Sensors, Signals and Noise

COURSE OUTLINE

- Introduction
- Signals and Noise
- Filtering: OPF2 Optimum Filtering 2
- Sensors and associated electronics

Optimum Filtering for High-Impedance Sensors

- High impedance sensors and low-noise preamplifiers
- Noise whitening filter
- Matched filter
- Optimum filtering for measuring the charge of pulse signals
- Optimum Filtering with Finite Readout Time
- Practical approximations of the optimum filtering

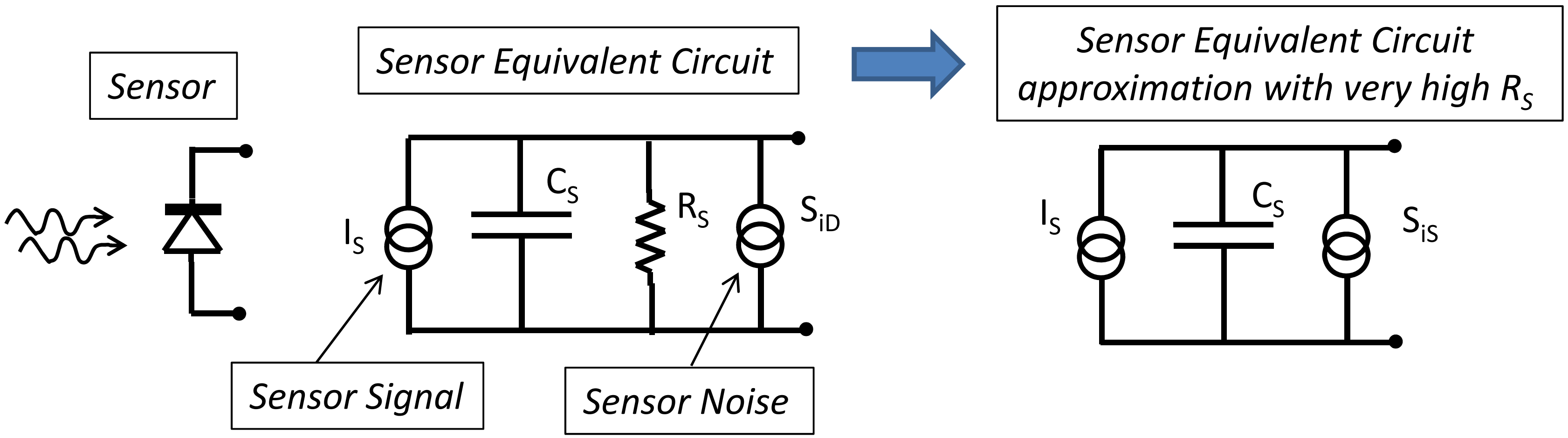
and for those who want to gain a better insight

- Appendix 1 : Noise whitening filter in case of finite resistance

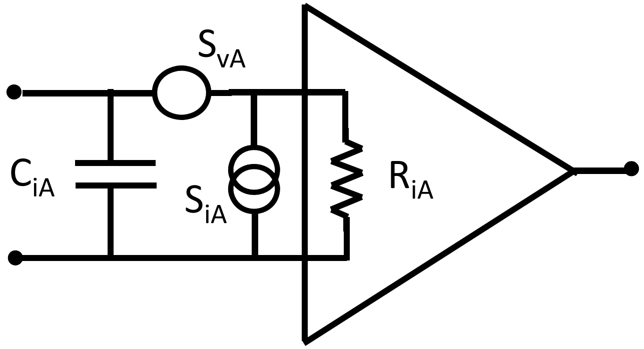
High Impedance Sensors and Low-Noise Preamplifiers

High-Impedance Sensors

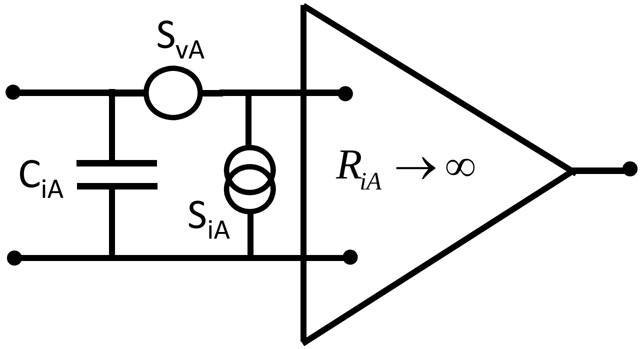
- Let us consider sensors that are seen by the circuits connected to their terminals as generators of current signals with high internal impedance (typically a small capacitance C_S with a high resistance R_S in parallel)
- Typical examples are: p-i-n junction photodiodes and other photodetectors (CCDs, vacuum tube photodiodes, etc.); Detectors of Ionizing Radiation (semiconductor devices and gas-filled chambers); piezoelectric Force Sensors in quartz or other piezoelectric ceramic materials
- They have internal noise sources (e.g. shot current noise of a junction reverse current) modeled by a current noise generator in parallel to the signal generator



High-Impedance Sensors and Low-Noise Preamplifiers



Preamplifier equivalent circuit



Approximation with very high R_{iA}

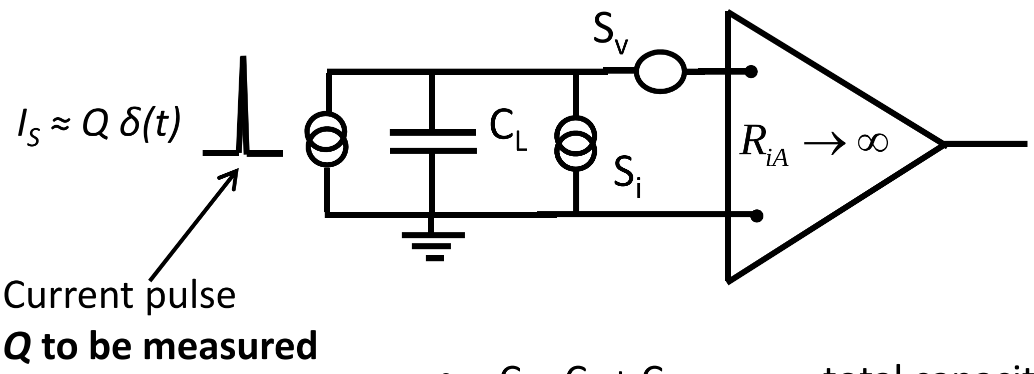
- R_{iA} = **true physical resistance** between the input terminals (NOT the dynamic input resistance modified by the feedback in the amplifier; e.g. not the low dynamic resistance of the virtual ground of an operational amplifier)
- Besides shot noise of bias currents, the S_{iA} includes Johnson resistor noise of R_{iA}

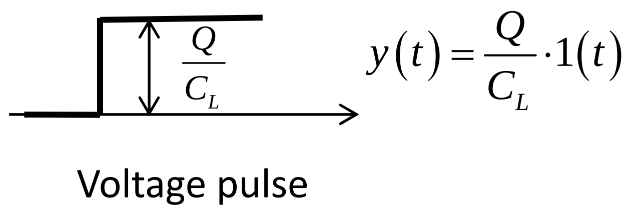
  $$\boldsymbol{S_{iR} = \frac{4kT}{R_i}}$$

- The current noise directly faces the sensor current signal I_S
  if R_{iA} is small the S_{iR} is overwhelming (e.g. with $R_{iA} = 50\,\Omega$ it is $\sqrt{S_{iR}} \approx 18\,pA/\sqrt{Hz}$)
  and other components of $\sqrt{S_{iA}}$ are much lower (about $1\,pA/\sqrt{Hz}$ or lower)
- Conclusion: for **low-noise** operation of **high-impedance sensors**,
  it is **mandatory to employ a preamplifier with high input resistance R_{iA}**

High-Impedance Sensors and Low-Noise Preamplifiers

Equivalent circuit of high-impedance Sensor and Preamplifier
(approximation valid for **very high** sensor resistance $R_S \to \infty$)





- $C_L = C_S + C_{iA}$ total capacitance load
- $S_v = S_{vA}$ voltage noise generator (wideband white spectrum)
- $S_i = S_{iD} + S_{iA}$ current noise generator (wideband white spectrum)

At the preamplifier output:

- The voltage noise spectrum S_n has two components, it is **NOT white**

$$\boxed{S_n(\omega) = S_V + \frac{S_i}{\omega^2 C_L^2}}$$

- The voltage signal is a step with amplitude Q/C_L

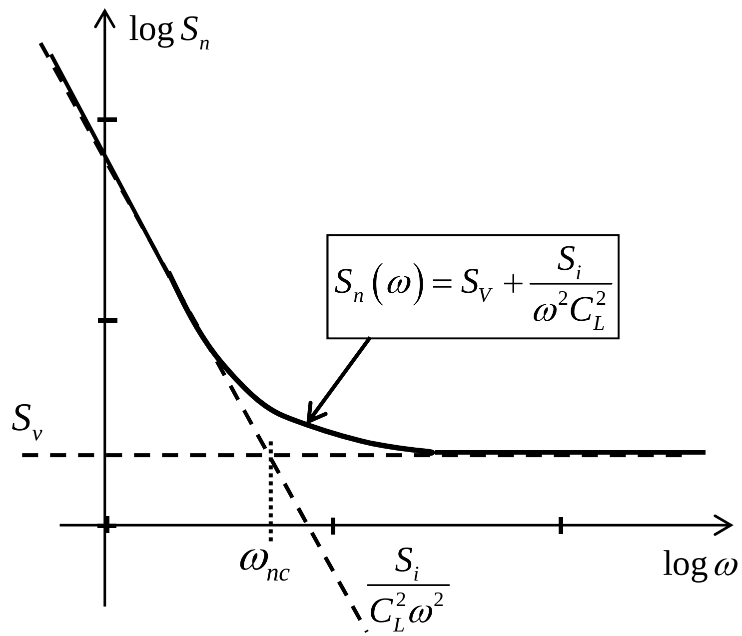
Noise Spectrum at Preamplifier Output

Crossing of the component defines
ω_{nc} Noise-Corner angular frequency

$$S_v = \frac{S_i}{C_L^2 \omega_{nc}^2} \Rightarrow \boxed{\omega_{nc} = \frac{\sqrt{S_i}}{C_L \sqrt{S_v}}}$$

