Sensors, Signals and Noise

COURSE OUTLINE

- Introduction
- Signals and Noise
- Filtering: 1/f Noise and High-Pass Filters 1 - HPF1
- Sensors and associated electronics

1/f Noise and High-Pass Filters 1

- 1/f Noise features
- 1/f Noise band-limits and power
- 1/f Noise Filtering
- Intrinsic High-Pass Filtering by Correlated Double Sampling (CDS)
- Correlated Double Sampling with Filtered Baseline (CDS-FB)
- Correlated Double Filtering (CDF)
- Appendix1: the rise transient of 1/f noise
- Appendix 2: checking CDF weighting in frequency

1/f Noise features

1/f Noise

Random fluctuations with power spectral density

$$S(f) \propto \frac{1}{|f|}$$

- first reported in 1925 as «flicker noise» in electronic vacuum tubes
- ubiquitous, observed in all electronic devices
- with very **different intensity in different devices**:
  very strong in MOSFETs; moderate in Bipolar Transistors BJTs;
  moderate in carbon resistors; ultra-weak in metal-film resistors; etc.
- observed in many cases also **outside electronics:**
  cell membrane potential; insulin level in diabetic blood; brownian motion;
  solar activity; intensity of white dwarf stars; ocean current flux;
  frequency of atomic clocks; ... and many others
- Basic **distinction** between 1/f and white noise:
  time span of interdependence between samples
  for **white** noise: samples are **uncorrelated even at short** time distance
  for **1/f** noise: samples are strongly **correlated even at long** time distance

1/f Noise features

- The real observed power density at low frequency is often not exactly $\propto \frac{1}{f}$
  but rather $\propto \frac{1}{|f|^{\alpha}}$ with α close to unity, i.e. 0,8 < α <1,2
  anyway the behavior of such noise is **well approximated by *1/f*** density
- 1/f noise arises from physical processes that generate a **random superposition** of elementary pulses with **random pulse duration** ranging from **very short to very long**.

E.g. in MOSFETs 1/f noise arises because:

a) carriers traveling in the conduction channel are randomly captured by local trap levels in the oxide, stop traveling and stop contributing to the current
b) trapped carriers are later released by the level with a random delay
c) the level lifetime (=mean delay) strongly depends on how far-off is from the silicon surface (= from the conduction channel) is the level in the oxide
d) trap levels are distributed from very near to very far from silicon, lifetimes are correspondingly distributed from very short to very long

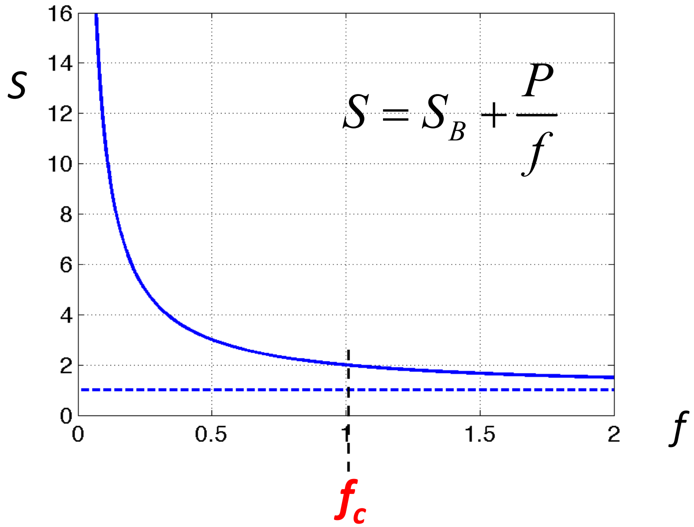
1/f Noise specification

- Spectral density $S_f(f) = \dfrac{P}{f}$ noise power $\overline{n_f^2} = \displaystyle\int_0^{+\infty} \frac{P}{f}\, df$ (with unilateral S_f)

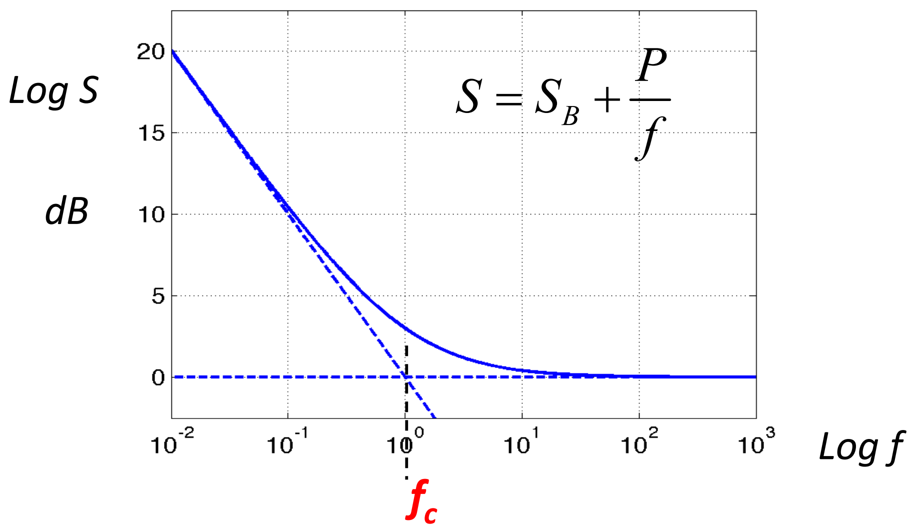
  NB: ***P*** dimension = noise power dimension (e.g. Volt^2 for voltage noise)

- circuits and devices have both 1/f noise S_f and white noise S_B

Linear diagram



Bode diagram



- S_f is specified in relative terms referred to the white noise S_B by specifing the «corner frequency» f_c at which $\boldsymbol{S_f = S_B}$

1/f Noise specification

- The **1/f noise corner** frequency f_c is defined by

$$\frac{P}{|f_c|} = S_B \qquad \text{hence} \qquad P = S_B\, f_c$$

NB: the **higher** is frequency f_c
the **stronger** is the role of 1/f noise
and for a given S_B , the higher is the intensity P

Typical values for low-noise voltage amplifiers :

- S_B a few 10^{-18} V^2/Hz → $\sqrt{S_B}$ a few $nV/\sqrt{Hz}$
- f_c 10Hz to 10kHz, that is
- P a few 10^{-17} to a few 10^{-14} V^2 → $\sqrt{P}$ from a few nV to a few 100 nV

1/f Noise band-limits and power

1/f Noise bandlimits and power

The ideal 1/f noise spectrum runs from f = 0 to f → ∞ and has divergent power $\overline{n_f^2} \rightarrow \infty$ (recall that also the ideal white spectrum has $\overline{n_B^2} \rightarrow \infty$)

$$\overline{n_f^2} = \int_0^\infty \frac{P}{f}\, df \rightarrow \infty$$

A **real** 1/f noise spectrum has **span limited** at both ends and is **not divergent**.

If there is **wide spacing** between the high-frequency and low-frequency limitations they can be **approximated by sharp cutoff** at low frequency f_i and high frequency $f_s \gg f_i$ and the noise power can be evaluated as *

$$\overline{n_f^2} \approx \int_{f_i}^{f_s} \frac{P}{f}\, df = P \ln\left(\frac{f_s}{f_i}\right) = S_B f_c \ln\left(\frac{f_s}{f_i}\right)$$

The actual 1/f bandlimits f_s and/or f_i of given filter types will be illustrated later.

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* Beware !

ONLY if $f_S \gg f_i$ the sharp cutoff gives a **GOOD APPROXIMATION** of the noise power !

1/f Noise bandlimits and power

In cases with widely spaced bandlimits $f_S >> f_i$ the 1/f noise power $\overline{n_f^2}$ is

$$\overline{n_f^2} \approx \int_{f_i}^{f_s} \frac{P}{f}\, df = S_B f_c \ln\left(\frac{f_s}{f_i}\right)$$

NOTE THAT:

- $\overline{n_f^2}$ is divergent for $f_s \rightarrow \infty$ (like white noise).
  A limit at high frequency is necessary for avoiding divergence,
  but in real cases a finite limit always exists.

- $\overline{n_f^2}$ is divergent for $f_i \rightarrow 0$ (like random-walk noise $1/f^2$).
  A limit at low frequency is necessary for avoiding divergence,
  but we will see that in real cases there is always a finite limit

- $\overline{n_f^2}$ depends on the **ratio f_s / f_i** and **NOT the absolute values fs and f_i**

1/f Noise bandlimits and power

$$\overline{n_f^2} \approx \int_{f_i}^{f_s} \frac{P}{f}\, df = S_B f_c \ln\left(\frac{f_s}{f_i}\right)$$

Note 1: $\overline{n_f^2}$ is **SLOWLY** divergent for $f_i \to 0$ or $f_s \to \infty$

Logarithmic dependence → $\overline{n_f^2}$ slowly increases with f_s/f_i

e.g : **x 10 multiplication** of f_s/f_i → **+ 2,3 addition** to $\ln(f_s/f_i)$

EXAMPLE: 1/f noise with $\sqrt{P} = \sqrt{S_B f_C} = 100\, nV$

a) filtered with f_i = 1kHz and f_s = 10kHz (f_s / f_i = **10**)

$$\sqrt{\overline{n_{f,a}^2}} = \sqrt{2,3}\,\sqrt{S_B f_c} = 151\, nV$$

b) filtered with f_i = 1 Hz and f_s = 10MHz (f_s / f_i = **10^7**, i.e. **x 10^6 higher**)

$$\sqrt{\overline{n_{f,b}^2}} = \sqrt{7 \cdot 2,3}\,\sqrt{S_B f_c} = 401\, nV$$ (just **x 2,7 higher**)

1/f Noise bandlimits and power

$$\overline{n_f^2} \approx \int_{f_i}^{f_s} \frac{P}{f}\, df = S_B f_c \ln\left(\frac{f_s}{f_i}\right)$$

Note 2 : reasonably approximate bandlimits are adequate for estimating $\overline{n_f^2}$

it is **not necessary to know very precisely** f_s and f_i !!

