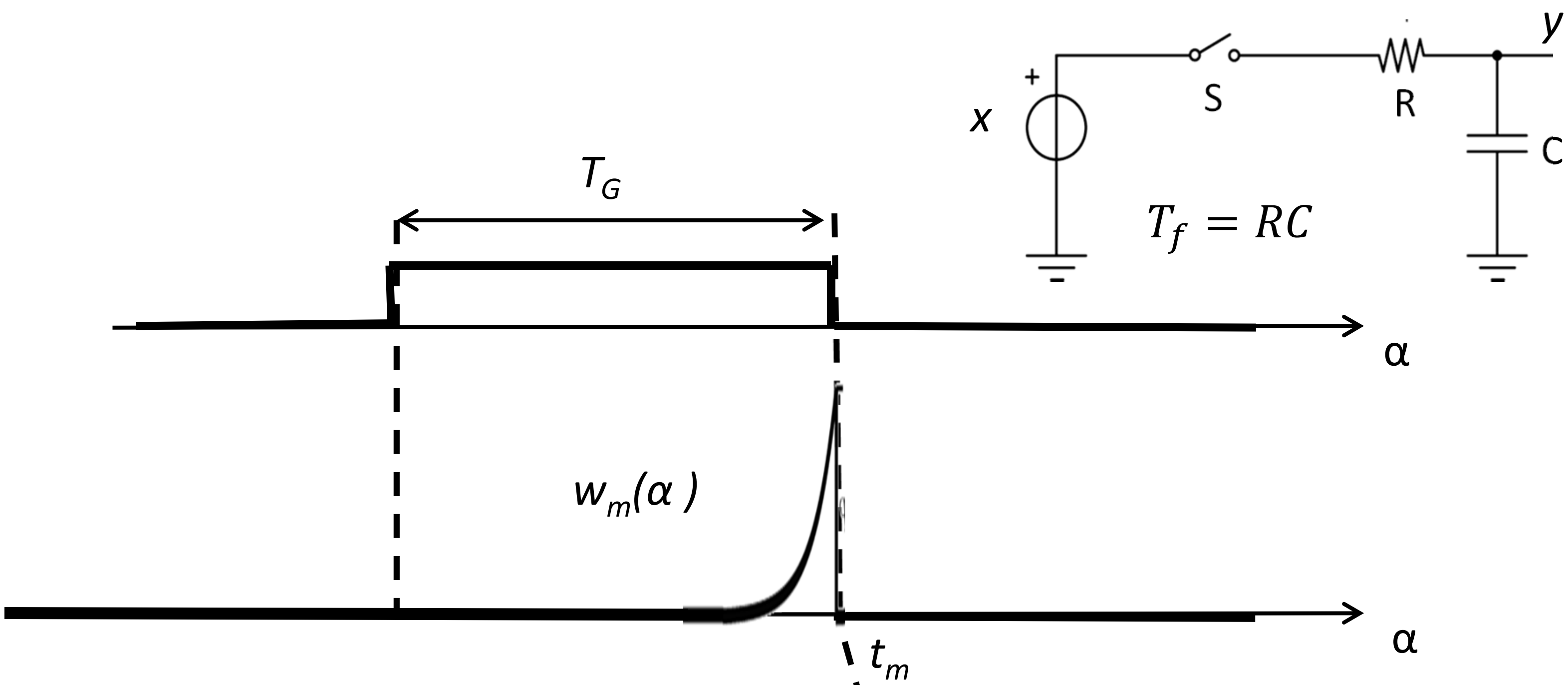
$$\overline{y_n^{\,2}} = S_b \cdot \frac{1}{2T_f}$$