$T_{nc} = 1/\omega_{nc}$ Noise-Corner time constant

$$\boxed{T_{nc} = \frac{1}{\omega_{nc}} = \frac{\sqrt{S_v}}{\sqrt{S_i}} C_L}$$



T_{nc} and ω_{nc} are fundamental parameters of the optimum filter: we will see that T_{nc} rules the duration of the filter weighting and ω_{nc} the filter bandlimit

We define the **Noise Corner resistance** $\boxed{R_{nc} = \sqrt{S_v} / \sqrt{S_i}}$ so that $\boxed{T_{nc} = R_{nc} C_L}$

- with $\sqrt{S_v}$ a few $nV/\sqrt{Hz}$ and $\sqrt{S_i}$ ranging from a few 0,1 to 0,01 $pA/\sqrt{Hz}$

  ***R_{nc}* ranges from tens to hundreds of kOhms**

- with C_L from 0,1 pF to a few pF

  ***T_{nc}* ranges from a few nanoseconds to some hundreds of nanoseconds**

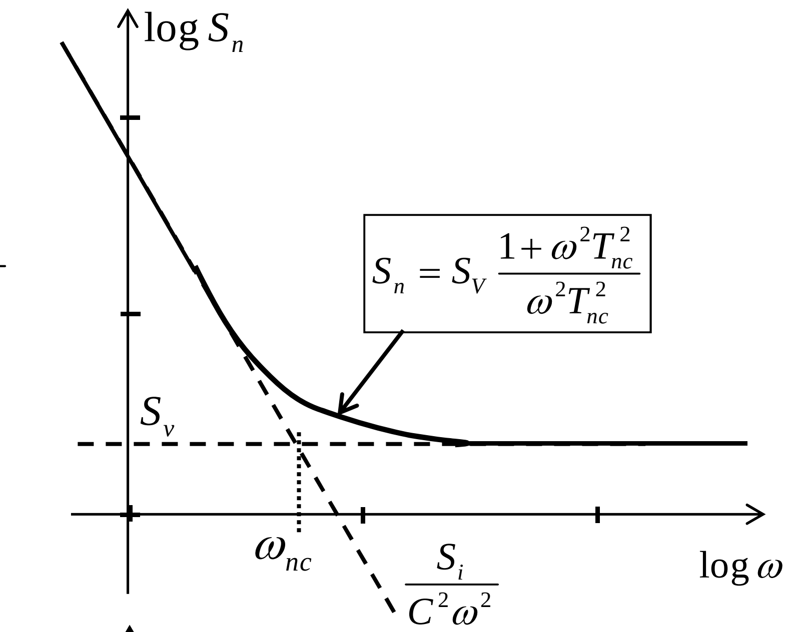
Noise whitening filter

Noise-Whitening Filter

The noise spectrum has

- a pole at $\omega_p = 0$
- a zero at $\omega_z = \omega_{nc} = 1/T_{nc}$

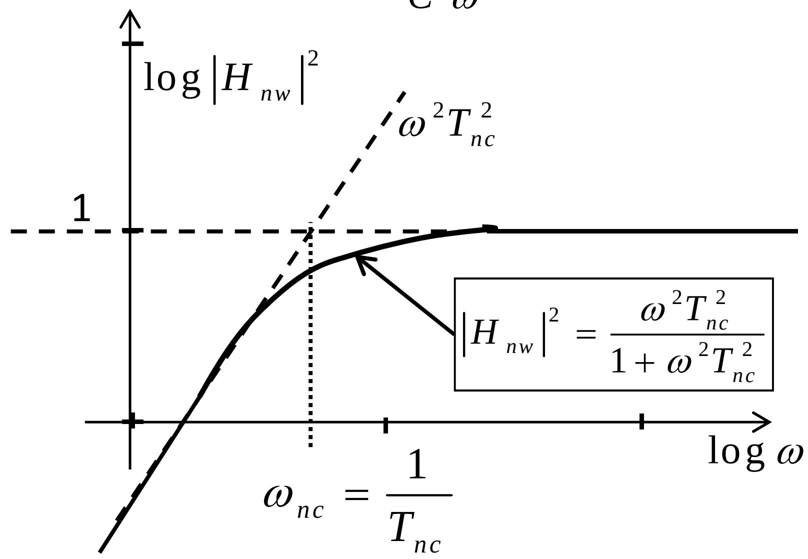
$$S_n(\omega) = S_V\left(1+\frac{S_i}{\omega^2 S_V C_L^2}\right) = S_V\left(1+\frac{1}{\omega^2 T_{nc}^2}\right) = S_V\,\frac{1+\omega^2 T_{nc}^2}{\omega^2 T_{nc}^2}$$



The **noise whitening filter H_{nw}** must

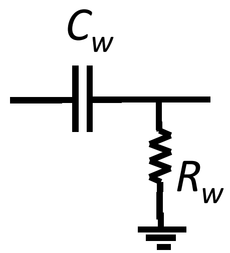
- cancel the pole with a zero at $\omega = 0$
- cancel the zero with a pole at $\omega = \omega_{nc} = 1/T_{nc}$

$$\boxed{\left|H_{nw}(\omega)\right|^2 = \frac{\omega^2 T_{nc}^2}{1+\omega^2 T_{nc}^2}}$$



It is a simple high-pass filter

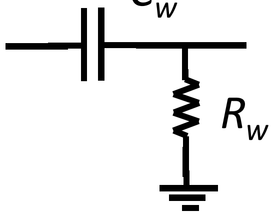
$$H_{nw}(\omega) = \frac{j\omega R_w C_w}{1+j\omega R_w C_w}$$



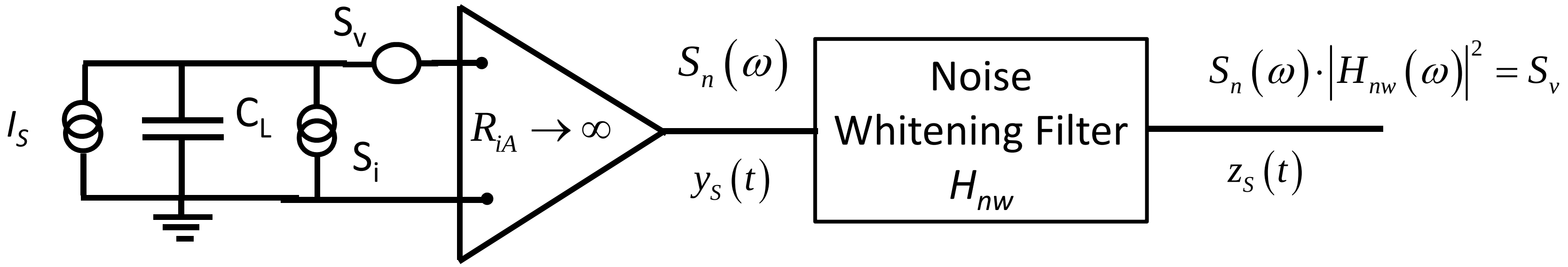
$$\boxed{\textbf{with}\quad R_w C_w = T_{nc}}$$

Action of the Noise-Whitening Filter

Simple high-pass CR filter



$$\boxed{H_{nw}(s) = \frac{sR_wC_w}{1+sR_wC_w}} \quad \text{with} \quad \boxed{R_wC_w = T_{nc}}$$

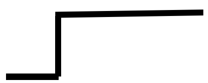


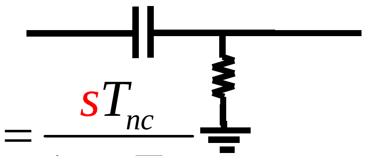
it makes **white** the noise at its **output**

and changes the signal into a short **exponential pulse with time-constant T_{nc}**