EXAMPLE: for 1/f noise with $\sqrt{P} = \sqrt{S_B f_C} = 100\, nV$ we estimate

a) with bandlimits f_i = 1kHz and f_s = 10kHz

$$\sqrt{\overline{n_{f,a}^2}} = \sqrt{2,3}\,\sqrt{S_B f_c} = 151\, nV$$

b) with bandlimit f_s corrected to $f_{sn} = \frac{\pi}{2} f_s = 15,7\ kHz$ (50% higher)

$$\sqrt{\overline{n_{f,b}^2}} = \sqrt{15,7}\,\sqrt{S_B f_c} = 166\, nV$$ (just 10 % higher)

1/f Noise Filtering

1/f Noise filtering and power

- For filters that limit 1/f noise, in the approximation with sharp cutoffs f_s and f_i

  $\overline{n_f^2}$ depends on the **ratio** $\frac{f_s}{f_i}$ **, NOT on the individual** values f_s and f_i

- This stems from a fundamental property expressed in terms of the characteristic parameters of W(f) (poles and zeros of constant parameter filters) :

  $\overline{n_f^2}$ depends on the **ratio of the parameters, NOT** on the **individual** values

- In order to verify it, let us consider to change all the characteristic frequency parameters of W(f) of a filter without changing their ratio, that is, to **multiply all parameters by a constant factor β.**

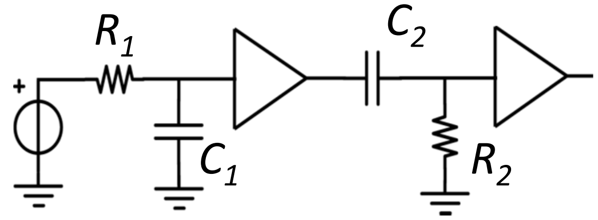
1/f Noise filtering and power

Multiplying all the parameters of W(f) **by a factor β** is equivalent

to **divide by β the frequency scale** of W(f), as shown in the following example

W(f) with the original parameters

$$|W|^2 = \frac{1}{1+\left(\dfrac{f}{f_{p1}}\right)^2} \cdot \frac{\left(\dfrac{f}{f_{p2}}\right)^2}{1+\left(\dfrac{f}{f_{p2}}\right)^2}$$



$f_{p1} = 1/2\pi R_1 C_1$ $\quad$ $f_{p2} = 1/2\pi R_2 C_2$

We can change the poles $f_{p1} \rightarrow \beta f_{p1}$ and $f_{p2} \rightarrow \beta f_{p2}$

e.g. by changing the resistors $R_1 \rightarrow R_1/\beta$ and $R_2 \rightarrow R_2/\beta$

W(f) with poles multiplied x β

$$|W_\beta|^2 = \frac{1}{1+\left[\dfrac{f}{\beta f_{p1}}\right]^2} \cdot \frac{\left[\dfrac{f}{\beta f_{p2}}\right]^2}{1+\left[\dfrac{f}{\beta f_{p2}}\right]^2} = \frac{1}{1+\left[\dfrac{(f/\beta)}{f_{p1}}\right]^2} \cdot \frac{\left[\dfrac{(f/\beta)}{f_{p2}}\right]^2}{1+\left[\dfrac{(f/\beta)}{f_{p2}}\right]^2}$$

1/f Noise filtering and power

To multiply by a factor β all the characteristic frequency-parameters of W(f) is equivalent to divide **by β the frequency scale** of W(f). In this condition the 1/f noise power is

$$\overline{n^2_{f,\beta}} = S_B\ f_c \int_0^\infty \left| W\left(\frac{f}{\beta}\right) \right|^2 \frac{1}{f}\, df$$

and by setting z = f/β we have $\frac{dz}{z} = \frac{df}{f}$

we can thus check that the **1/f output power is NOT changed by the change of scale**

$$\overline{n^2_{f,\beta}} = S_B\ f_c \int_0^\infty |W(z)|^2 \frac{1}{z}\, dz = S_B\ f_c \int_0^\infty |W(f)|^2 \frac{1}{f}\, df = \overline{n^2_f}$$

whereas the **white noise output power is multiplied by β**

$$\overline{n^2_{B,\beta}} = S_B \int_0^\infty \left| W\left(\frac{f}{\beta}\right) \right|^2 df = S_B \int_0^\infty |W(z)|^2 \beta\, dz = \beta \cdot \overline{n^2_B}$$

CONCLUSION: with a given filter type (a given W(f)), if the scale of parameters is changed

- for **white** noise the output noise power **is changed**
- for ***1/f*** noise the output noise power **is NOT changed**;
  for changing it, one must change the type of filter, not just the parameter scale

1/f Noise filtering and power

$$\overline{n_f^2} = S_B \ f_c \int_0^{\infty} |W(f)|^2 \frac{df}{f} = S_B \ f_c \int_{-\infty}^{\infty} |W(\ln f)|^2 \, d(\ln f)$$

Filtering of *1/f* noise can be better understood by changing variable from f to *lnf*
(beware: it's NOT A BODE diagram: the vertical scale is linear !!)

- **1/f** noise: filtered power $\overline{\boldsymbol{n_f^2}}$ $\propto$ **area of $|\mathbf{W}|^2$ plot in logarithmic frequency** scale

which is different from the case of

- **white** noise: filtered power $\overline{\boldsymbol{n_B^2}}$ $\propto$ **area of $|\mathbf{W}|^2$ plot in linear frequency** scale

In both cases the noise power depends mainly on the **frequency span covered** by $|\mathbf{W}|^2$, delimited by upper and lower bounds in frequency. However, the frequency span is **measured differently** :

- for **white** noise, by the **difference** of the bounds
- for **1/f** noise, by the **logarithmic difference**, i.e. by the **ratio** of the bounds

1/f Noise filtering and power

- The **band-limits of a filter for white noise** are well visualized in the **linear-linear diagram** of the weighting function $|W(\boldsymbol{f})|^2$:

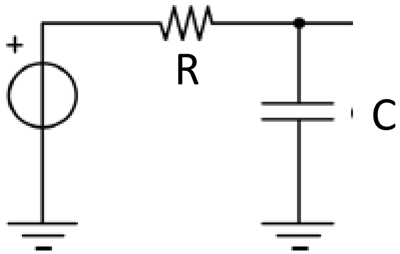
  the simple equivalent weighting function is rectangular with area and height equal to the true weighting $|W(\boldsymbol{f})|^2$

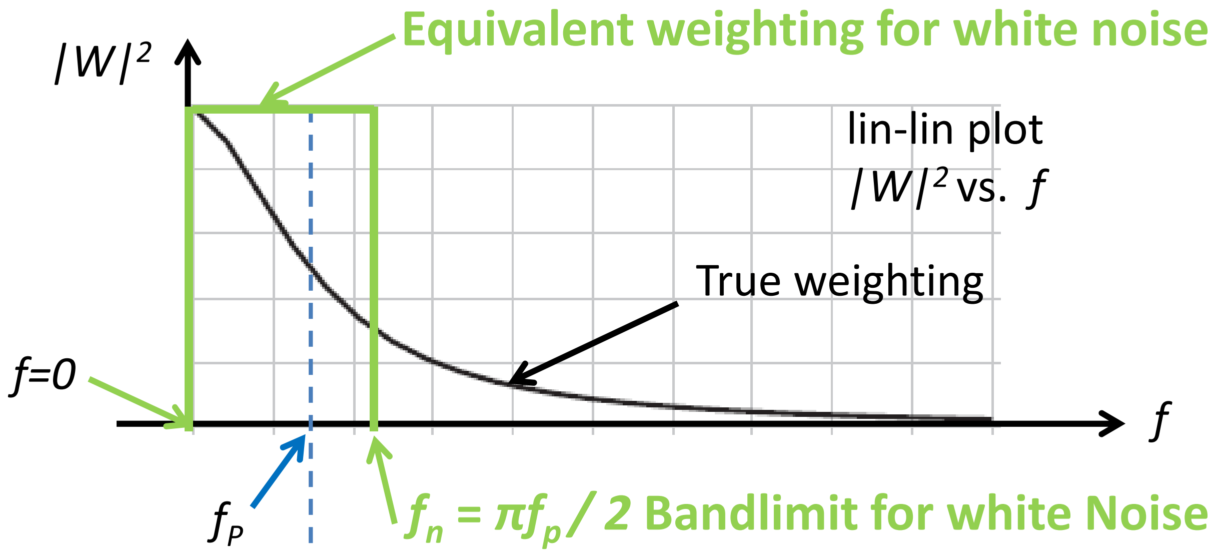
- The **band-limits of a filter for 1/f noise** are well visualized in the **linear-log diagram** of the weighting function $|W(\mathbf{ln}\boldsymbol{f})|^2$:

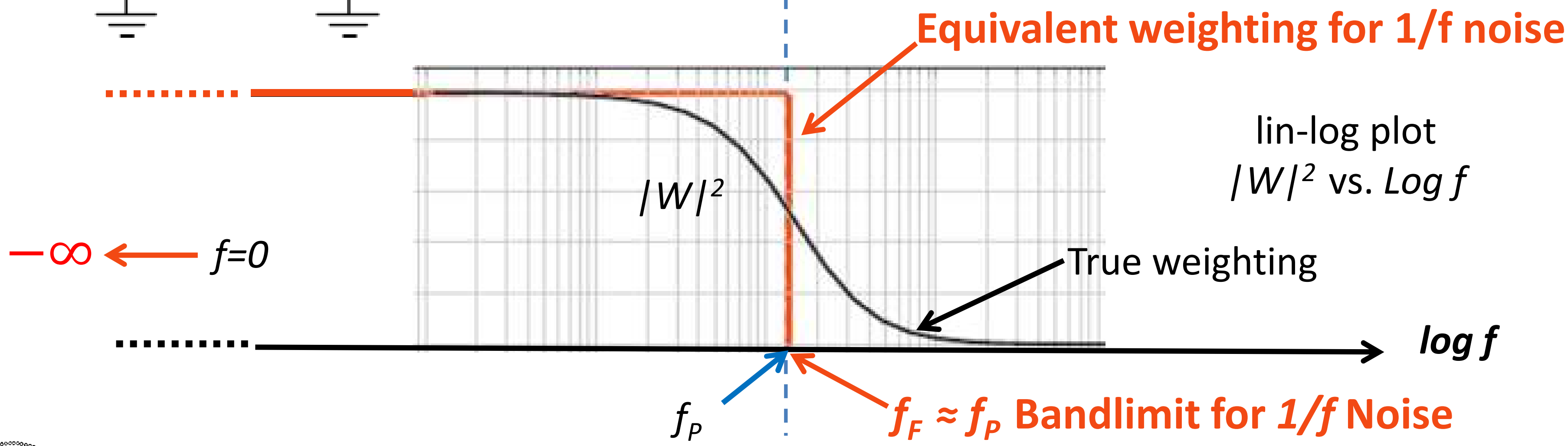
  the simple equivalent weighting function is rectangular with area and height equal to the true weighting $|W(\mathbf{ln}\boldsymbol{f})|^2$