Real vs ideal S&H

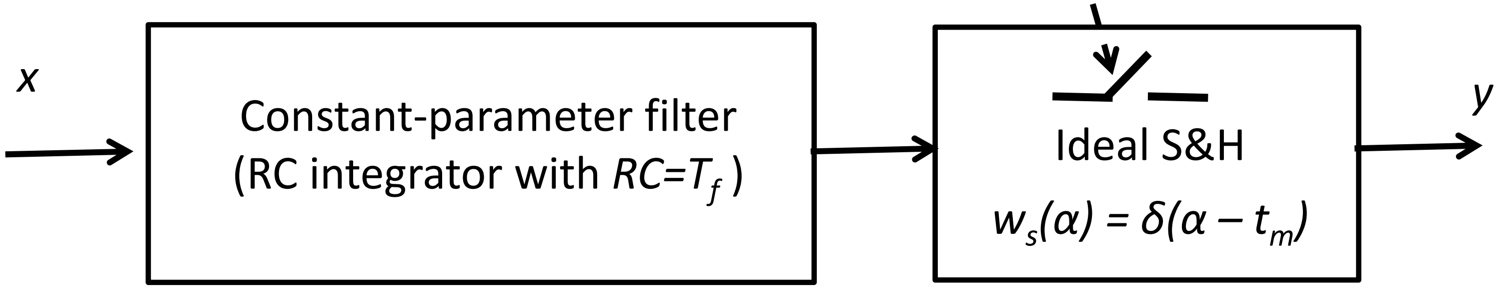


- The minimum available T_f is limited by the technology of devices and circuits (finite R values of fast switching devices and C values required for holding information)
- S&H acquisition time = time for reaching the full output value ≈ a few T_f , i.e. currently some tens of nanoseconds in discrete-component circuits
  some tens of picoseconds in integrated circuits with minimized capacitances

S&H equivalent model



The output of a real S&H is equivalent to
(and can be modeled as) the cascade of two stages:



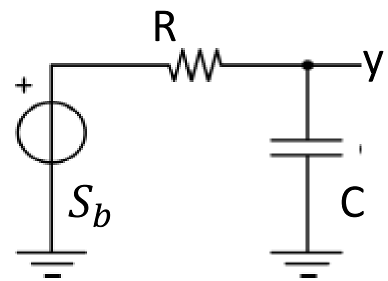
S&H Readout Noise

- READOUT NOISE of a sampling circuit is the contribution to the output noise due to the internal noise sources in the sampling circuit itself
- In the S&H the main source of readout noise is the wide-band Johnson noise of R with spectral density $S_{bB} = 2kTR$ (bilateral)

Since

$$w(\alpha) = \frac{1}{T_f} e^{-\frac{(t_m-\alpha)}{T_f}} 1(t_m - \alpha) \quad \text{and} \quad k_{ww}(\tau) = \frac{1}{2T_f} e^{-\frac{|\tau|}{T_f}}$$

the readout noise is



$$\overline{y_R{}^2} = S_{bB} \cdot k_{ww}\,(0) = 2kTR \cdot \frac{1}{2T_f} = 2kTR \cdot \frac{1}{2RC}$$

$$\boxed{\overline{y_R^2} = \frac{kT}{C}}$$

this is just the noise generated and self-filtered by a constant parameter RC filter and is INDEPENDENT OF THE R VALUE, in agreement wth the S&H circuit model.

- Note that this noise can be directly compared with the input signal, because the S&H has unity DC gain, it brings to the output the full amplitude of the sampled signal

S&H Readout Noise

The equation

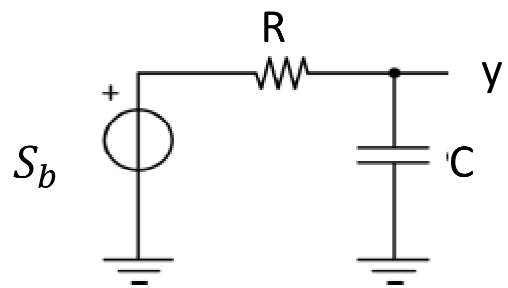
$$\overline{y_R^{\,2}} = \frac{kT}{C}$$

of the self-generated output noise of a constant parameter RC filter
is consistent with the laws of thermodinamics. In fact:

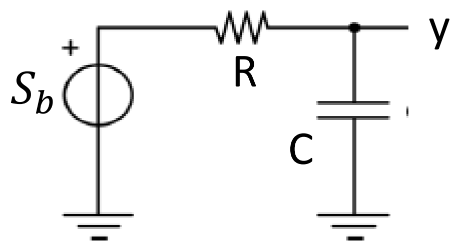
- In a system in thermal equilibrium, the mean thermal energy is $\frac{kT}{2}$ for each degree of freedom;
- the RC circuit has one degree of freedom (one C, i.e, one time constant)