NW Filter

$$y_S(t) = \frac{Q}{C_L}\cdot 1(t)$$





$$H = \frac{sT_{nc}}{1+sT_{nc}}$$

$$z_S(t) = \frac{Q}{C_L}\cdot 1(t)\cdot \exp\left(-\frac{t}{T_{nc}}\right)$$

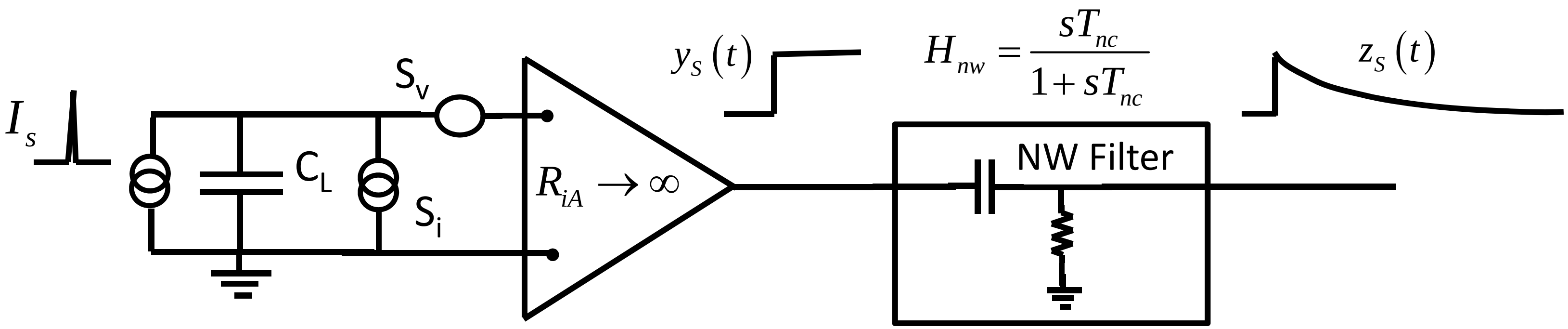
$$Y_S(s) = \frac{Q}{C_L}\cdot\frac{1}{s}$$

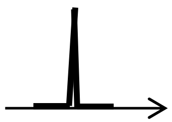
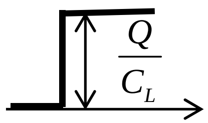
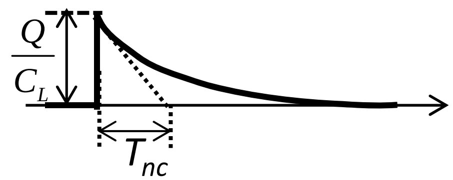
$$Z_S(s) = \frac{Q}{C_L}T_{nc}\cdot\frac{1}{1+sT_{nc}}$$

Cancels pole at *s=0*

and replaces it by pole at *s = - 1/T_{nc}*

Signal at the output of the Noise-Whitening Filter

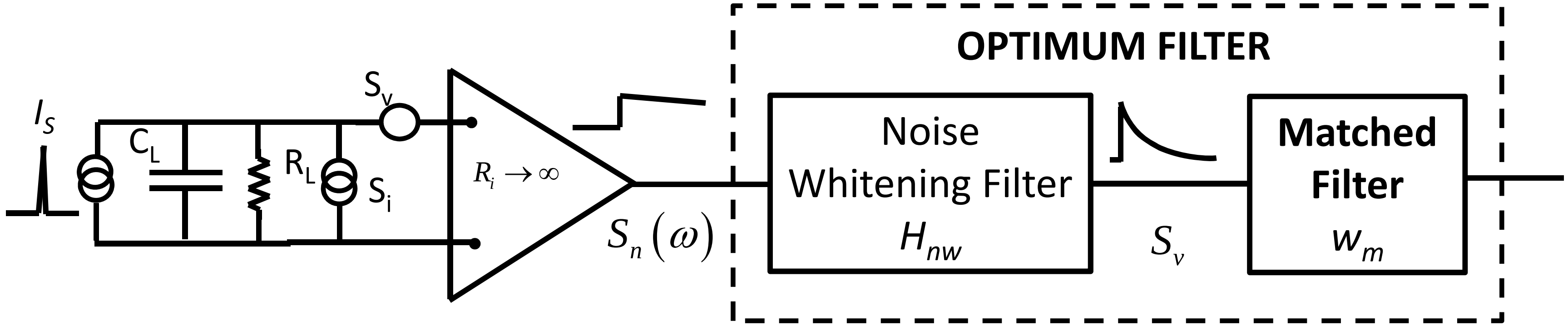


Input (current)	Preamp Output (voltage)	NW Filter Output (voltage)
δ – pulse 	Step pulse 	Exponential pulse 
$I_s(t) = Q \cdot \delta(t)$	$y_S(t) = \frac{Q}{C_L} \cdot 1(t)$	$z_S(t) = \frac{Q}{C_L} \cdot 1(t) \cdot \exp\left(-\frac{t}{T_{nc}}\right)$
$I_s(s) = Q$	$Y_S(s) = \frac{Q}{C_L} \cdot \frac{1}{s}$	$Z_S(s) = \frac{QT_{nc}}{C_L} \frac{1}{1+sT_{nc}}$

The **matched** filter has to be tailored to the signal at the whitening filter output, i.e. it must have **weighting function exponential with time constant T_{nc}**

Matched Filter

The Matched Filter completes the Optimum Filtering



The case with finite load resistance R_L is treated in Appendix 1 , showing that the whitening filter is different but the output signal produced is the same as with $R_L \rightarrow \infty$

$$z_S(t) = \frac{Q}{C_L} \cdot 1(t) \cdot \exp\left(-\frac{t}{T_{nc}}\right)$$

Therefore, the matched filter is the same in the two cases

$$\boxed{w_M = 1(t) \cdot \frac{1}{T_{nc}} \exp\left(-\frac{t}{T_{nc}}\right)}$$

and gives the same result

$$\eta_o^2 = \left(\frac{S}{N}\right)^2_{opt} = \frac{\left[\int_{-\infty}^{\infty} z_s(\alpha) w_m(\alpha) d\alpha\right]^2}{S_v \cdot \int_{-\infty}^{\infty} w_m^2(\alpha) d\alpha} = \frac{Q^2}{C_L^2} \frac{T_{nc}^2}{S_v} \int_{-\infty}^{\infty} w_m^2(\alpha) d\alpha = \frac{Q^2}{C_L^2} \frac{1}{2} \frac{T_{nc}}{S_v}$$

Optimum Filtering

At the output of the optimum filter (i.e. of the matched filter) we have

Signal $$s_o = \int_0^\infty z_s(\alpha)\, w_m(\alpha)\, d\alpha = \frac{Q}{C_L}\frac{1}{T_{nc}}\int_0^\infty \exp\left(-\frac{2\alpha}{T_{nc}}\right) d\alpha = \frac{1}{2}\frac{Q}{C_L}$$

Noise $$\sqrt{\overline{n_o^2}} = \sqrt{S_v}\cdot\sqrt{k_{ww}(0)} = \sqrt{S_v}\,\frac{1}{T_{nc}}\sqrt{\int_0^\infty \exp\left(-\frac{2\alpha}{T_{nc}}\right) d\alpha} = \sqrt{\frac{S_v}{2T_{nc}}}$$

S/N $$\boxed{\eta_o = \frac{s_o}{\sqrt{\overline{n_o^2}}} = \frac{1}{2}\frac{Q}{C_L}\sqrt{\frac{2T_{nc}}{S_v}}}$$

- The optimum filter theory specifies what is the best S/N physically obtainable and the kind of filter required for attaining it
- The optimum filter can be implemented in reality, but the filter design turns out to be quite complex
- Furthermore, the optimum filter takes infinite time (after the signal onset) to complete its action, which is not acceptable in practice
- It is possible, however, to consider and evaluate fairly simple filters that approximate the optimum filter and closely approach its performance

Optimized Noise Charge

We want measure the signal charge Q , so we consider the noise in terms of charge q_{no}^2

$$\eta_o^2 = Q^2 \frac{T_{nc}}{2C_L^2 S_v} = \frac{Q^2}{q_{no}^2}$$

We have

$$\boxed{q_{no}^2 = \frac{2C_L^2 S_v}{T_{nc}}}$$

Recalling $T_{nc} = C_L R_{nc} = C_L \left(\sqrt{S_v} \middle/ \sqrt{S_i}\right)$ we can express q_{no}^2 in terms of the main parameters of the sensor and preamplifier

$$\boxed{q_{no}^2 = 2C_L \cdot \sqrt{S_v} \cdot \sqrt{S_i}}$$

q_{no}^2 is the charge pulse for which S/N= 1. It represents the minimum measurable pulse Q_{min} and the minimum difference between two distinguishable pulses.