Noise bandlimit: RC integrator

$$|W|^2 = \frac{1}{1+\left(2\pi fRC\right)^2}$$

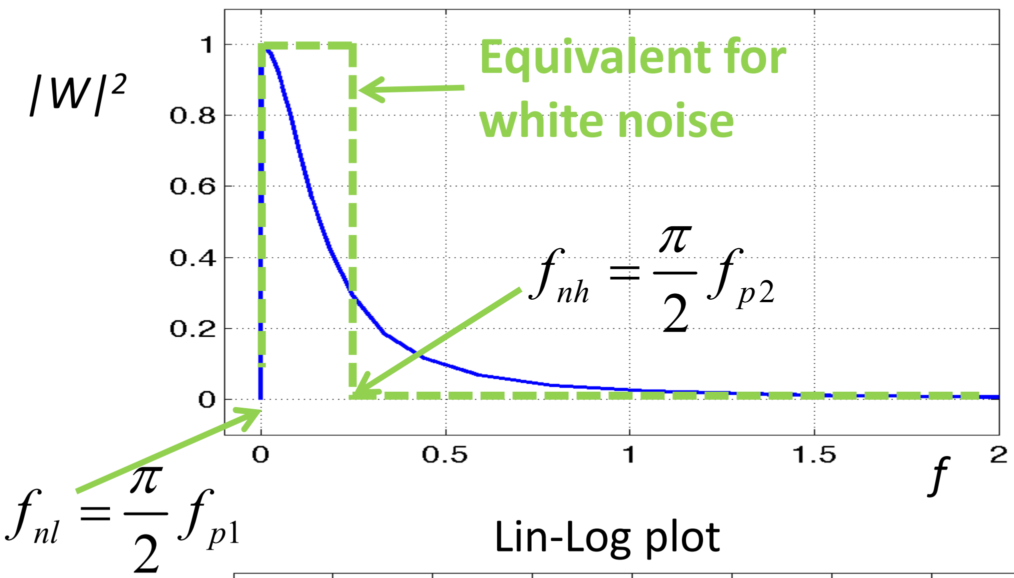


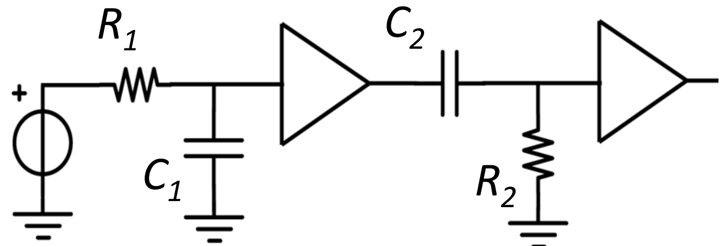




Noise bandlimits: RC lowpass plus CR highpass

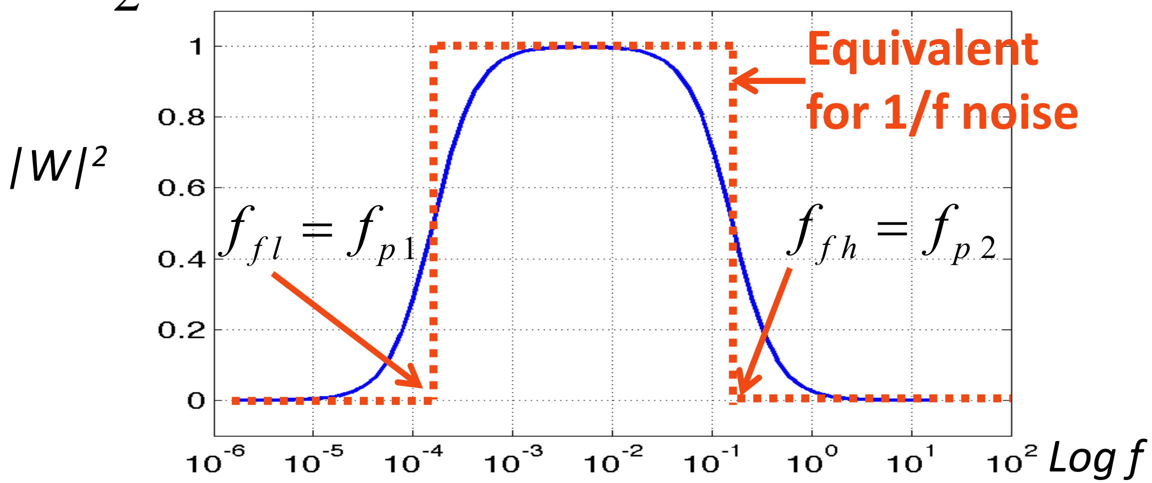
Lin-Lin plot





$$|W|^2 = \frac{1}{1+(2\pi f R_1 C_1)^2} \cdot \frac{(2\pi f R_2 C_2)^2}{1+(2\pi f R_2 C_2)^2}$$

Lin-Log plot



Cascaded two-cell filter:
low-pass $T_{RC} = R_1 C_1$
high-pass $T_{CR} = R_2 C_2$

Example plotted with
$T_{RC} = 1$
$T_{CR} = 1000$

Intrinsic High-Pass Filtering
by
Correlated Double Sampling CDS

Intrinsic High-Pass Filtering

- In all real cases, even with DC coupled electronics:

  weighting is inherently NOT extended down to zero frequency,

  because an intrinsic high-pass filtering is present in any real operation.

- The intrinsic filtering action arises because:

  a) operation is **started at some time before** the acquisition of the measure and

  b) operation is **started from zero** value

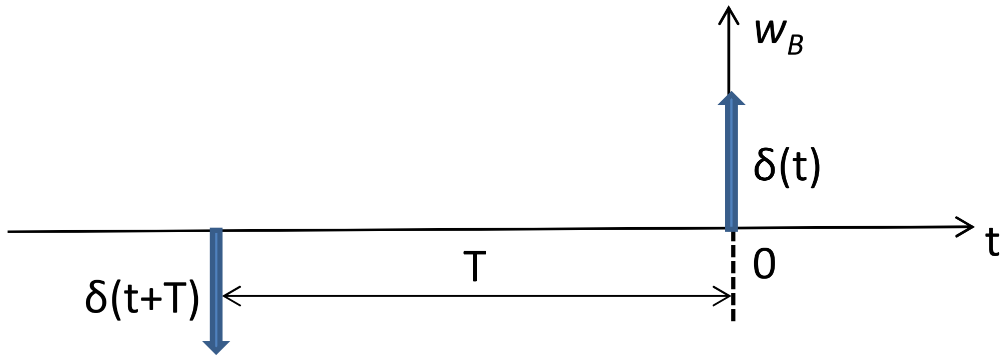
- EXAMPLE: measurement of amplitude of the output signal of a DC amplifier.
  Zero-setting is mandatory: the baseline voltage is preliminarly adjusted to zero, or it is measured, recorded and then subtracted from the measured signal.
  It may be done a long time before the signal measurements (e.g. when the amplifier is switched on) or repeated before each measurement; it may be done manually or automated, but it must be done anyway.
  Zero-setting produces a high-pass filtering: let us analyze why and how

Zero-setting by Correlated Double Sampling (CDS)

Baseline sample subtracted from signal sample, **both** acquired with **instant sampling**

Time-domain weighting

$$w_B(t) = \delta(t) - \delta(t+T)$$



Frequency-domain weighting

$$W_B(\omega) = F[w_B(t)] = 1 - e^{i\omega T} = 1 - \cos\omega T - i\ \sin\omega T$$

For noise $$\left|W_B(\omega)\right|^2 = \left[1-\cos\omega T\right]^2 + \sin^2\omega T = 2\ [1-\cos\omega T]$$

We can also write $$\left|W_B(\omega)\right|^2 = 4\sin^2\left(\frac{\omega T}{2}\right)$$ [since it is *(1-cosx) = 2 sin*^{2}*(x/2)*]

At ωT<< 1 a **low frequency cutoff** is produced

$$\left|W_B(\omega)\right|^2 \approx \omega^2 T^2$$ (for x << 1 it is *sinx≈x* and *cosx ≈ 1 – x*^{2}*/2*)

CDS vs. CR High-Pass Filter: Cut-Off

Baseline subtraction with delay T

$$|W_B(\omega)|^2 = 4\sin^2\left(\frac{\omega T}{2}\right)$$

at low-frequency ω << 1/T

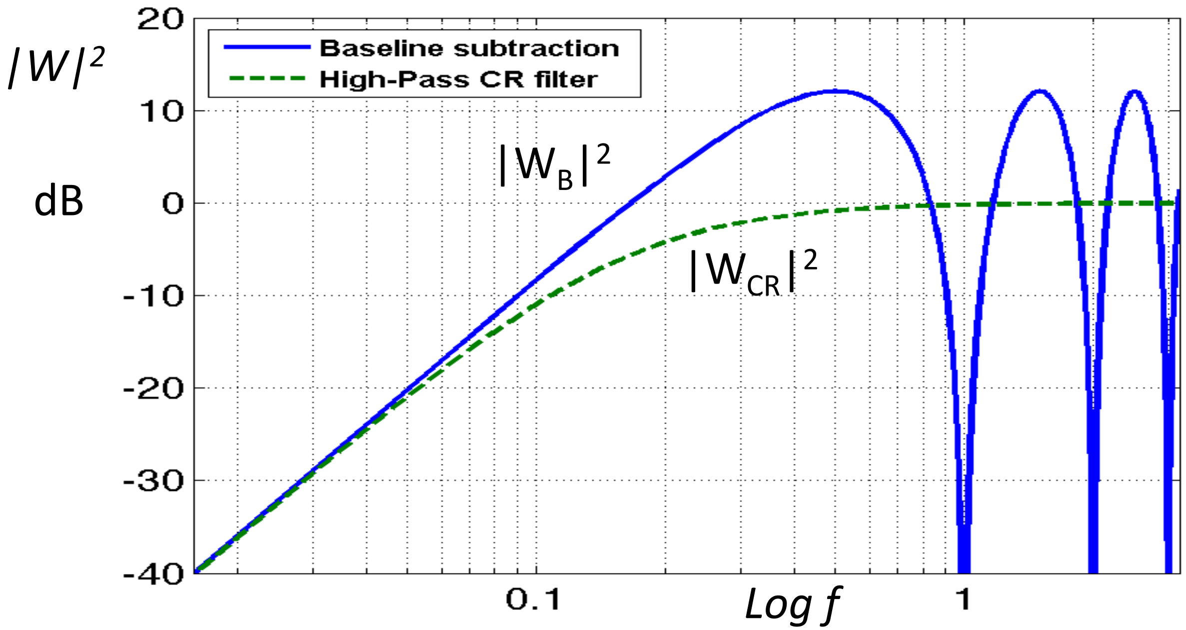
$$|W_B(\omega)|^2 \approx \omega^2 T^2$$

High-Pass CR filter (differentiator)

$$|W_{CR}(\omega)|^2 = \frac{\omega^2 R^2 C^2}{1+\omega^2 R^2 C^2}$$

at low-frequency ω << 1/RC

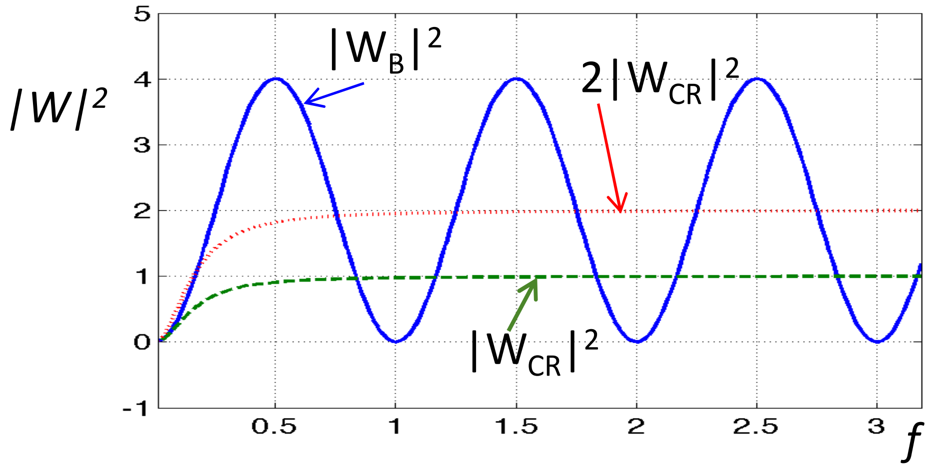
$$|W_{CR}(\omega)|^2 \approx \omega^2 R^2 C^2$$



BODE DIAGRAM
highlights
the low-freq cutoff

Examples with
equal cutoff T=RC
plotted for T=1

CDS vs. CR High-Pass Filter: White Noise



LIN –LIN DIAGRAM
white noise power ∝ area of $|W|^2$

NB: examples with
equal cutoff T=RC
plotted for T=1

White noise $\overline{n_B^2}$ limited also by a low-pass f_s , but with $f_s >> 1/T$ and $f_s >> 1/RC$

$$\overline{n_B^2} = S_B \int_0^{f_S} \left|W(f)\right|^2 df$$

CDS: $|W_B|^2$ oscillates around 2; its area is **exactly the same** as for a constant $|W_B|^2 = 2$

CR: $|W_{CR}|^2$ has a cutoff at low frequency $f < f_i$ =1/4RC; at higher frequency it is $|W_{CR}|^2 \approx 1$

Therefore, for white noise the output power of the CDS is double of the unfiltered noise and approximately double of the filtered output of the CR (actually even more than double !)