Therefore:

$$\frac{1}{2} C \overline{y_R^{\,2}} = \frac{kT}{2}$$



S&H Readout Noise



The **readout noise voltage** evaluated at room temperature is

$$\sqrt{\overline{y_R{}^2}} = \sqrt{\frac{kT}{q}\frac{q}{C}} \cong \frac{63\mu V}{\sqrt{C(in\, pF)}}$$

In various applications (e.g. CCD imaging photodetectors) the signal is a not a given voltage but a given charge, to be compared with the **readout noise charge** in C

$$\overline{Q_n{}^2} = C^2\overline{y_R{}^2} = kTC$$

In terms of number of electrons N_q the noise is (NB: *q = electron charge*)

$$\overline{N_q{}^2} = \frac{\overline{Q_n{}^2}}{q^2} = \frac{kTC}{q^2} = \frac{kT}{q}\cdot\frac{C}{q}$$

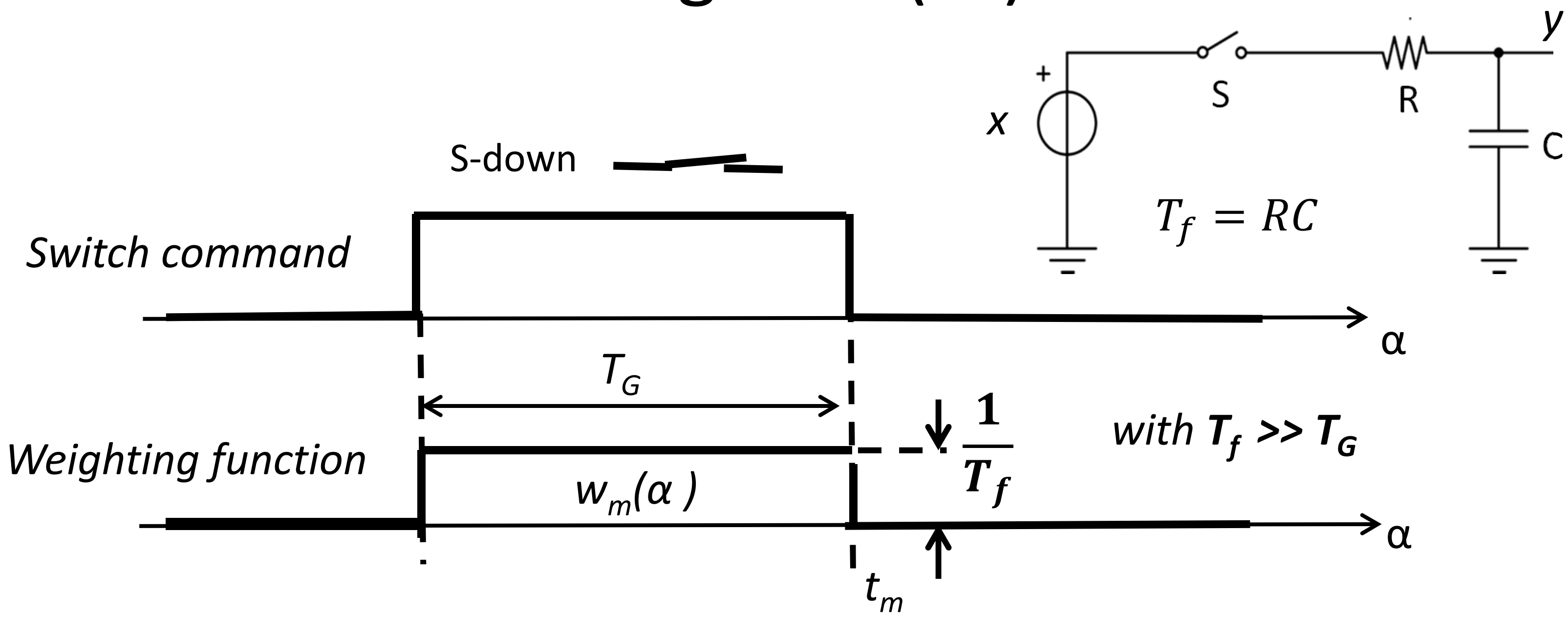
Therefore, the rms fluctuation of the electron number evaluated at room temperature is

$$\sqrt{\overline{N_q{}^2}} = \sqrt{\frac{kT}{q}\cdot\frac{C}{q}} \cong 125\cdot\sqrt{C[in\, pF]} \quad \textit{electrons}$$

e.g. with C = 0,1 pF the rms is about 40 electrons

Gated Integrator GI

Gated Integrator (GI)