The very small size of q_{no}^2 is better appreciated if expressed in number of electron charges (NB electron charge $e = 1{,}6 \cdot 10^{-19}$ *C)*

$$\sqrt{N_{eo}^2} = \frac{\sqrt{q_{no}^2}}{e} = \frac{\sqrt{2C_L \sqrt{S_v} \sqrt{S_i}}}{e} = 278 \cdot \sqrt{C_L \left[pF \right] \cdot S_v^{1/2} \left[nV \middle/ \sqrt{Hz} \right] \cdot S_i^{1/2} \left[pA \middle/ \sqrt{Hz} \right]}$$

Optimized Noise Charge Examples

It is instructive to evaluate and compare typical values of the optimized noise charge in typical cases of PIN photodiodes coupled to high-impedance preamplifiers.

a) PIN with **wide** sensitive area (≈1 mm diameter) and discrete-component preamplifier with **moderately low-noise**. Fairly high capacitance of PIN & connections ($C_L \approx 10pF$) and not-so-low preamp noise (typical $S_v^{1/2} \approx 4\, nV/\sqrt{Hz}$; $S_i^{1/2} \approx 1\, pA/\sqrt{Hz}$) set a fairly high limit

$$\sqrt{\overline{N_{eo}^2}} \approx 1800 \text{ electrons}$$

b) PIN with **reduced** sensitive area (≈ 100 µm diameter) and discrete-component preamplifier with **low-noise**. Reduced capacitance of PIN & connections ($C_L \approx 1pF$) and reduced preamp noise (typical $S_v^{1/2} \approx 2\, nV/\sqrt{Hz}$; $S_i^{1/2} \approx 0{,}1\, pA/\sqrt{Hz}$) bring a strong reduction

$$\sqrt{\overline{N_{eo}^2}} \approx 120 \text{ electrons}$$

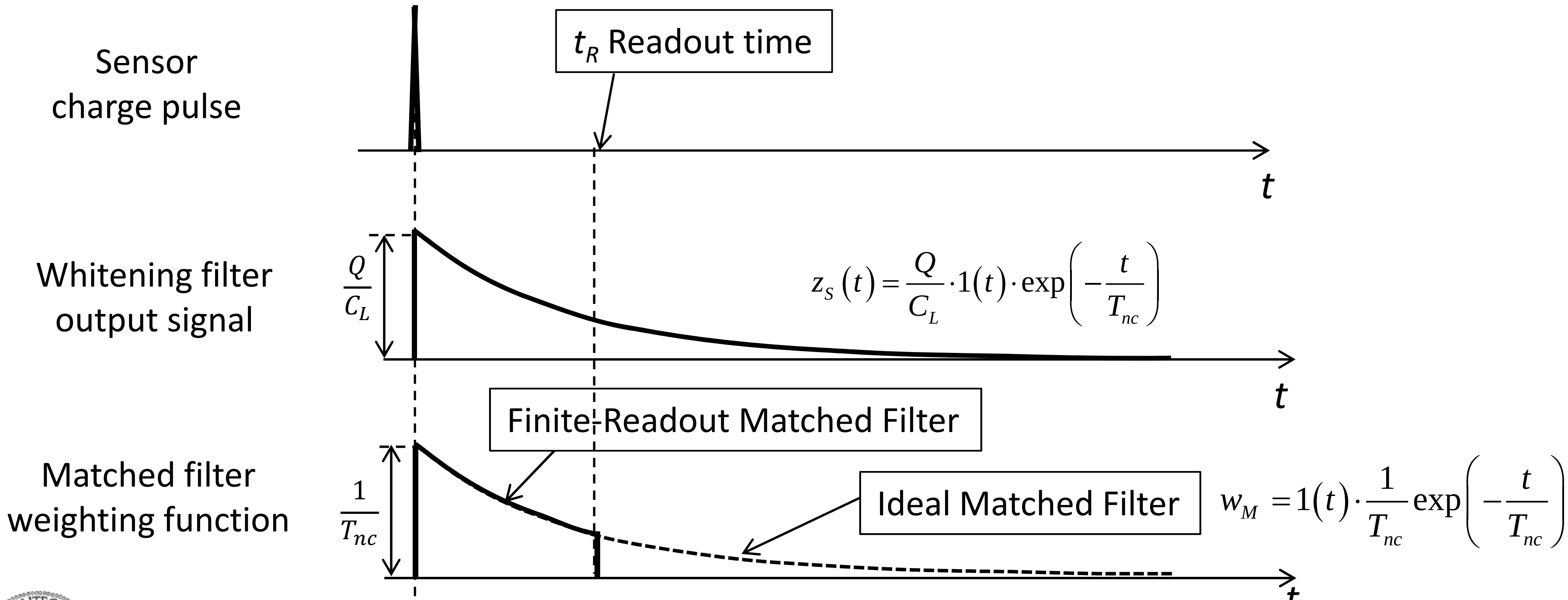
c) PIN with **small** sensitive area (≈ 20 µm diameter) **integrated** with a **very low-noise preamplifier**. Capacitance minimization by monolithic integration ($C_L \approx 0{,}1pF$) and very low preamp noise ($S_v^{1/2} \approx 2\, nV/\sqrt{Hz}$; $S_i^{1/2} \approx 0{,}01\, pA/\sqrt{Hz}$) bring further progress

$$\sqrt{\overline{N_{eo}^2}} \approx 10 \text{ electrons}$$

Optimum Filtering with Finite Readout Time

Optimum Filtering with Finite Readout Time

- The exponential weighting function w(t) of the matched filter reaches zero only at t → ∞, so that the measurement of the pulse in principle is available with infinite delay after the pulse onset.
- However, most of the weight of the matched filter is over a finite time interval of a few time constants T_{nc} . We can introduce the constraint of terminating the weight at a finite time t_R and evaluate the result of this finite-time-readout matched filter



Optimum Filtering with Finite Readout Time

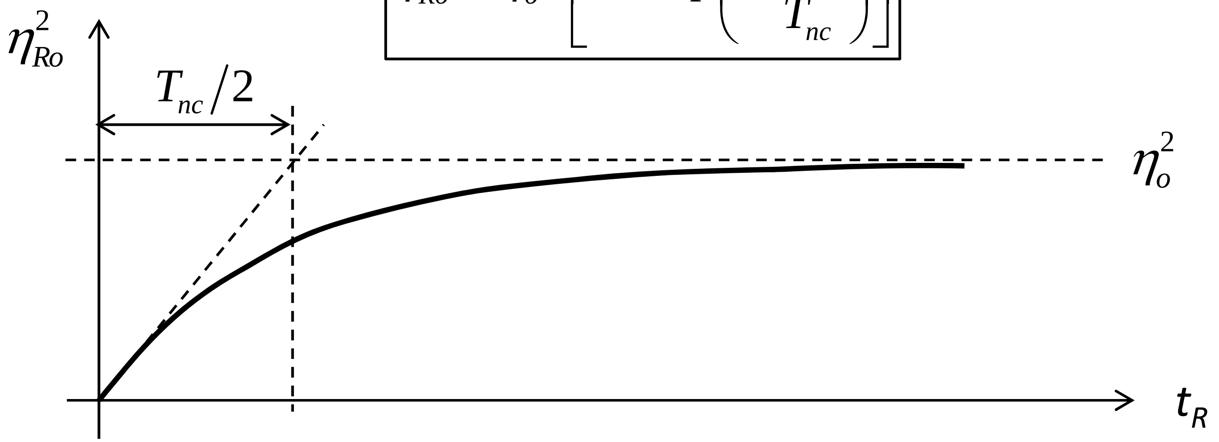
The constraint of ensuring a finite readout time t_R forces to terminate the weighting at t_R and leads to

NB: No more ∞ !!

$$\eta_R^2 = \left(\frac{S}{N}\right)^2_{opt,R} = \frac{Q^2}{C_L^2}\frac{T_{nc}^2}{S_v}\int_{-\infty}^{t_R} w_m^2(\alpha)\,d\alpha = \frac{Q^2}{C_L^2}\frac{1}{S_v}\int_0^{t_R}\exp\left(-\frac{2\alpha}{T_{nc}}\right)d\alpha = \frac{Q^2}{C_L^2}\frac{T_{nc}}{2S_v}\left[1-\exp\left(-\frac{2t_R}{T_{nc}}\right)\right]$$

that is

$$\boxed{\eta_{Ro}^2 = \eta_o^2\cdot\left[1-\exp\left(-\frac{2t_R}{T_{nc}}\right)\right]}$$