CDS vs. CR High-Pass Filter: White Noise

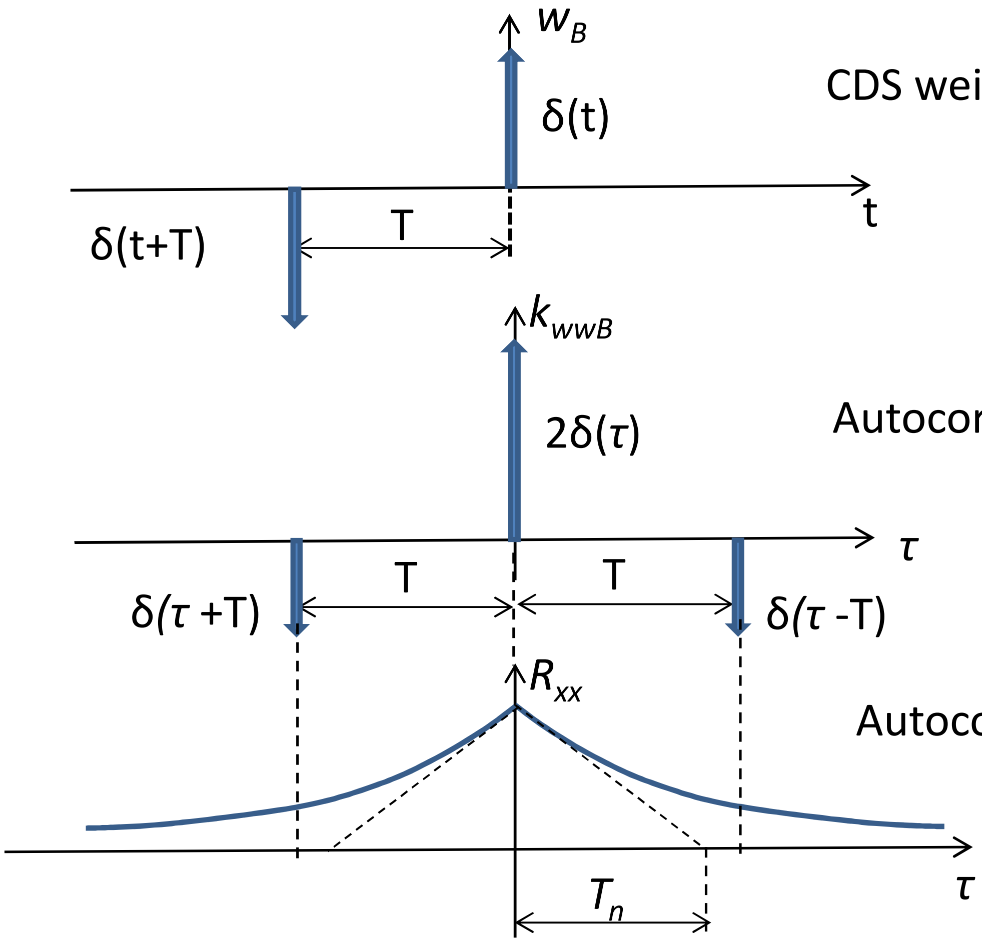
$$\overline{n_B^2} = S_B \int_0^{f_S} |W(f)|^2 df$$

With Baseline sampling & subtraction	With CR high-pass filter
$\overline{n_B^2} = S_B \int_0^{f_S} 2 \cdot [1 - \cos \omega T]\, df$ that is $\overline{n_B^2} = 2\, S_B f_S$	$\overline{n_B^2} = S_B (f_S - f_i)$ and since $f_s >> f_i$ $\overline{n_B^2} \approx S_B f_S$

Double White noise power, as intuitive because:

1. white noise is acquired twice, in the baseline sampling and in the signal sampling.
2. The two noise samples are uncorrelated, hence their power is quadratically added.

Filtering Band-Limited White Noise by CDS: time-domain analysis gives further insight



CDS weighting function

Autocorrelation of CDS weighting function

Autocorrelation of band-limited white noise

$$R_{xx}(\tau) = \overline{n_x^2} e^{-\frac{\tau}{T_n}}$$

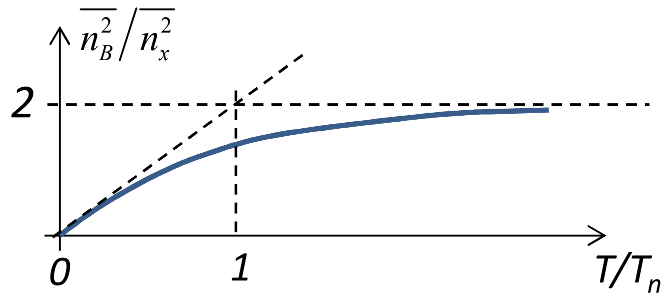
T_n noise autocorrelation time

$f_n = 1/4T_n$ noise band-limit

Filtering band-limited White Noise by CDS

$$\overline{n_B^2} = \int_{-\infty}^{\infty} R_{xx}(\tau) k_{wwB}(\tau) d\tau = 2\overline{n_x^2} - R_{xx}(T) - R_{xx}(-T)$$

$$\boxed{\overline{n_B^2} = 2\overline{n_x^2} \cdot \left(1 - e^{-\frac{T}{T_n}}\right)}$$



- Noise with **very short correlation** time (i.e. very high band-limit) is **doubled**:

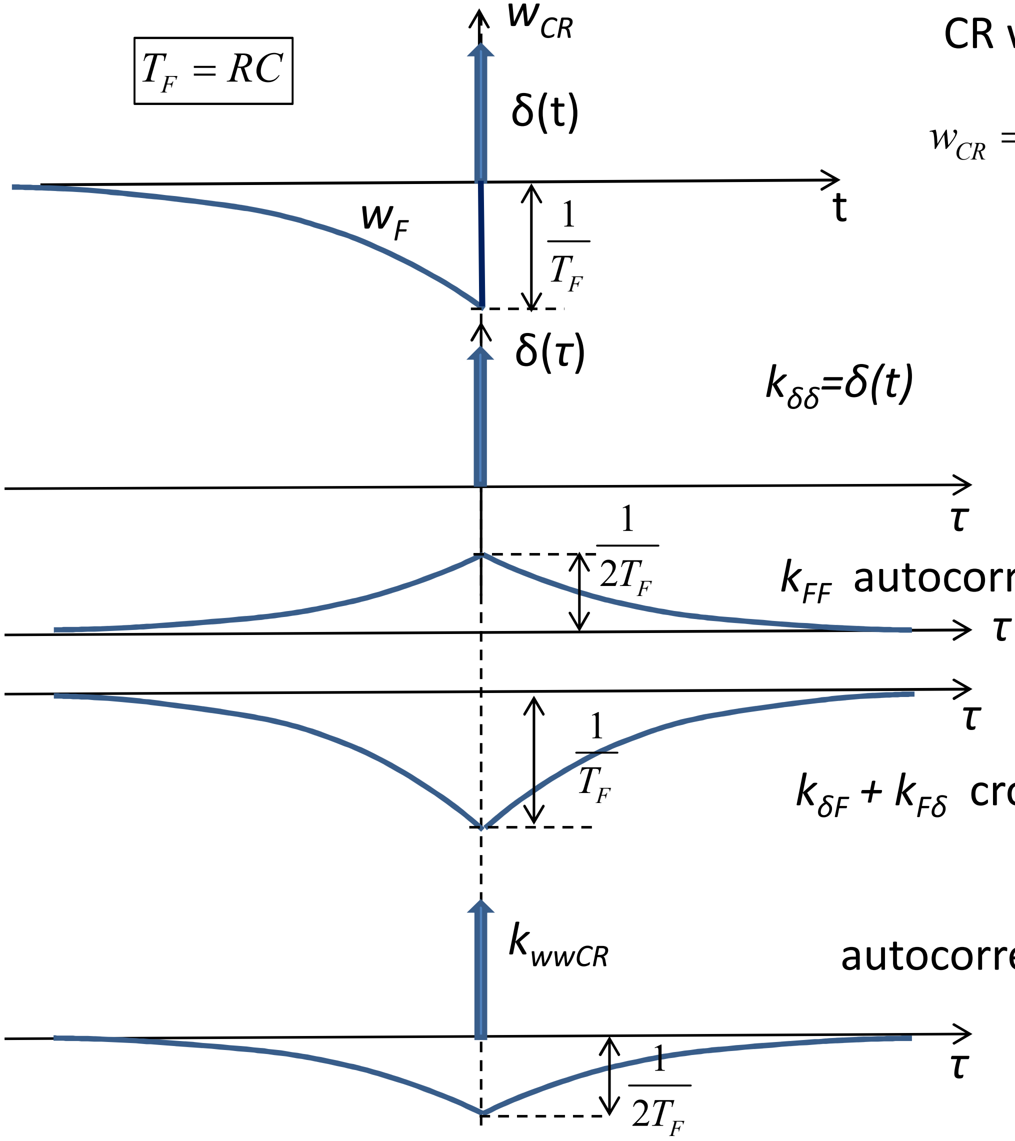
  if $T_n \ll T$ we have $\overline{n_B^2} \approx 2\overline{n_x^2}$

- Noise with **long correlation time** (i.e. very low band-limit) is **strongly attenuated**:

  if $T_n \gg T$ we have $\overline{n_B^2} \approx \overline{n_x^2} \cdot 2\dfrac{T}{T_n} \ll \overline{n_x^2}$

Time-domain analysis clearly shows how with band-limited white noise the output noise power of CDS is double of that of a CR constant-parameter filter with equal cutoff, i.e. with $T_F = RC = T$

Filtering band-limited White Noise by CR



CR weighting function

$$w_{CR} = \delta(t) - w_F(t) = \delta(t) - \frac{1}{T_F}\exp\left(-\frac{|t|}{T_F}\right)$$

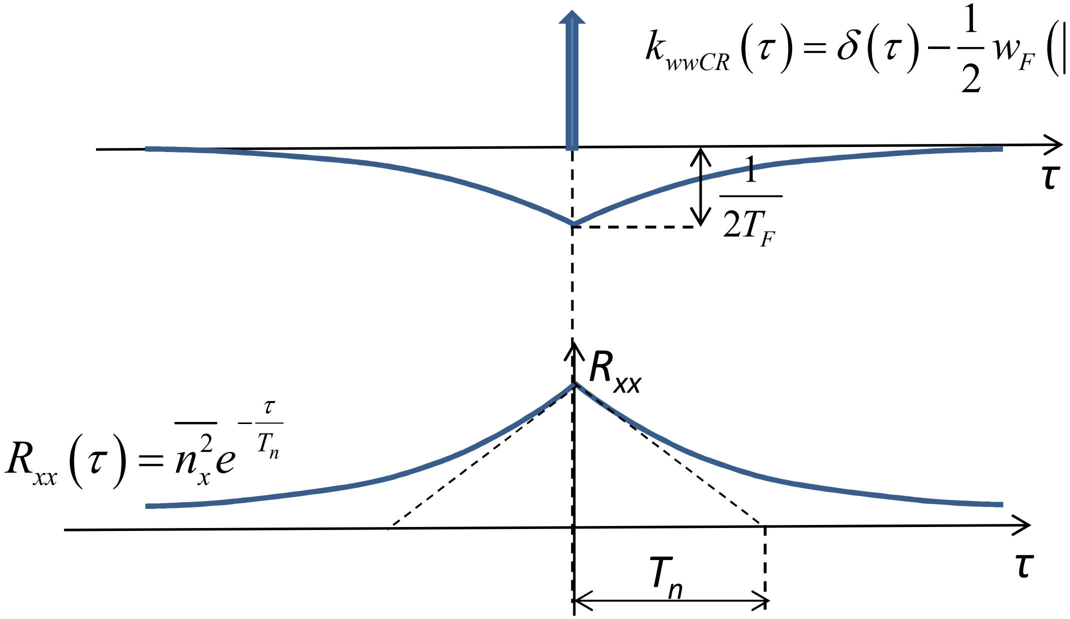
k_{FF} autocorrelation of w_F

$k_{\delta F} + k_{F\delta}$ crosscorrelations δw_F and $w_F\delta$

autocorrelation of the CR weighting function

$$k_{wwCR} = k_{\delta\delta} + k_{FF} + k_{F\delta} + k_{\delta F}$$

Filtering band-limited White Noise by CR



$$k_{wwCR}(\tau) = \delta(\tau) - \frac{1}{2} w_F(|\tau|) = \delta(\tau) - \frac{1}{2T_F} \exp\left(-\frac{|\tau|}{T_F}\right)$$

$$\overline{n_B^2} = \int_{-\infty}^{\infty} R_{xx}(\tau) k_{wwCR}(\tau) d\tau =$$