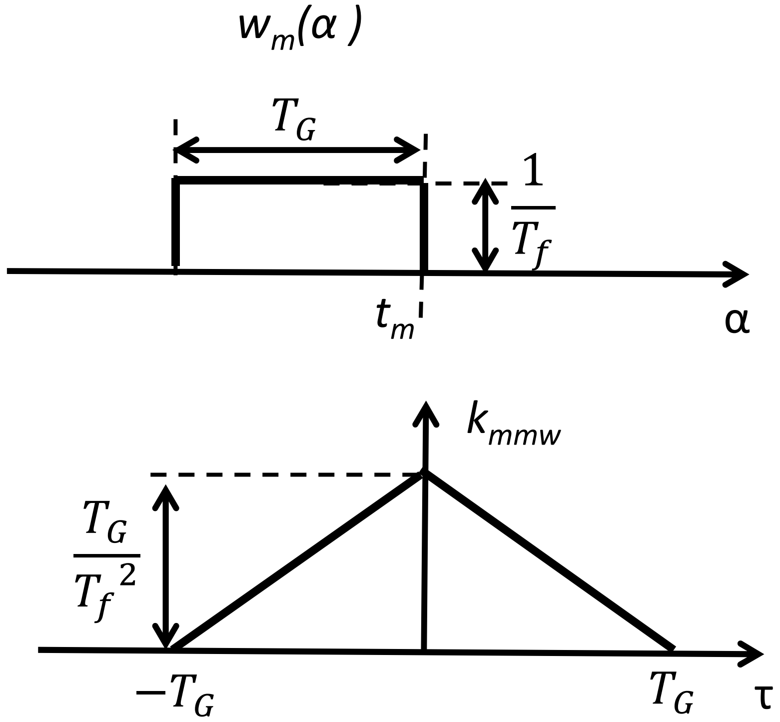
- For behaving as GI (uniform weight in T_G) the circuit must have $T_f >> T_G$
- Therefore, the **DC gain G is inherently much less than unity**

$$G = W_m(0) = \int_0^\infty w_m(\alpha) d\alpha = \frac{T_G}{T_f} \ll 1$$

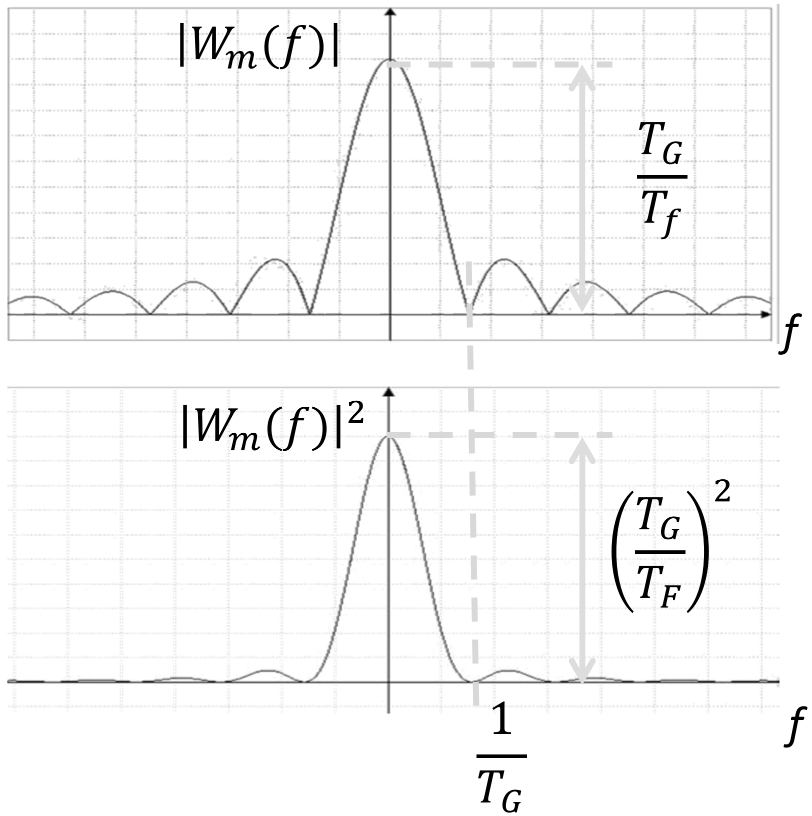
- A GI has remarkable filtering action on a wide-band input noise, that is, on noise with autocorrelation width much shorter than the gate duraton T_G .
- Long gate duration T_G is well feasible in practice, much better than a long averaging interval T_a in a mobile-mean filter