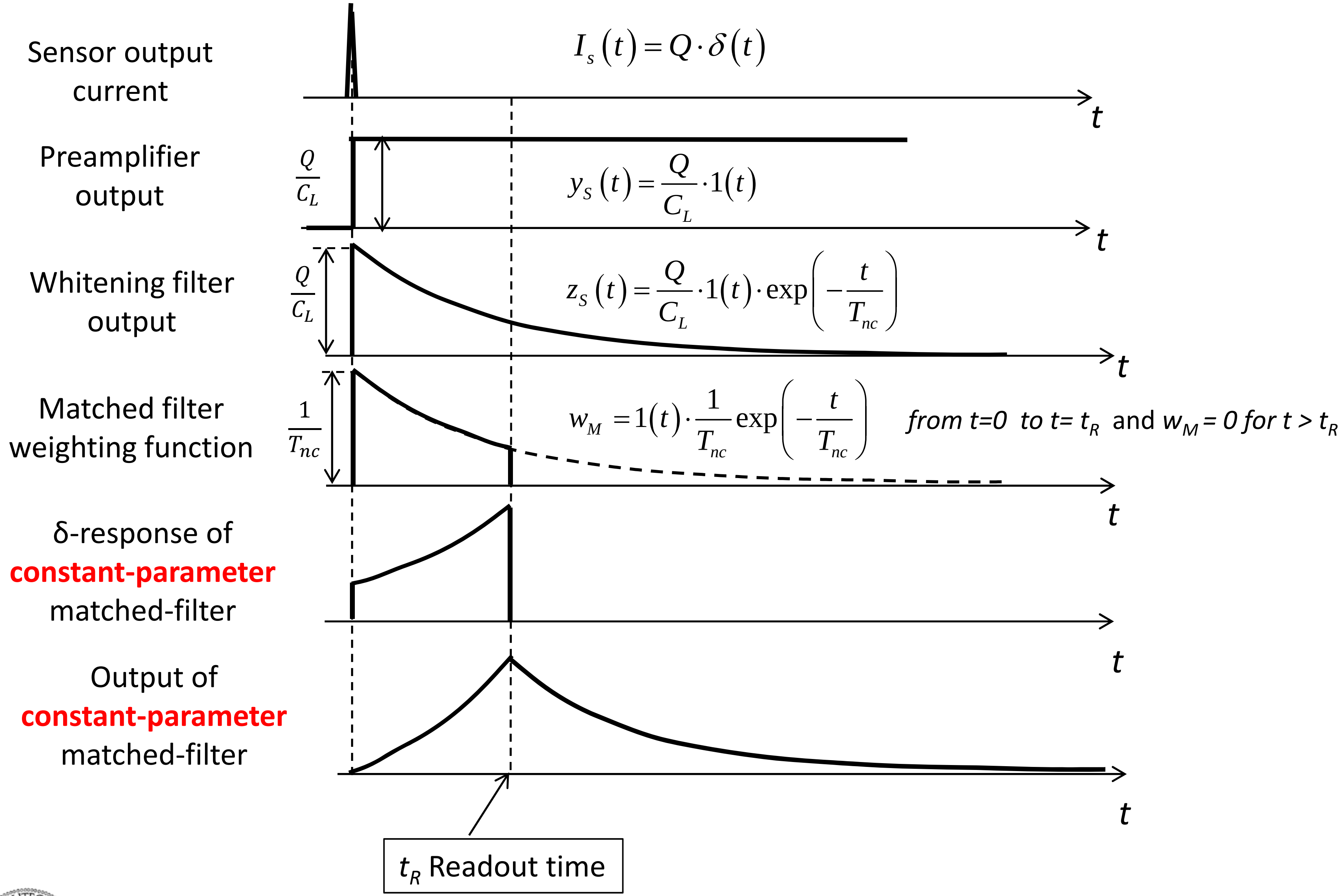
The absolute optimum η_o is closely approximated in a reasonable readout time t_R :

at $t_R \approx T_{nc}$ $\quad \eta_{Ro} \approx \eta_o \cdot 0{,}929$ $\quad$ i.e. η_{Ro} ≈ 7 % lower than η_o

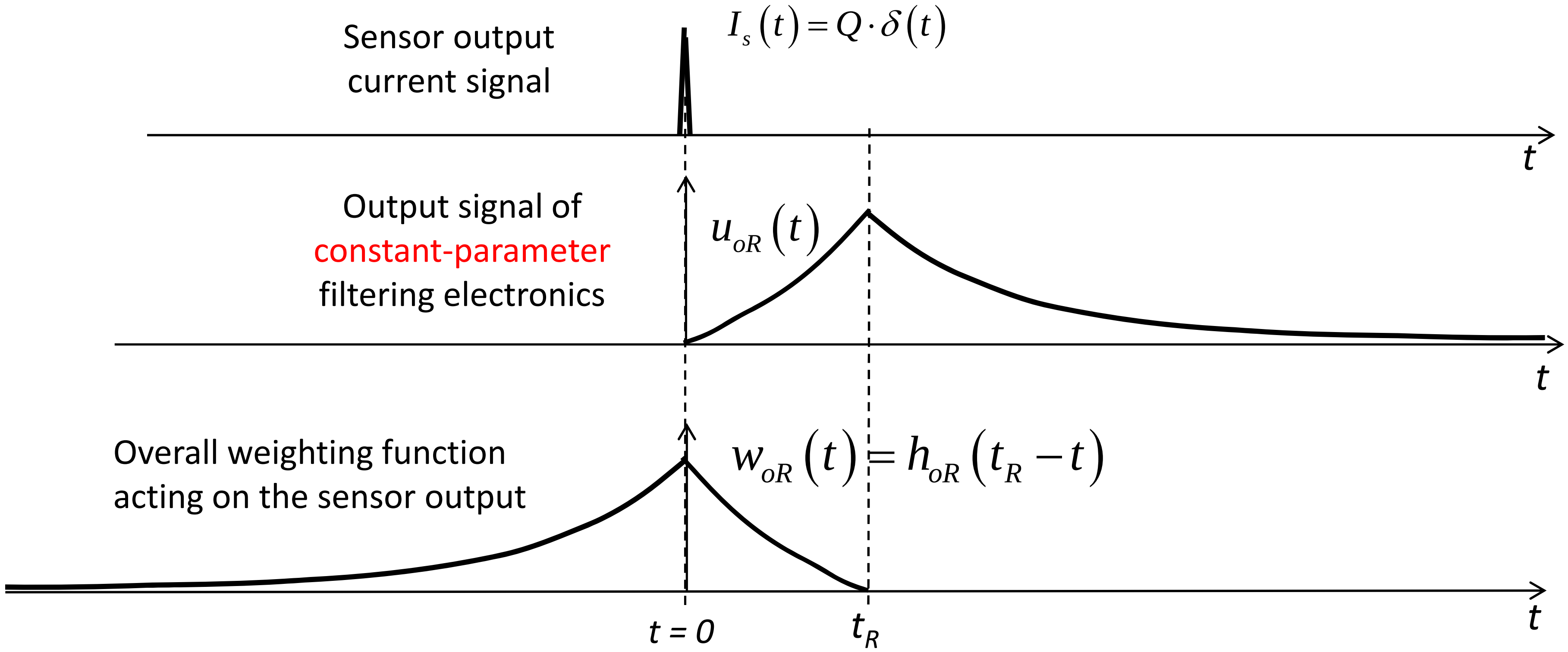
at $t_R \approx 2\,T_{nc}$ $\quad \eta_{Ro} \approx \eta_o \cdot 0{,}991$ $\quad$ i.e. η_{Ro} ≈ 1 % lower than η_o

at $t_R \approx 4\,T_{nc}$ $\quad \eta_{Ro} \approx \eta_o \cdot 0{,}9999$ $\quad$ i.e. η_{Ro} ≈ 0,01 % lower than η_o

Optimum Filtering with Finite Readout Time



Optimum Filtering with Finite Readout Time



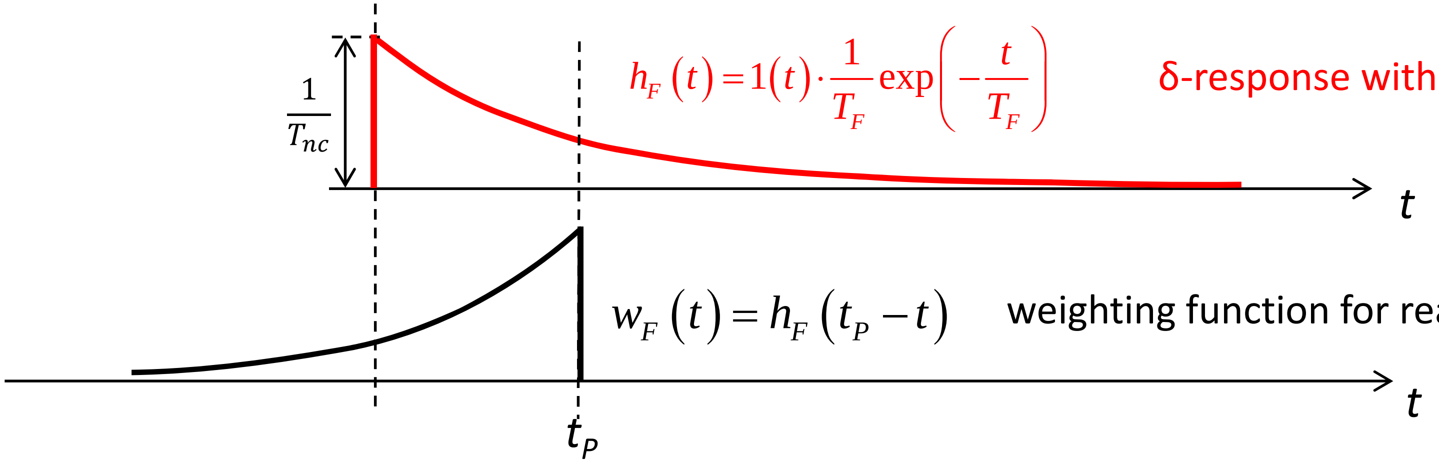
Note that:

- The weight is maximum at *t = 0*, i.e. in correspondence to the sensor pulse
- Weighting is limited to t_R on the $t > 0$ side
- Weighting is not limited on the t < 0 side