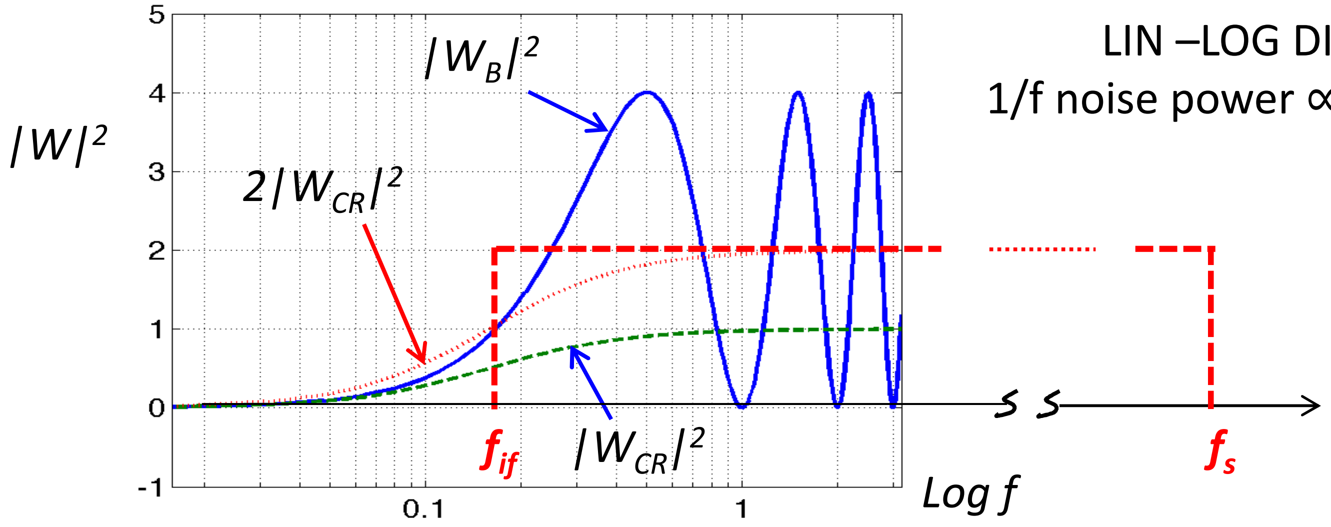
$$= R_{xx}(0) - 2\int_0^{\infty} R_{xx}(\tau) \frac{1}{2} w_F(|\tau|) d\tau =$$

$$= \overline{n_x^2} - \overline{n_x^2} \int_0^{\infty} \frac{1}{T_F} \exp\left[-\tau\left(\frac{1}{T_n} + \frac{1}{T_F}\right)\right] d\tau$$

$$\boxed{\overline{n_B^2} = \overline{n_x^2}\left(1 - \frac{T_n}{T_F + T_n}\right) = \overline{n_x^2} \frac{T_F}{T_F + T_n}}$$

- Noise with **very short correlation time** (i.e. very high band-limit) is practically **passed as it is, not doubled** as for CDS: if $T_n \ll T_F$ we have $\overline{n_B^2} \approx \overline{n_x^2}$
- Noise with **long correlation time** (i.e. very low band-limit) is strongly **attenuated at half the level of CDS**: if $T_n \gg T_F$ we have $\overline{n_B^2} \approx \overline{n_x^2} \cdot \frac{T_F}{T_n} \ll \overline{n_x^2}$

CDS vs. CR High-Pass Filter: 1/f noise



LIN –LOG DIAGRAM

1/f noise power ∝ area of $|W|^2$

Examples with equal cutoff T=RC plotted for T=1

1/f noise power $\overline{n_f^2}$ limited also by a low-pass f_s , but with $f_s >> 1/T$ and $f_s >> 1/RC$

$$\overline{n_f^2} = \int_0^{f_S} \left|W(f)\right|^2 \frac{S_B f_C}{f} df = S_B f_C \int_0^{f_S} \left|W(f)\right|^2 d\left(\ln f\right)$$

At low frequency $f<<1/T$ the $|W_B|^2$ and $|W_{CR}|^2$ have the same cutoff (with $T=RC$).

At higher frequency W_{CR} is constant $|W_{CR}|^2 \approx 1$ whereas the $|W_B|^2$ oscillates around a mean value 2, so that :

$$\int_0^{f_S} \left|W_B(f)\right|^2 d\left(\ln f\right) \approx 2 \int_0^{f_S} \left|W_{CR}(f)\right|^2 d\left(\ln f\right)$$

CDS vs. CR High-Pass Filter: 1/f noise

Therefore

$$\overline{n^2_{f,B}} \approx 2\,\overline{n^2_{f,CR}}$$

the 1/f noise power output of CDS is approximately double (actually even more than double!) with respect to a CR high-pass with equal cutoff, i.e. with RC=T

For the CR filter it will be shown that the high-pass band-limit for 1/f noise is

$$f_{if} \approx f_p = \frac{1}{2\pi RC} \qquad \text{and} \qquad \overline{n^2_{f,CR}} = S_B f_C \ln\left(\frac{f_S}{f_{if}}\right)$$

By comparing the cut-off behavior of CDS and CR, we can conclude that for CDS

$$f_{if} \approx \frac{1}{2\pi T} \qquad \text{and} \qquad \overline{n^2_{f,B}} \approx 2\, S_B f_C \ln\left(\frac{f_S}{f_{if}}\right)$$

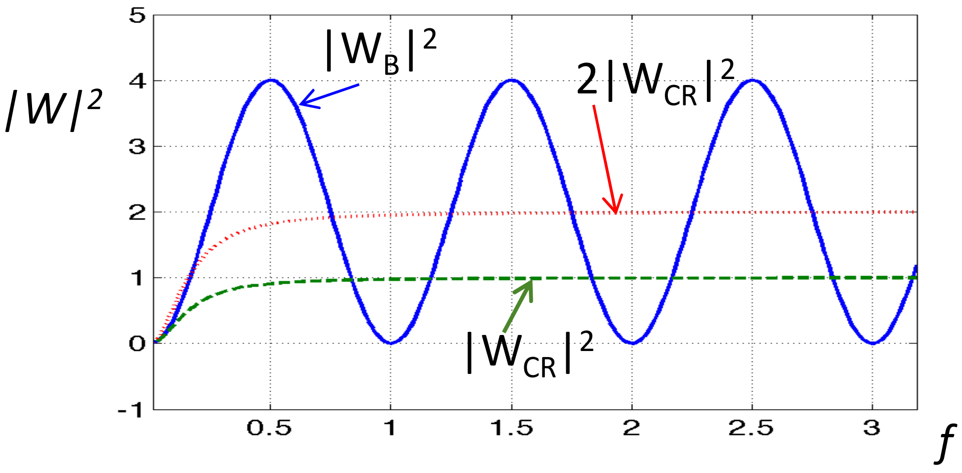
Zero-setting by CDS: conclusions

- Zero-setting by **correlated double sampling (CDS)** produces a high-pass filtering action that limits the power of 1/f noise.
- The **interval T** between zero setting and measure in most real cases is quite **long** (from a few seconds to several minutes) so that the high-pass **band-limit $\boldsymbol{f_{if}}$** is quite **low.** This is a main drawback: the filtering is **not very effective** since the 1/f noise power is limited just to a moderately low level, which may be higher than that of white noise.
- Further drawback: **with respect to CR** high-pass filter with equal bandlimit f_{if} the output noise power is **approximately double** . This occurs because in the baseline sampling all frequency components are acquired, but in the subtraction only those with $f \ll 1/T$ are really effective for reducing the 1/f noise. At higher frequencies

  - components with $f = (2n+1) / 2T$ (n integer) have power enhanced x 4

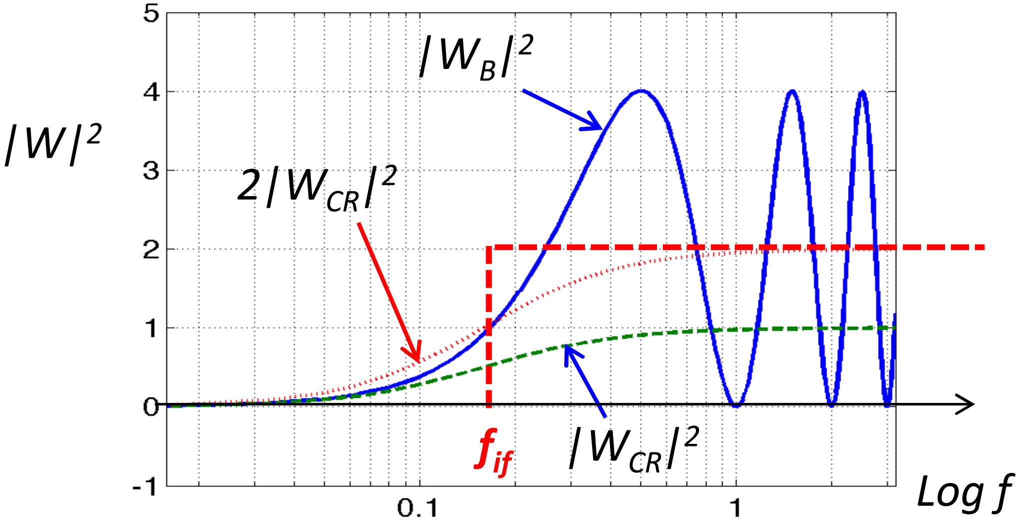
  - components with $f = n\, 1/T$ (n integer) are canceled, power is zero

  - at the intermediate frequencies the power varies between zero and x 4 (see diagrams) for convenience, the diagrams reported in slides 25 and 31 are here repeated