Gated Integrator (GI)

TIME DOMAIN



FREQUENCY DOMAIN



$$W_m(f) = \frac{T_G}{T_f} \cdot \frac{\sin \pi f T_G}{\pi f T_G}$$

Filtering and S/N enhancement by GI

INPUT:

- signal x_s constant in T_G (DC signal)
- wide-band noise S_b (bandwidth f_n >> $1/T_G$ and autocorrelation width T_n << T_G)

$$\overline{x_n{}^2} = S_b 2 f_n = S_b / 2T_n$$

OUTPUT:

Signal $y_s = x_s \cdot \frac{T_G}{T_f} = x_s G$ i.e. with gain $\boxed{G = \frac{T_G}{T_f} \ll 1}$

Noise $\overline{y_n{}^2} = S_b \cdot \frac{T_G}{T_f{}^2} = \frac{S_b}{T_G} \cdot \left(\frac{T_G}{T_f}\right)^2 = \frac{S_b}{T_G} \cdot G^2 =$

$$= \frac{S_b}{2T_n} \frac{2T_n}{T_G} G^2 = \overline{x_n{}^2} \cdot \frac{2T_n}{T_G} \cdot G^2$$

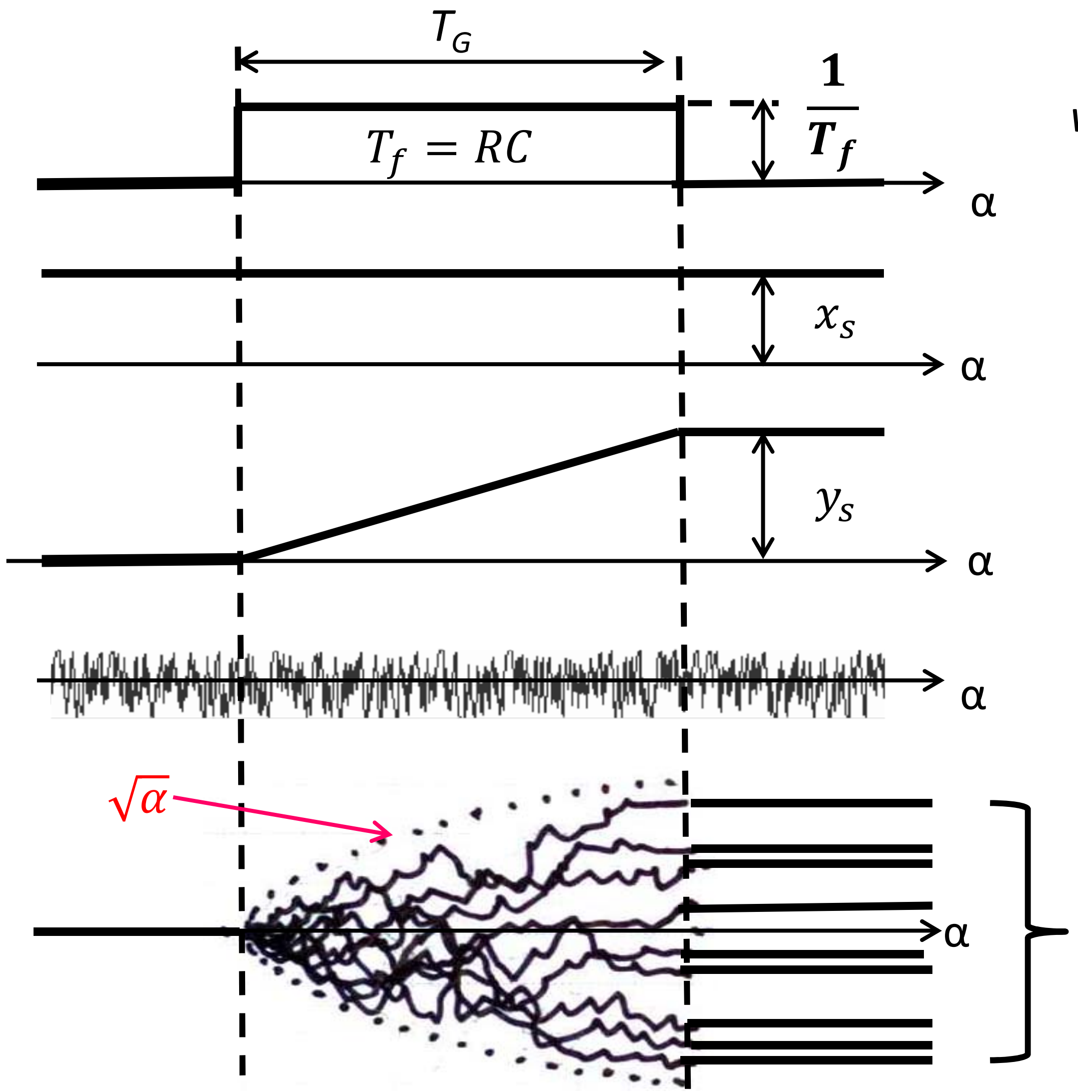
Signal-to-noise ratio

$$\left(\frac{S}{N}\right)_y = \frac{y_s}{\sqrt{\overline{y_n{}^2}}} = \frac{x_s}{\sqrt{\overline{x_n{}^2}}} \cdot \sqrt{\frac{T_G}{2T_n}} = \left(\frac{S}{N}\right)_x \cdot \sqrt{\frac{T_G}{2T_n}}$$

NB: the output signal increases as T_G and the noise as $\sqrt{T_G}$, therefore

the S/N increases as the square root of the gate time $\sqrt{T_G}$

Output Signal and Noise of GI



weighting function $w_m(\alpha)$

Input DC signal x_s

Output signal $y_s = x_s \cdot \frac{T_G}{T_f} \propto T_G$

Wide-band input noise x_n

Output noise

$$\sqrt{\overline{y_n{}^2}} = \sqrt{S_b \cdot \frac{T_G}{T_f{}^2}} \propto \sqrt{T_G}$$

Gaining Insight in the GI output noise