Practical approximations of the optimum filtering

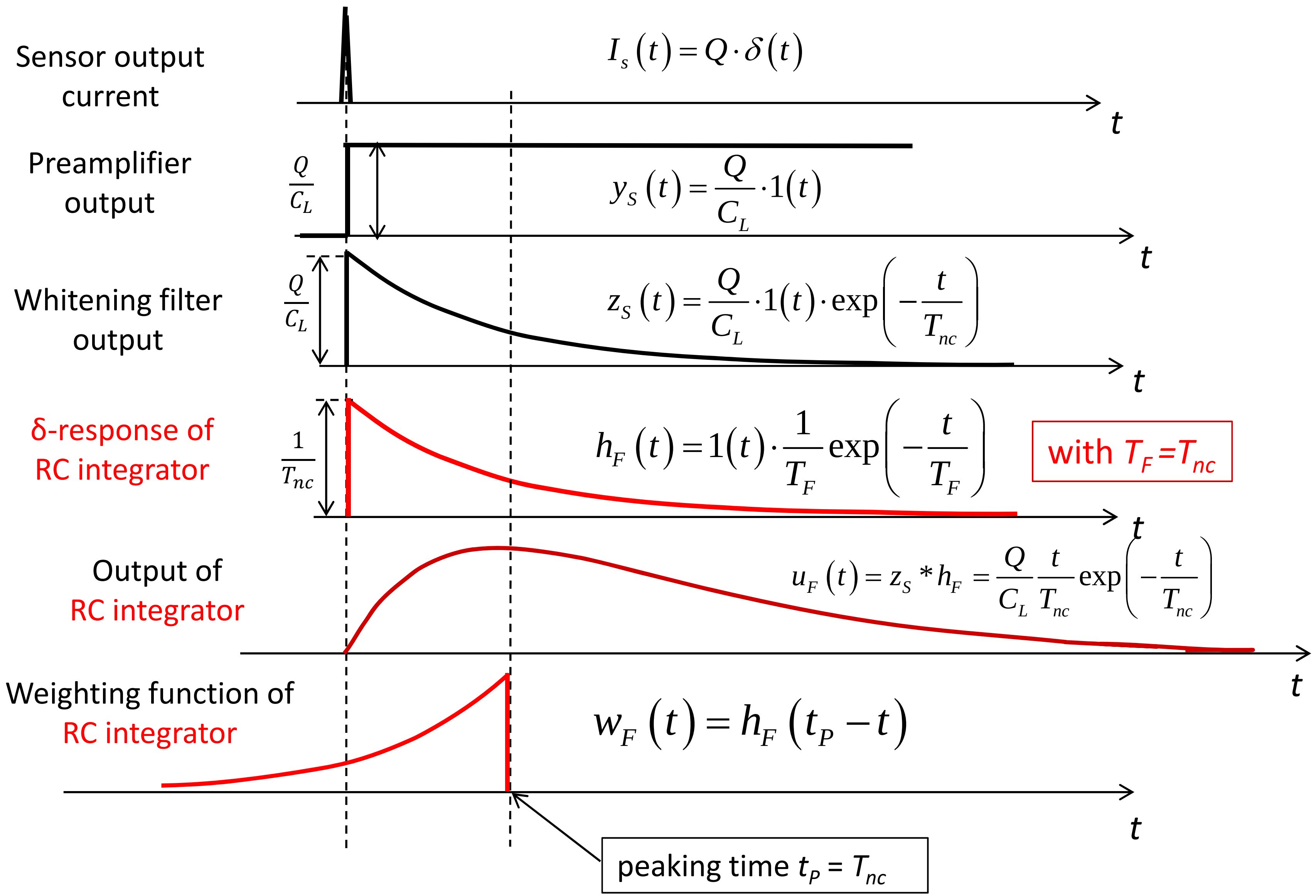
RC integrator Approximation of the Matched Filter

- The whitening filter is simple and easily and exactly implemented. For completing the optimum filter it is sufficient to find out how to approximate the matched filter.
- The features of the matched filter weighting function observed in time and in frequency point out that it is a low-pass filter
- A simple **RC integrator** (single-pole low-pass filter) can be an **approximation** of the matched filter. With $RC = T_{nc}$ its δ-response $h_F(t)$ is identical to the weighting function $w_M(t)$ of the matched filter. The **RC weighting** $w_F(t)$ has the same shape as $w_M(t)$ of the matched filter, but it's not fully correct because it is **reversed in time**!

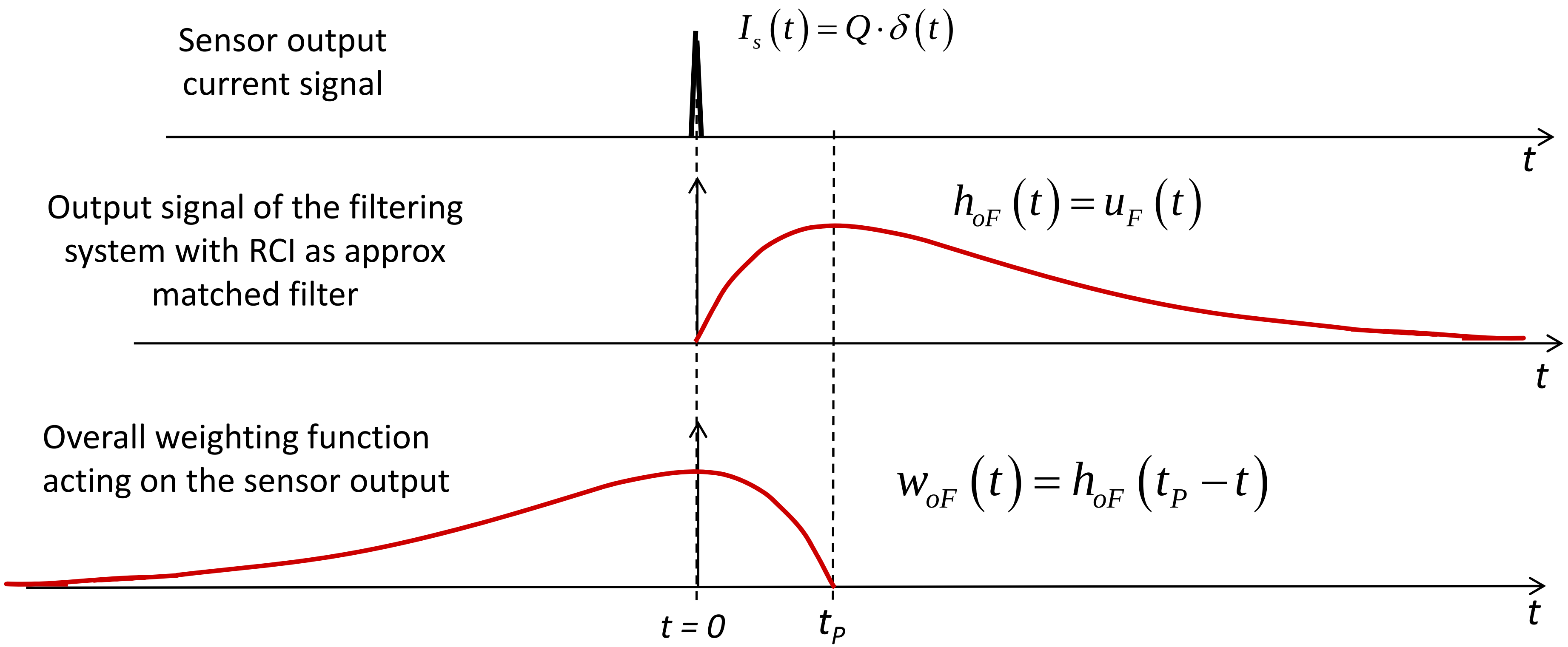


- **Noise** filtering is **equal to the matched filter**, since it is unaffected by time-inversion; the output is white noise with band-limit set by a simple pole with time-constant T_F .
- **Signal** filtering is **different** from the matched filter, since it is modified by time-inversion

RC integrator Approximation of the Matched Filter



Filtering system with RC as approx matched filter



Note that:

- The weight is maximum at *t = 0*, i.e. in correspondence to the sensor pulse
- Weighting is limited to t_P on the $t > 0$ side
- Weighting is not limited on the t < 0 side

(see for comparison the optimum filtering in slide 26)

RC integrator Approximation compared to the Optimum Filter

The RC output signal waveform is $u_F(t) = \frac{Q}{C_L} \frac{t}{T_{nc}} \exp\left(-\frac{t}{T_{nc}}\right)$

Signal peak value (at $t = T_{nc}$) $s_F = u_F(T_{nc}) = \frac{1}{e} \frac{Q}{C_L}$

Noise $\sqrt{\overline{n_F^2}} = \sqrt{S_v} \cdot \sqrt{k_{hh}(0)} = \sqrt{\frac{S_v}{2T_{nc}}}$

S/N $\eta_F = \frac{s_F}{\sqrt{\overline{n_F^2}}} = \frac{1}{e} \frac{Q}{C_L} \sqrt{\frac{2T_{nc}}{S_v}}$

Comparing the RC approximation with the ideal optimum filter system we see that

$s_F = \frac{2}{e} s_o \approx 0,736 \cdot s_o$ the signal is lower

$\sqrt{\overline{n_F^2}} = \sqrt{\overline{n_o^2}}$ the noise is equal

$\eta_F = \frac{2}{e} \eta_o \approx 0,736 \cdot \eta_o$ the S/N is lower

the performance of the filter system with RC approximation of matched filter is about 27% worse than the absolute optimum.
Note that the loss is due to bad exploitation of the signal

RC integrator Approximation compared to Finite-Readout Optimum Filter

- The filter system with RC approximation of the matched filter has finite readout time $t_R = T_{nc}$ (the output pulse peak amplitude is measured at $t = T_{nc}$).
- The ideal optimum filter has infinite readout time.
- A more objective assessment of the RC approximation is obtained by taking as reference the optimum filter with the constraint of finite readout time $t_R = T_{nc}$.
- The optimum filter with the constraint of finite readout time t_R gives

$$\eta_{Ro}(t_R) = \eta_o \cdot \sqrt{1 - \exp\left(-\frac{2t_R}{T_{nc}}\right)}$$