LIN –LIN DIAGRAM
white noise power ∝ area of $|W|^2$

NB: examples with equal cutoff T=RC plotted for T=1



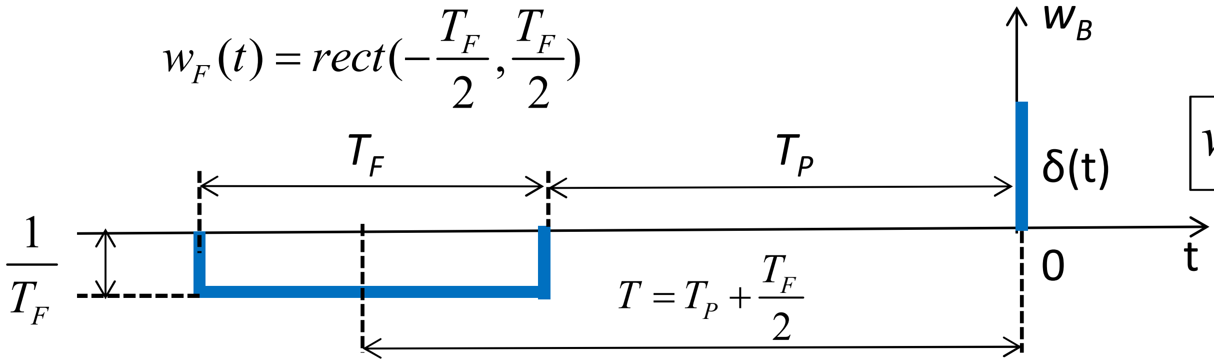
LIN –LOG DIAGRAM
1/f noise power ∝ area of $|W|^2$

Correlated Double Sampling with Filtered Baseline CDS-FB

Correlated Double Sampling with Filtered Baseline CDS-FB

- Baseline sampling is intended to acquire the contributions of the low-frequency components that we want subtract from the measurement.
- However, instant sampling acquires all frequency components at low and high frequency; by subtracting them all, we double the noise passed above the CDS cutoff.
- Remedy: modify baseline sampling for acquiring only the low-frequency components; that is, sample with a low-pass weighting function $w_F(t)$ with band-limit f_{Fn} , which includes only the frequencies to be subtracted.
  Example: noise with upper bandlimit f_S and baseline acquired by a Gated Integrator with narrower filtering band $f_{fn} << f_S$ (recall $f_{f\,n} = 1/2T_F$ with gate duration T_F)

$$w_F(t) = rect(-\frac{T_F}{2}, \frac{T_F}{2})$$



$$\boxed{w_{BI}(t) = \delta(t) - w_F(t+T)}$$

NB: we still consider cases with **long interval $T_P >> T_F$** from zero-setting to measurement

CDS-FB : Cut-off and Noise Filtering

$$W_{BI}(\omega) = F[w_{BI}(t)] = F\left[\delta(t) - w_F(t+T)\right] = 1 - e^{i\omega T} W_F(\omega)$$

since $W_F(\omega) = \text{sinc}\left(\frac{\omega\, T_F}{2}\right)$ is real at any ω , we have

$$W_{BI}(\omega) = 1 - W_F(\omega)\cos\omega T - i\,W_F(\omega)\sin\omega T$$

$$\left|W_{BI}(\omega)\right|^2 = 1 + W_F^2(\omega) - 2W_F(\omega)\ \cos\omega T$$

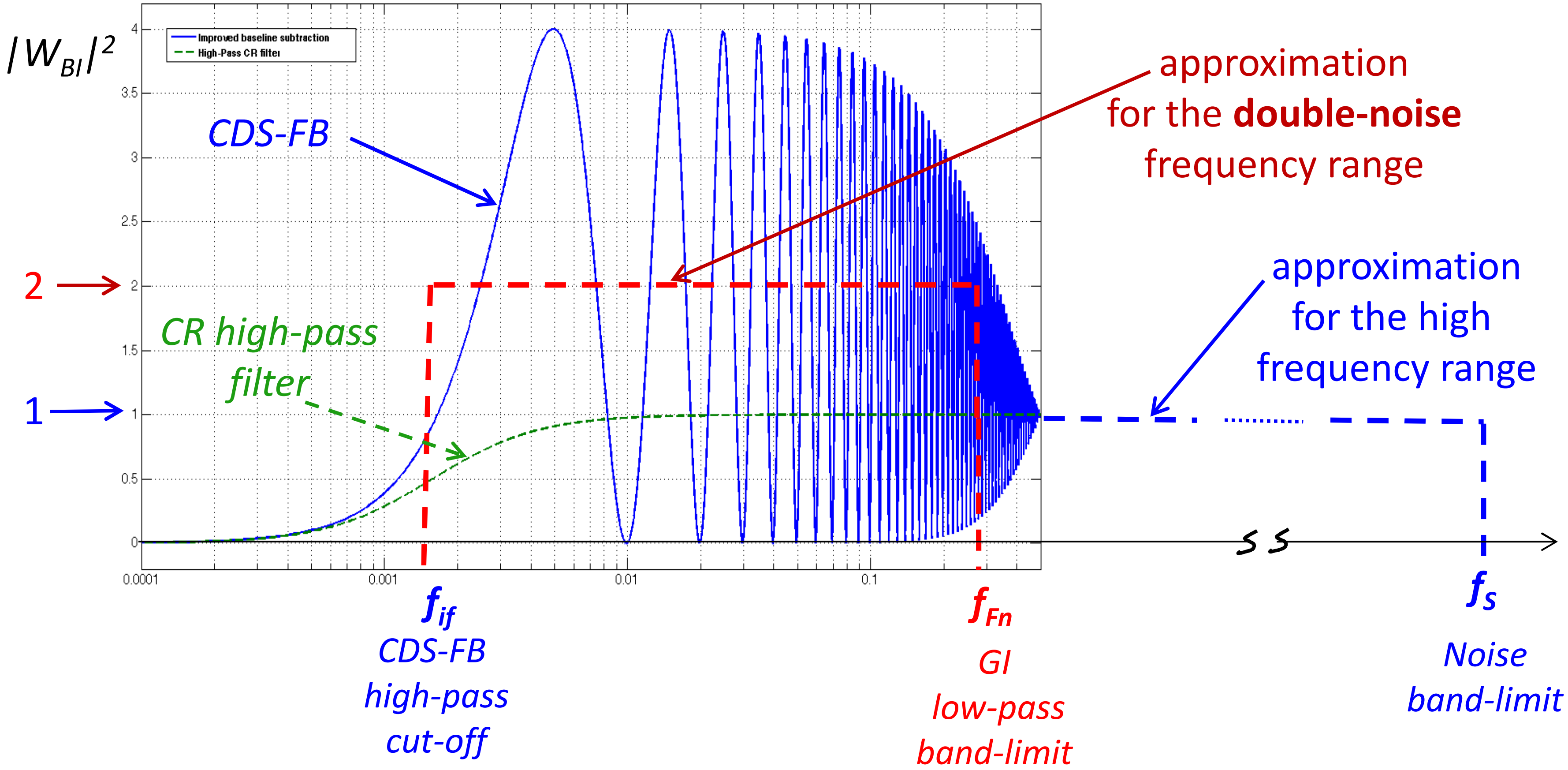
- At low frequency (*f << 1/T)* it is $W_F(f) \approx 1$ and W_{BI} has a high-pass cutoff equivalent to a CR differentiator with *RC=T*

$$\boxed{\left|W_B(\omega)\right|^2 \approx \omega^2 T^2 = \omega^2\left(T_P + \frac{T_F}{2}\right)^2}$$

$$\boxed{f_{if} \approx \frac{1}{2\pi T} = 1\Big/ 2\pi\left(T_P + \frac{T_F}{2}\right)}$$ cutoff frequency

- At high frequency above the GI low-pass cutoff (*f >> f_n =1/2T_F*) it is $|W_F(f)| \approx 0$ so that $|W_{BI}(f)|^2 \approx 1$
- In the intermediate range (*1/T<< f << 1/2T_F*) it is roughly $W_F(f) \approx 1$ so that roughly it is $|W_{BI}(f)|^2 \approx 2(1 - \cos 2\pi f T)$. In this range the average value is about $|W_{BI}(f)|^2 \approx 2$, hence we can denote it as **double-noise range**

CDS-FB : Cut-off and Noise Filtering



Example of CDS-FB with $T_P = 101$ and $T_F = 2$

for comparison, a CR filter with equal cutoff $RC = T = T_P + T_F$ is reported

CDS-FB : output noise power

$$\boxed{\overline{n^2} = \int_0^{f_S} S(f)\left|W_{BI}(\omega)\right|^2 df = \int_0^{f_S} S(f)\left[1 + W_F^2 - 2W_F \cos 2\pi f T\right] df}$$

By approximating W_{BI} as outlined, the noise power can be approximately evaluated

1/f noise

$$\overline{n^2_{f,BI}} \approx S_B f_C \ln\left(\frac{f_S}{f_{if}}\right) + S_B f_C \ln\left(\frac{f_{Fn}}{f_{if}}\right)$$

white noise

$$\overline{n^2_{B,BI}} \approx S_B\left(f_S - f_i\right) + S_B\left(f_{Fn} - f_i\right) \approx S_B f_S + S_B f_{Fn}$$

In CDS-FB the noise-doubling effect is strongly reduced with respect to the simple CDS : it occurs only in the range from the low-frequency cutoff to the GI filtering band-limit.

In cases where the GI band-limit is much smaller than the noise band-limit ($f_S >> f_{Fn}$) the effect of noise doubling is practically negligible

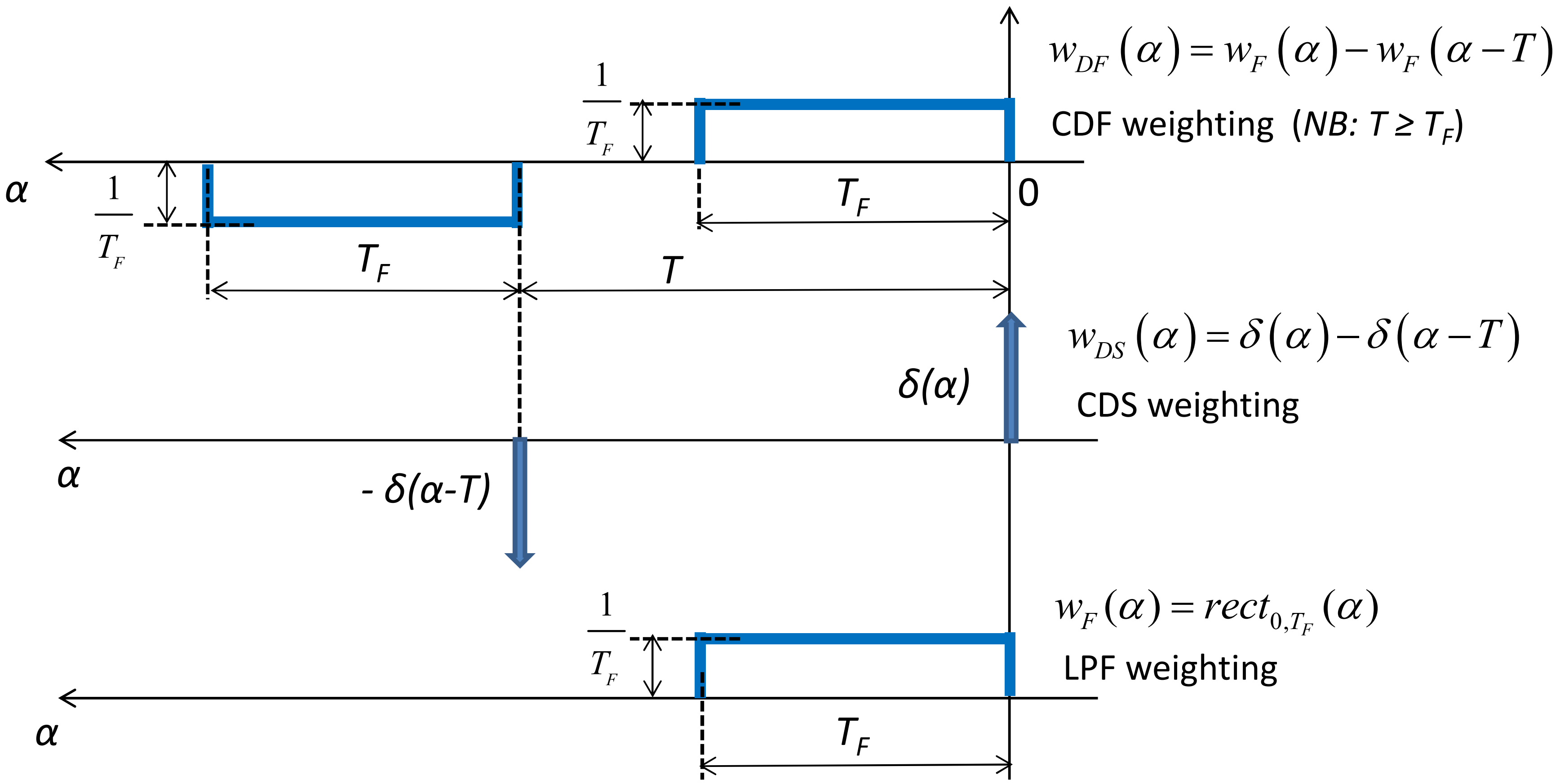
$$\overline{n^2_{f,BI}} \approx S_B f_C \ln\left(\frac{f_S}{f_{if}}\right) \qquad\qquad \overline{n^2_{B,BI}} \approx S_B f_S$$