- Poisson Noise model: two random sequences of elementary pulses with positive and negative polarity and equal rate $n_+ = n_- = n$
- White input noise: the elementary pulses are δ-like with area *+q* and *-q*
- Output noise of the GI: the elementary pulses are steps with amplitude *+q* and *–q*
- Mean numbers of pulses in T_G :
  positive $\overline{N_+} = n_+ T_G$ and negative $\overline{N_-} = n_- T_G$; therefore $\overline{N_+} = \overline{N_-} = N = nT_G$
- Fluctuating output amplitude : $y_n = N_+ q - N_- q$ (with mean value $\overline{y_n} = 0$)
- Mean square numbers of pulses in T_G (Poisson statistics): $\overline{N_+{}^2} = \overline{N_-{}^2} = N$
- **Mean square** amplitude of the output noise
  $$\overline{y_n{}^2} = \overline{N_+{}^2} q^2 + \overline{N_-{}^2} q^2 = 2Nq^2 = 2nq^2 \cdot T_G$$
- **Root-mean-square** amplitude of the output noise

$$\sqrt{\overline{y_n{}^2}} = q\sqrt{2nT_G} \propto \sqrt{\boldsymbol{T_G}}$$

GI compared to other LPF

Fair comparison between different LPF with different DC gain G can be made by considering the value of the **filtered noise referred to the input** of the filter (and the input signal). This is equivalent to consider the **output with unity DC gain** (if necessary, by considering to add further gain stages).
For a GI this noise is

$$\left(\overline{x_n{}^2}\right)_{GI} = \frac{\left(\overline{y_n{}^2}\right)_{GI}}{G^2} = \frac{S_b}{T_G}$$