- in our case with $t_R = T_{nc}$ it is

$$\eta_{Ro}(T_{nc}) = \eta_o \cdot 0{,}929$$

- In conclusion

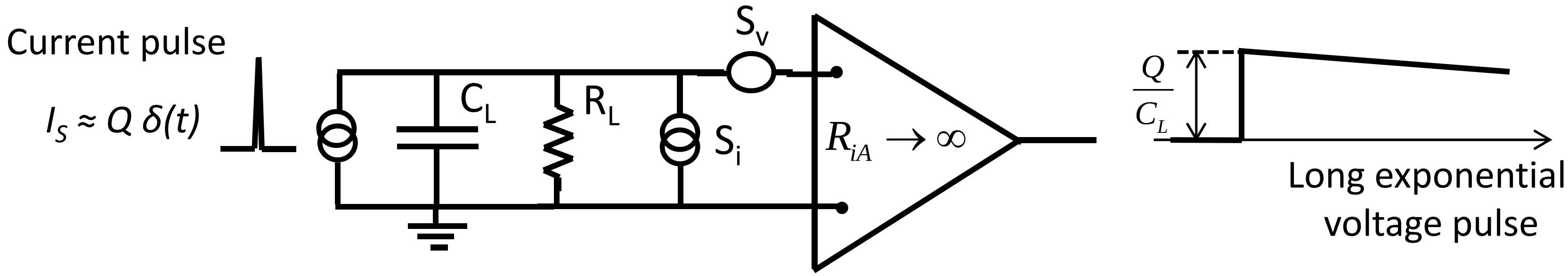
$$\eta_F = \frac{2}{e}\eta_o \approx 0{,}736 \cdot \eta_o = 0{,}79 \cdot \eta_{Ro}(T_{nc})$$

  the performance of the RCI approximation is about 21% worse than the optimum filter with the constraint of finite readout time $t_R = T_{nc}$
- This performance is remarkably lower than the optimum, but the RC is just a crude approximation: better results can be obtained with more sophisticated filter design

Appendix 1
Noise whitening filter:
case with finite load resistance

High-Impedance Sensor and Preamplifier

Equivalent circuit of a high-impedance Sensor and Preamplifier configuration with finite load resistance R_L



R_L = total resistance of sensor and load

$T_L = R_L C_L$ time constant of the load

At the preamplifier output:

- The voltage noise spectrum S_n has two components and is **NOT white**

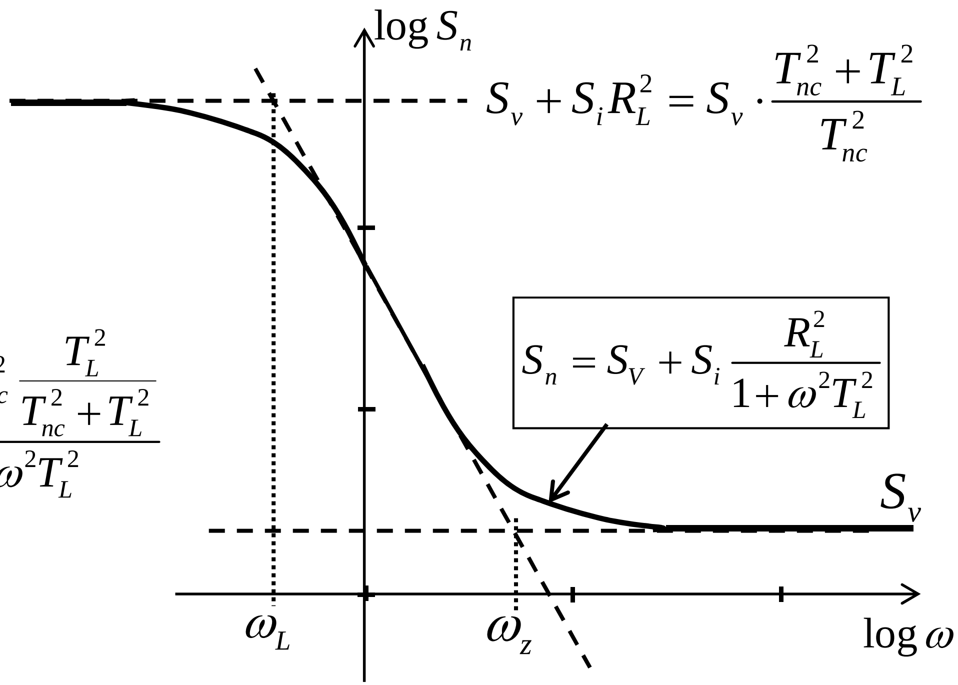
$$\boxed{S_n(\omega) = S_V + S_i \frac{R_L^2}{1+\omega^2 R_L^2 C_L^2} = S_V + S_i \frac{R_L^2}{1+\omega^2 T_L^2}}$$

- The output voltage signal is an exponential pulse with long time constant $T_L = R_L C_L$

$$y(t) = \frac{Q}{C_L} \cdot 1(t) \cdot \exp\left(-\frac{t}{T_L}\right)$$

Noise Spectrum at Preamplifier Output

$$\boxed{S_n = S_V + S_i \frac{R_L^2}{1+\omega^2 T_L^2}}$$



recalling $T_{nc} = C_L \cdot \left(\sqrt{S_v} / \sqrt{S_i}\right) = C_L R_{nc}$

$$S_n = S_V \left(1 + \frac{1}{T_{nc}^2} \frac{T_L^2}{1+\omega^2 T_L^2}\right) = S_V \cdot \frac{T_{nc}^2 + T_L^2}{T_{nc}^2} \cdot \frac{1+\omega^2 T_{nc}^2 \dfrac{T_L^2}{T_{nc}^2 + T_L^2}}{1+\omega^2 T_L^2}$$

The **noise spectrum** has

- a pole at $\omega_L^2 = 1/T_L^2$
- a zero at $\omega_z^2 = \dfrac{1}{T_z^2} = \dfrac{1}{T_{nc}^2}\left(1 + \dfrac{T_{nc}^2}{T_L^2}\right)$

$$\Rightarrow \boxed{S_n = S_V \frac{T_L^2}{T_z^2} \cdot \frac{1+\omega^2 T_z^2}{1+\omega^2 T_L^2}}$$

With **high load resistance *R_L***, namely with $R_L / R_{nc} = T_L / T_{nc} > 10$, the zero is practically the same as with infinite load resistance $\boxed{\omega_z^2 \approx \dfrac{1}{T_{nc}^2} = \omega_{nc}^2}$

Noise Spectrum at Preamplifier Output

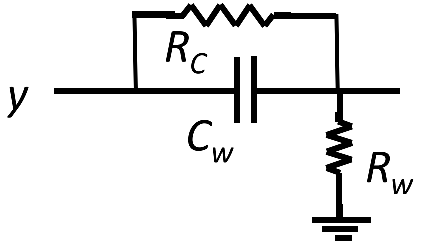
$$\boxed{S_n = S_V \frac{T_L^2}{T_z^2} \cdot \frac{1+\omega^2 T_z^2}{1+\omega^2 T_L^2}}$$

with $T_z^2 = T_{nc}^2 \dfrac{T_L^2}{T_{nc}^2 + T_L^2}$ and $T_L = R_L C_L$

The noise **whitening filter *H_{nw}*** must

- cancel the pole with a zero at $\omega_L = 1/T_L$
- cancel the zero with a pole at $\omega_p = \dfrac{1}{T_z} \approx \dfrac{1}{T_{nc}}$