Correlated Double Filtering
CDF

Correlated Double **Filtering** CDF

- In various cases of pulse-amplitude measurements, filtering by gated integrator (GI) is quite efficient for the white noise component, but not for the 1/f component.
- An improvement is obtained by subtracting from the GI acquisition of the pulse another GI acquistion over an equal interval before the pulse (or after it, anyway outside the pulse)
- This approach has the same conceptual foundation as CDS, but has the two samples filtered by the GI: it is therefore called «Correlated Double **Filtering**» CDF
- The approach can be extended to cases where a constant-parameter low-pass filter LPF is employed for filtering the white noise component and a 1/f component is also present
- In such cases, the measure can be obtained as a difference of two samples of the LPF output: a sample taken at the pulse peak and a sample taken before the pulse (or after it, anyhow outside the pulse)

Correlated Double Filtering CDF



CDF weighting = convolution of CDS weighting with LPF weighting

$$w_{DF}(\alpha) = w_{DS}(\alpha) * w_F(\alpha)$$

Weighting in frequency by CDF

Since in time domain $w_{DF}(\alpha) = w_{DS}(\alpha) * w_F(\alpha)$

in frequency domain it is $W_{DF}(\omega) = W_{DS}(\omega) \cdot W_F(\omega)$

for noise computation $|W_{DF}|^2 = |W_{DS}|^2 \cdot |W_F|^2$

and since $|W_{DS}|^2 = 2(1 - \cos\omega T) = 4sin^2(\omega T/2)$

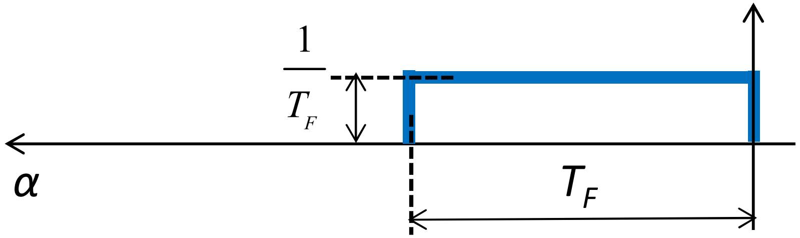
we have

$$\boxed{\left|W_{DF}\right|^2 = 2\left(1-\cos\omega T\right)\cdot\left|W_F\right|^2 = 4\sin^2\left(\frac{\omega T}{2}\right)\cdot\left|W_F\right|^2}$$

The main features of CDS reflect the fact that it is a combination of CDS and LPF :

1. The LPF cuts the noise at high frequencies with its LPF band-limit f_F
2. The CDS cuts the noise at low frequencies with its HPF band-limit $f_{iD} \approx 1/2\pi T$
3. The CDS enhances the noise in the passband between the band-limits (with enhancement factor roughly 2)

Weighting in frequency by CDF with GI



$w_F(\alpha) = rect_{0,T_F}(\alpha)$

LPF weighting

$$w_F(\alpha) = rect_{0,T_F}(\alpha) = rect_{-\frac{T_F}{2},\frac{T_F}{2}}(\alpha - \frac{T_F}{2}) \rightleftarrows W_F(\omega) = Sinc\left(\frac{\omega T_F}{2}\right) e^{-j\omega\frac{T_F}{2}}$$

but the module does not depend on the phase factor (i.e. on the time shift)

$$\left|W_F(\omega)\right| = \left|Sinc\left(\frac{\omega T_F}{2}\right)\right| = \left|sin\left(\frac{\omega T_F}{2}\right) \Big/ \frac{\omega T_F}{2}\right|$$

Therefore

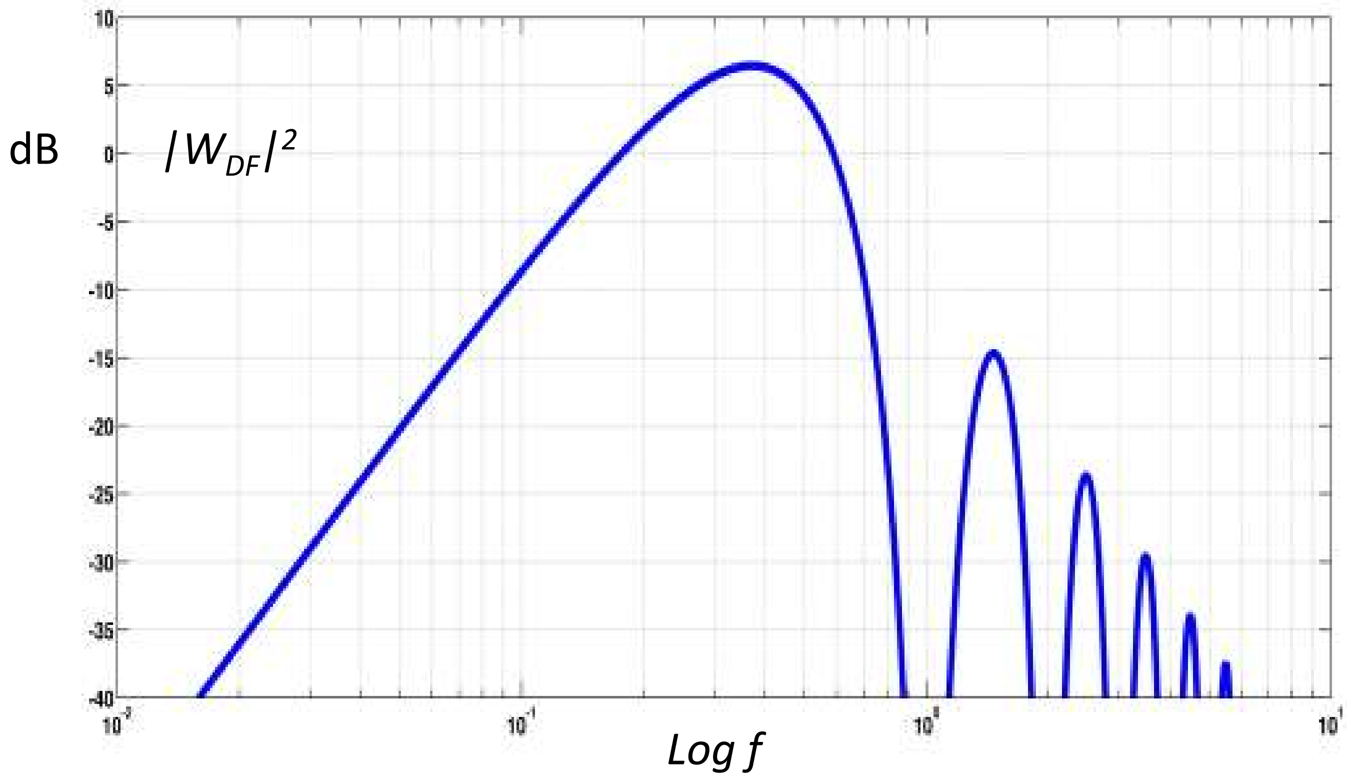
$$\left|W_{DF}\right|^2 = 2\left(1-\cos\omega T\right)\cdot\left|W_F\right|^2 = 2\left(1-\cos\omega T\right)\cdot Sinc^2\left(\frac{\omega T_F}{2}\right) = 4\sin^2\left(\frac{\omega T}{2}\right)\cdot Sinc^2\left(\frac{\omega T_F}{2}\right)$$

that is

$$\boxed{\left|W_{DF}\right|^2 = 2\left(1-\cos\omega T\right)\cdot\frac{sin^2\left(\frac{\omega T_F}{2}\right)}{\left(\frac{\omega T_F}{2}\right)^2} = 4\frac{sin^4\left(\frac{\omega T_F}{2}\right)}{\left(\frac{\omega T_F}{2}\right)^2}}$$

Example of noise filtering by CDF with GI

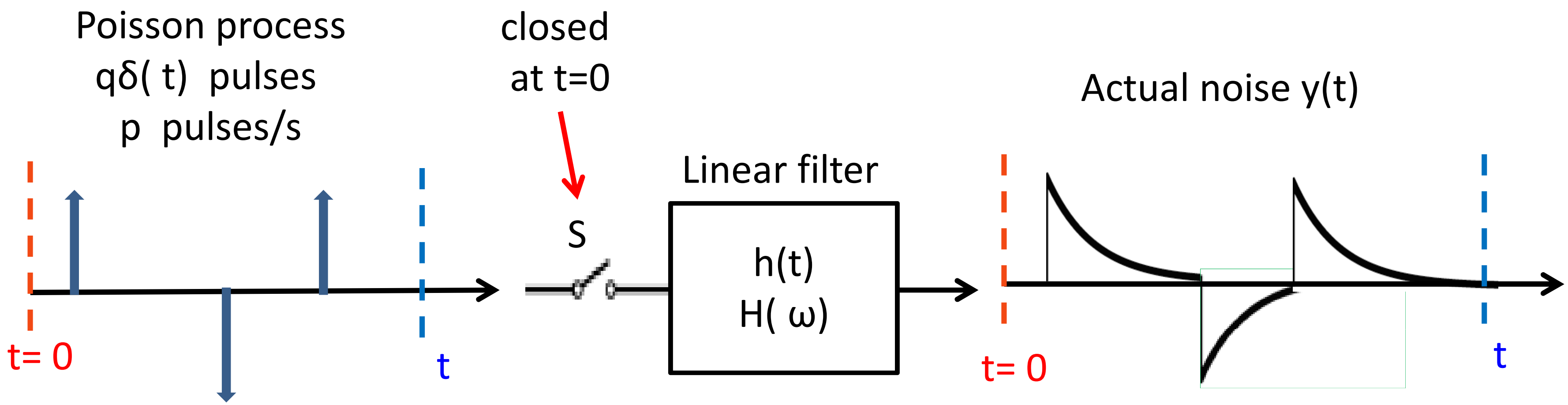
Computed for the case of time shift $T = T_F$ integration time = 1



Appendix 1:
the rise transient of 1/f noise better illustrates why 1/f noise in reality is not divergent !