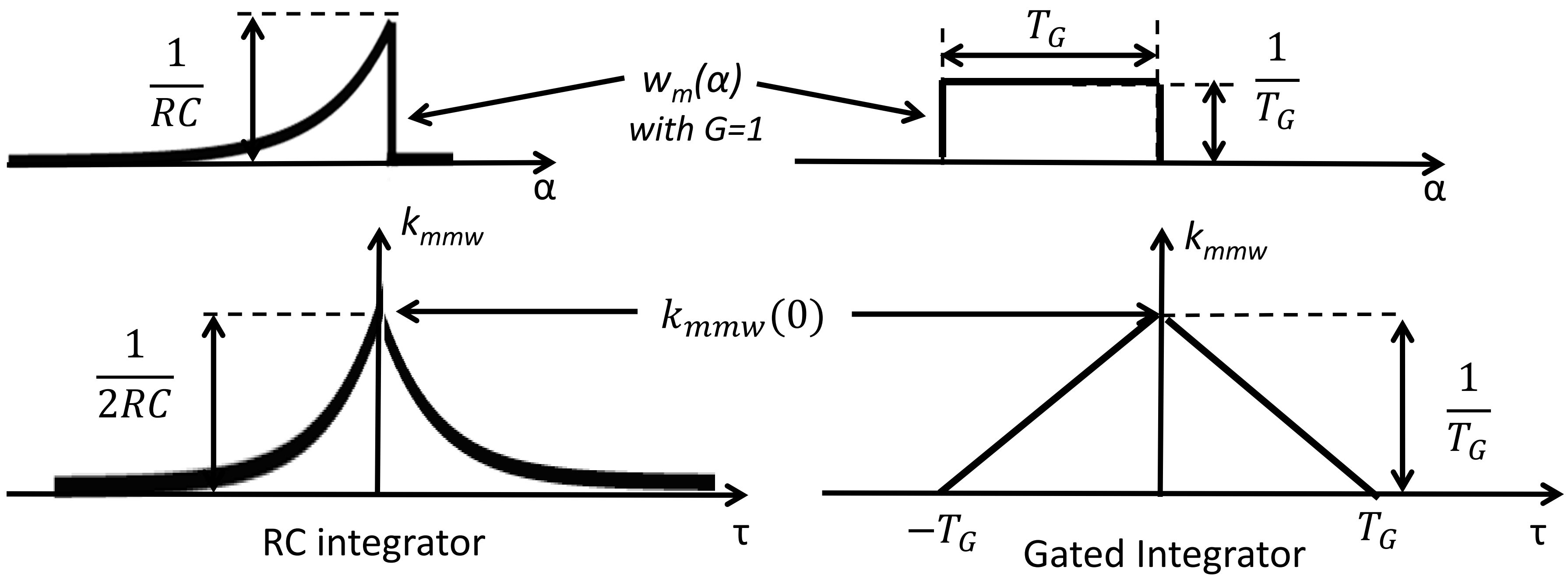
For a constant-parameter RC (inherently with G=1) that filters the same wide-band noise S_b it is

$$\left(\overline{x_n{}^2}\right)_{RC} = \left(\overline{y_n{}^2}\right)_{RC} = \frac{S_b}{2RC}$$

Therefore, as concerns the S/N obtained for input DC signals accompanied by wide-band noise, GI and RC integrator are equivalent if

$$T_G = 2RC$$

GI and equivalent RC-integrator



We consider here filters with **equal DC gain of unity**, hence with equal output signal.