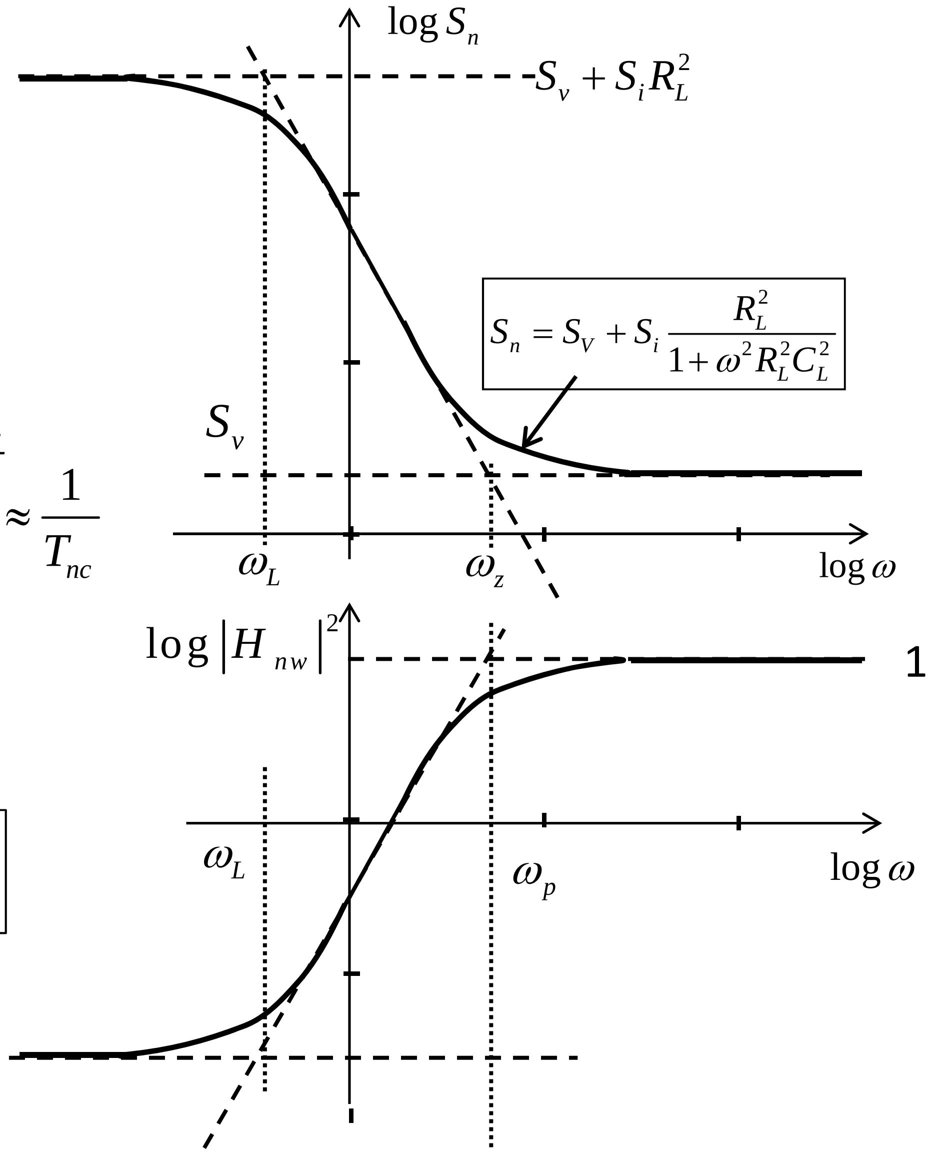
It's a quasi-high-pass CR with finite DC attenuation



$$\boxed{R_C C_w = T_L}$$

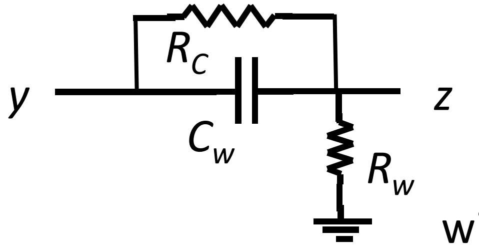
$$\boxed{\frac{R_C R_w}{R_C + R_w} C_w = T_z}$$

$$H_{nw}(\omega) = \frac{T_z}{T_L}\frac{1+j\omega T_L}{1+j\omega T_z} \approx \frac{T_{nc}}{T_L}\frac{1+j\omega T_L}{1+j\omega T_{nc}}$$



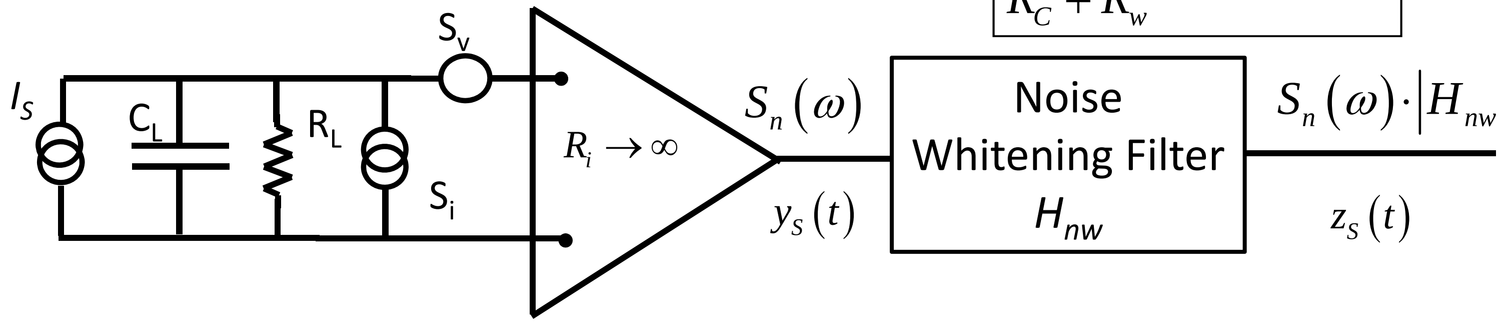
Action of the Noise-Whitening Filter

Quasi-high-pass CR filter



$$H_{nw}(s)=\frac{T_z}{T_L}\frac{1+sT_L}{1+sT_z}$$

with $\boxed{\frac{R_C R_w}{R_C+R_w}C_w=T_z\approx T_{nc}}$ and $\boxed{R_L C_w=T_L}$



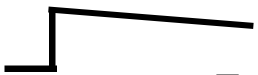
a) it makes **white** the noise at its **output** and

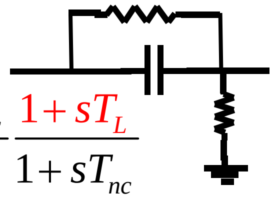
b) it changes the signal into a short **exponential pulse with time-constant T_{nc}** .

Note that this **«post-whitening» signal is the same found with $R_L\to\infty$**

NW Filter

$$y_S(t)=\frac{Q}{C_L}\cdot 1(t)\cdot\exp\left(-\frac{t}{T_L}\right)$$





$$\frac{T_{nc}}{T_L}\frac{1+sT_L}{1+sT_{nc}}$$

$$z_S(t)=\frac{Q}{C_L}\cdot 1(t)\cdot\exp\left(-\frac{t}{T_{nc}}\right)$$

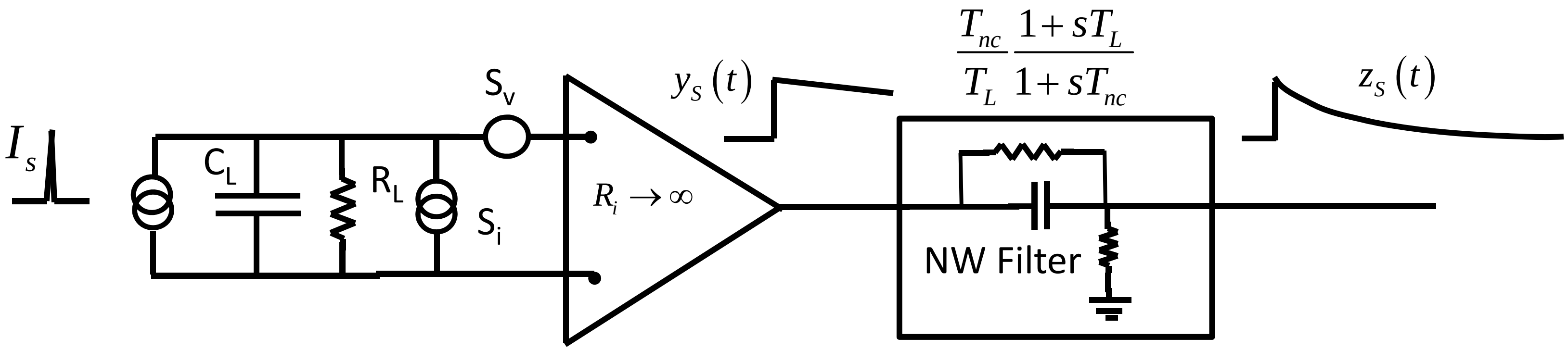
$$Y_S(s)=\frac{Q}{C_L}\cdot T_L\frac{1}{1+sT_L}$$

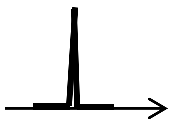
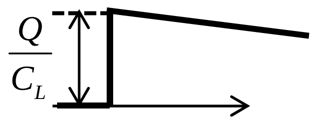
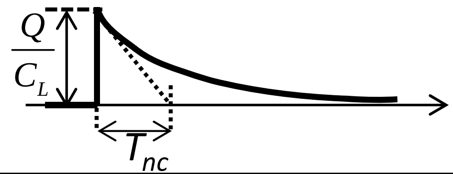
$$Z_S(s)=\frac{Q}{C_L}T_{nc}\cdot\frac{1}{1+sT_{nc}}$$

Cancels pole at s = $-1/T_L$

and replaces it by pole at $s=-1/T_{nc}$

Signal at the Noise-Whitening Filter Output



Input (current)	Preamp Output (voltage)	NW Filter Output (voltage)
δ – pulse 	Long exponential pulse 	Short exponential pulse 
$I_s(t) = Q \cdot \delta(t)$	$y_S(t) = \frac{Q}{C_L} \cdot 1(t) \cdot \exp\left(-\frac{t}{T_L}\right)$	$z_S(t) = \frac{Q}{C_L} \cdot 1(t) \cdot \exp\left(-\frac{t}{T_{nc}}\right)$
$I_s(s) = Q$	$Y_S(s) = \frac{QT_L}{C_L}\frac{1}{1+sT_L}$	$Z_S(s) = \frac{QT_{nc}}{C_L}\frac{1}{1+sT_{nc}}$

The **matched** filter has to be tailored to the signal at the whitening filter output, i.e. it must have **weighting function exponential with time constant *T_{nc}***