Sensors, Signals and Noise

COURSE OUTLINE

- Introduction
- Signals and Noise
- Filtering: High-Pass Filters 2 – HPF2
- Sensors and associated electronics

1/f Noise and High-Pass Filters 2

- Measuring pulse signals in presence of 1/f noise with constant-parameter filters
- Basic constant-parameter High-Pass Filter (CR differentiator)
- Constant-Parameter High-Pass Filters in measurements of pulses in sequence
- Switched-Parameter High-Pass Filter: the Baseline Restorer
- Appendix: Band-limit of the high-pass CR differentiator for 1/f noise

Measuring pulse signals in presence of 1/f noise

Pulse signals in presence of 1/f noise

Case: amplitude measurement of **pulse** signals with **1/f and wideband** noise.

The classic approach to optimum filtering (to find first a noise-whitening filter and then a matched filter) is arduous in this case because 1/f noise

- sets a remarkably difficult mathematical problem
- makes the whitening filter difficult to design, not implementable with lumped circuit components, but with distributed parameters (distributed RC delay lines, etc.)

However, by noting that

a) for **1/f noise** the filtered power

- mainly depends on the span of the band-pass measured by the **bandlimit ratio**, hence it is **markedly sensitive to the lower bandlimit level**
- **weakly** depends on the **shape** of the filter weighting function

b) for **wideband noise** the S/N

- depends on the span of the band-pass measured by the **bandlimit difference**, hence it is **weakly sensitive to the lower bandlimit level**
- markedly depends on the shape of the weighting function

an alternative approach leading to quasi-optimum filtering can be devised

Pulses and 1/f noise: filtering in two-steps

FIRST STEP:

- Design a **main filter** for signal and wideband noise only (that is, considering non-existent the 1/f noise) and then
- Take then into account the 1/f component and evaluate the **additional noise power** that 1/f noise brings to the main filter output.

In the (lucky) cases where this 1/f noise power is smaller than the wide-band noise (or at least comparable), the main filter may be considered sufficient without further filtering.

Otherwise, if the addition due to 1/f noise is excessive, proceed to the

SECOND STEP :

- design an **additional filter** for limiting the 1/f noise power without worsening excessively the filtering of the wideband noise.

It is obviously a high-pass filter, which must combine the goal of

a) reducing efficiently the 1/f noise power

with the further requirements of

b) limiting to tolerable level the increase of the filtered wide-band noise
c) limiting to tolerable level the reduction of the output **signal** amplitude

Filtering Pulses and 1/f Noise: First Step

The issue is better clarified by considering as FIRST STEP the **optimum filter for signal and wide-band noise (or its approximation)** composed by

- Noise-whitening filter, with output white noise S_B and pulse signal.
  Let f_S be the upper band-limit and A the center-band amplitude of the pulse transform.
- Matched filter, which has weighting function matched to the pulse signal from the whitening filter and is therefore a low-pass filter with upper bandlimit f_S.
  The output has a signal with amplitude roughly $V_S \approx A\, f_S$ and band-limited white noise with band-limit f_S and power

$$\overline{n_B^2} \approx S_B f_S$$

For focusing the ideas, let's consider a well known specific case: filtering of pulse-signals from a high impedance sensor with an approximately optimum filter, i.e. with matched filter approximated by a constant-parameter RC integrator.
In this case, the output noise corresponding to the input wide-band noise is a white noise spectrum with band-limit set by a pole with time constant $RC=T_{nc}$

Filtering Pulses and 1/f Noise: Second Step

Let's now take into account also a 1/f noise source, which brings at the whitening filter output a significant 1/f spectral density $S_B f_C / f$.

At high frquency, the 1/f component is limited by the upper bandlimit f_S of the matched filter.

At low frequency, the 1/f component can be limited by a lower band-limit f_i set by an additional constant-parameter filter. With $f_i << f_S$ the output power of the *1/f* noise can be evaluated as

$$\overline{n_{fn}^2} \approx S_B f_C \ln\left(\frac{f_S}{f_i}\right)$$

However, the constant-parameter high-pass filter operates also on the signal: it attenuates the low frequency components and thus causes a loss in pulse amplitude, hence a loss in S/N. The reduced amplitude is roughly evaluated as

$$V_S \approx A(f_S - f_i) = A f_S \left(1 - f_i / f_S\right)$$

For limiting the signal loss, f_i / f_S must be limited; e.g. for keeping loss < 5% it must be

$$f_i / f_S \leq 0,05 \quad \text{that is} \quad \ln\left(f_S / f_i\right) \geq 3$$