Rise in time of the 1/f Noise power

The time-domain divergence test applied to 1/f noise illustrates why and how in real cases the 1/f noise power is not divergent even for DC coupled electronics



Noise is «switched-on» at t=0 and measured at time t

the mean square output increases with time t as shown by Campbell's theorem

NB: no more up to ∞

$$\overline{n_y^2} = pq^2 \int_0^t h^2(\alpha)d\alpha$$

Let's compute this variance normalized to unit input spectrum ($pq^2 = 1$)

$$\sigma_y^2 = \int_0^t h^2(\alpha)d\alpha$$

Rise in time of the 1/f Noise power

For generating 1/f noise we must have $\left|H(\omega)\right|^2 = \frac{1}{\omega}$, that is $H\left(\omega\right) = \frac{1}{\sqrt{j\omega}}$

It can be shown mathematically that

$$h\left(t\right) = F^{-1}\left[\frac{1}{\sqrt{j\omega}}\right] = 1\left(t\right)\frac{1}{\sqrt{t}}$$

This is the h(t) required for generating an ideal 1/f noise spectrum without limit at high frequency, which however is unrealistic (like white noise without band-limit).

For generating a real 1/f noise with a high-frequency limit $f_S \approx 1/T_S$ we include then also a low-pass filter with short time constant T_S

$$f_S(t) = \frac{1}{T_S} e^{-\frac{t}{T_S}}$$

and we consider the filtered pulse

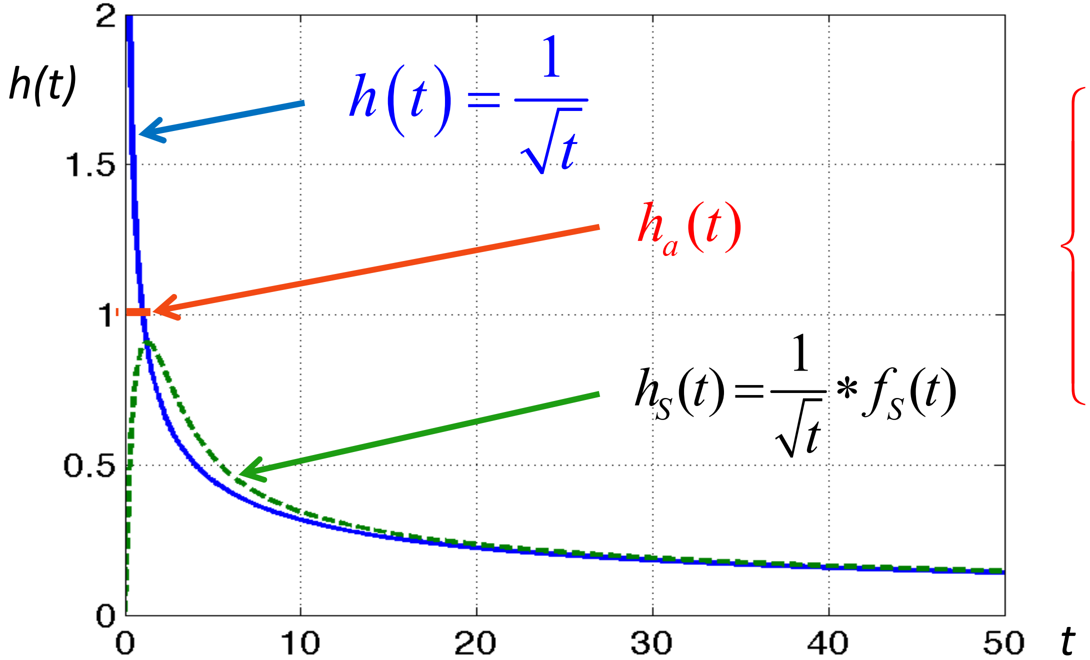
$$h_S(t) = h(t) * f_S(t) = \frac{1}{\sqrt{t}} * \frac{1}{T_S} e^{-\frac{t}{T_S}}$$

for computing the rise in time of the power of the high-frequency-limited 1/f noise

$$\sigma_y^2 = \int_0^t h_S^2(\alpha)\, d\alpha$$

Rise in time of the 1/f Noise power

We can simplify the computation of the σ_y^2 integral by using instead of $h_S(t)$ the function $h_a(t)$ obtained simply by **clipping** $h(t)$ at $t=T_S$, as shown below. The approximation is crude at short times $t \approx T_S$, but good at longer times, which is what we need for computing the rise accurately at long times $t \gg T_S$.

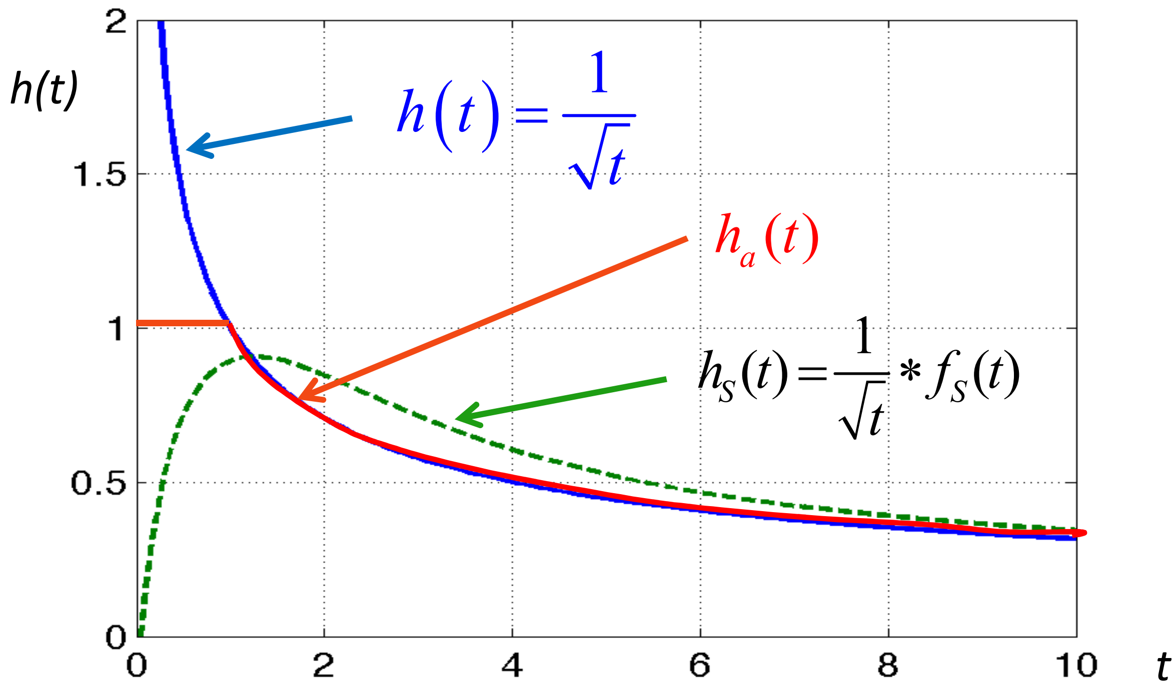


$$
\begin{cases}
h_a(t) = h(t) = \dfrac{1}{\sqrt{t}} & \text{for } t > T_S \\
h_a(t) = \dfrac{1}{\sqrt{T_S}} & \text{for } t \le T_S
\end{cases}
$$

NB: diagrams plotted for $T_s=1$

Rise in time of the 1/f Noise power

Expanded view of the pulse-response waveforms plotted for T_s=1



Rise in time of the 1/f Noise power

By employing as pulse response the waveform $h_a(t)$ clipped at $t=T_S$, we get for $t>T_S$

$$\sigma_y^2 \approx \int_0^t h_a^2(\alpha)d\alpha = \int_0^{T_S} \frac{1}{T_S} d\alpha + \int_{T_S}^t \frac{1}{\alpha} d\alpha = 1 + \ln\left(\frac{t}{T_S}\right)$$

which is a good approximation for longer times, say for $t>100\ T_S$.

For much longer times, say for $t>10000\ T_S$ we can further approximate

$$\boxed{\sigma_y^2 \approx \ln\left(t/T_S\right)}$$

This is indeed the total power over the 1/f noise spectrum limited as follows:

a) at high frequency by the low-pass filter with upper band-limit $f_s=1/2\pi T_s$

b) at low frequency by by a high-pass filtering with lower band-limit $f_i=1/2\pi t$ due to the fact that the noise waveform starts from zero at $t=0$ and is observed after a finite time t

Rise in time of the 1/f Noise power

We see that the rms noise rises in time **very slowly** $\sigma_y \approx \sqrt{1+\ln\left(t/T_S\right)}$

so that for long times $t \gg T_S$ we can estimate

$$\boxed{\sigma_y \approx \sqrt{\ln\left(t/T_S\right)}}$$

EXAMPLE:

let us consider 1/f noise with high frequency limit ≈ 1GHz, that is, with T_S = 1 ns.

If the noise is switched-on at t=0, we will have

- **σ_y ≈ 1** at t= T_S = **1 ns**
- σ_y ≈ 3,7 at t= $10^6 T_S$ = 1 ms
- σ_y ≈ 5,25 at t= $10^{12} T_S$ = 1000 s ≈ 16 minutes after
- **σ_y ≈ 6,43** at t= $10^{18} T_S$ = 10^6 s ≈ **12 days after**

CONCLUSION:

the 1/f noise power in DC coupled electronic systems rises steadily in time, but with progressively slower rate: the **increase is moderate even after a long time** from switch-on.

Appendix 2:
checking CDF weighting
in frequency

Appendix 2: checking CDF weighting in frequency

The CDF weighting can be directly computed in the frequency domain

LPF weighting $w_F(\alpha) \rightleftarrows W_F(\omega) = R(\omega) + jI(\omega)$

CDF weighting $w_{DF}(\alpha) = w_F(\alpha) - w_F(\alpha - T) \rightleftarrows W_{DF}(\omega) = W_F(\omega) - W_F(\omega) e^{-j\omega T}$

$$W_{DF}(\omega) = W_F(\omega) - W_F(\omega) e^{-j\omega T} = W_F(\omega)\left[1 - e^{-j\omega T}\right] =$$
$$= \left[R + jI\right]\left[1 - \cos\omega T + j\sin\omega T\right] =$$
$$= \left[R(1-\cos\omega T) - I\sin\omega T\right] + j\left[I(1-\cos\omega T) + R\sin\omega T\right]$$

CDF weighting for noise computations

$$\left|W_{DF}\right|^2 = \left[R(1-\cos\omega T) - I\sin\omega T\right]^2 + \left[I(1-\cos\omega T) + R\sin\omega T\right]^2 =$$
$$= R^2\left[(1-\cos\omega T)\right]^2 + I^2\left[\sin\omega T\right]^2 - 2RI(1-\cos\omega T)\sin\omega T +$$
$$+ I^2\left[(1-\cos\omega T)\right]^2 + R^2\left[\sin\omega T\right]^2 + 2RI(1-\cos\omega T)\sin\omega T =$$
$$= \left|W_F\right|^2\left[(1-\cos\omega T)\right]^2 + \left|W_F\right|^2\left[\sin\omega T\right]^2 =$$
$$= 2\left(1-\cos\omega T\right)\cdot\left|W_F\right|^2$$