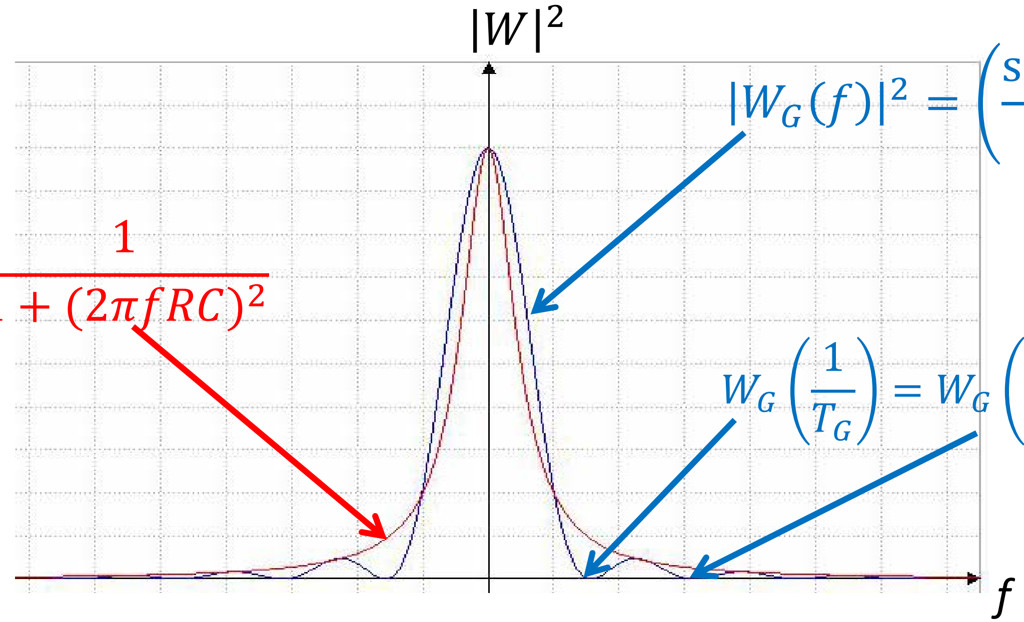
With wide-band input noise S_b the output noise is

$$\overline{y^2} = S_b \cdot k_{mmw}\ (0)$$

therefore, GI and RC have **equal output noise** if

$$\boxed{T_G = 2RC}$$

GI and equivalent RC-integrator



$$|W_{RC}(f)|^2 = \frac{1}{1 + (2\pi f RC)^2}$$

$$|W_G(f)|^2 = \left(\frac{\sin \pi f T_G}{\pi f T_G}\right)^2$$

$$W_G\left(\frac{1}{T_G}\right) = W_G\left(\frac{2}{T_G}\right) = \cdots = 0$$

With $T_G = 2RC$ they are equivalent for:

- the S/N obtained with wide-band noise and DC signal input
- the attenuation of high-frequency disturbances in general

However:

- The GI has zeros of $W_G(f)$ at $f_k = k/T_G$ that can be exploited to cancel specific disturbances at known frequencies (radio frequencies or mains frequency and harmonics)

GI as fast sampler - Readout Noise

Ultrafast samplers that acquire signals in time intervals much shorter than the acquisition times of S&H are in fact gated integrators with ultrashort gate time T_G . Consequently they have DC gain much smaller than unity G<<1, **typically G≈ 0,01**.

The READOUT NOISE of such GI-samplers has the same source as the S&H (the Johnson noise $S_b = 2kTR$ of the internal resistance R) but it has a much stronger effect. The readout noise at the **GI-sampler output** is

$$\overline{y_n^2} = S_b \frac{T_G}{T_f^2} = 2kTR \frac{T_G}{T_f^2} = 2\frac{kT}{C} G \qquad \text{(with } T_f = RC\text{)}$$

but the **noise referred to the input** is

$$\frac{\overline{y_n{}^2}}{G^2} = \frac{S_b}{T_G} = \frac{kT}{C} \cdot \frac{2}{G}$$

which because of the very low G value is **much higher than that of a S&H**

$$\frac{\overline{y_n^2}}{G^2} = \frac{kT}{C} \cdot \frac{2}{G} \gg \frac{kT}{C} \qquad \textbf{typically} \qquad \frac{\overline{y_n^2}}{G^2} \approx 100 \frac{kT}{C}$$