Filtering Pulses and 1/f Noise: Second Step

For reducing the 1/f noise to the white noise level or lower

$$S_B f_C \ln\left(f_S / f_i\right) \leq S_B f_S$$

We need that

$$f_C \leq \frac{f_S}{\ln\left(f_S / f_i\right)}$$

and since for keeping the signal loss <5% it must be $\ln\left(f_S / f_i\right) \geq 3$

we need to have

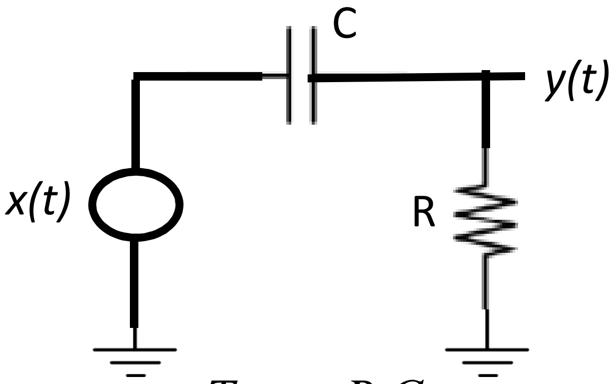
$$f_C < f_S / 3$$

This means that the goal can be achieved only if the 1/f noise component is low or moderate. Note that $\boldsymbol{f_C}$ **and** $\boldsymbol{f_S}$ **are data** of the problem, they cannot be changed. In cases where f_C exceeds the above limit, a constant-parameter high-pass filter is NOT a suitable solution for reducing the 1/f noise power.

CONCLUSION: constant-parameter high-pass filters can be useful as additional filter for limiting the 1/f noise, but just in cases with moderate 1/f noise intensity, because of their detrimental effect on the signal pulse amplitude.

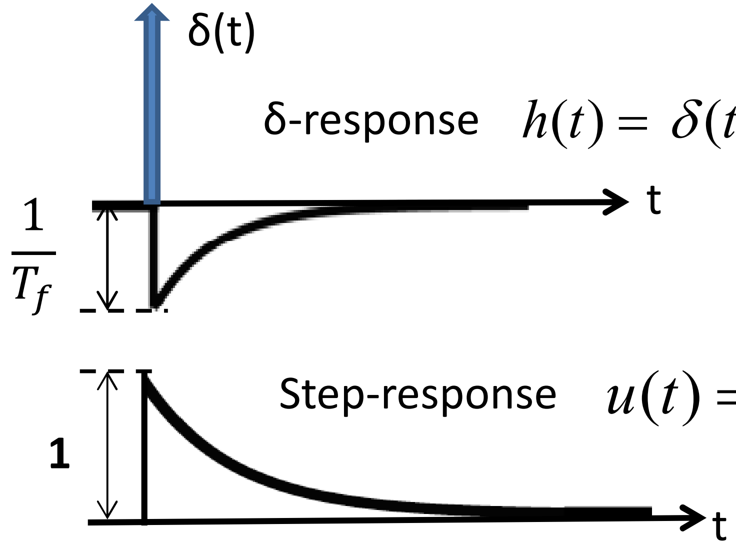
Basic constant-parameter High-Pass Filter (CR differentiator)

Basic High-Pass Filter (CR differentiator)



$$T_f = RC$$

$$f_p = \frac{1}{2\pi T_f}$$



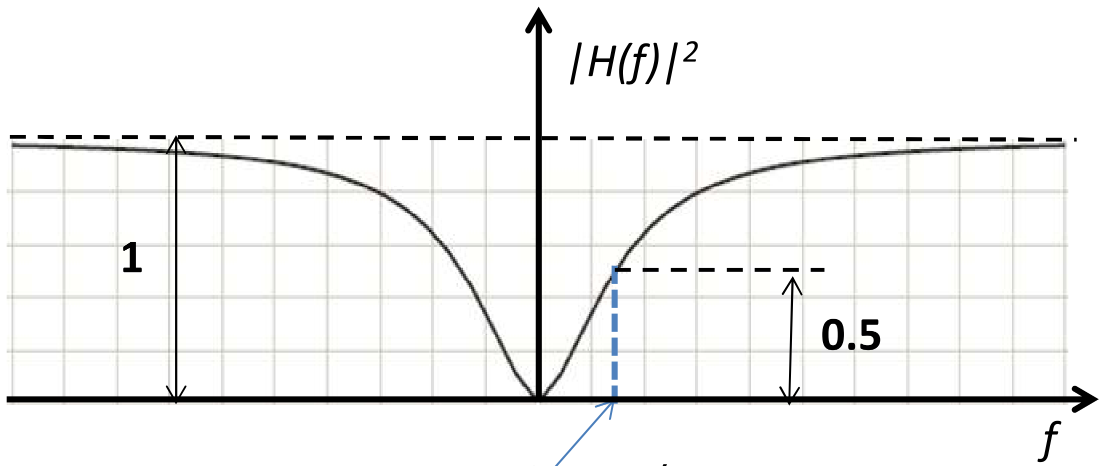
δ-response $h(t) = \delta(t) - 1(t)\frac{1}{T_f} e^{-\frac{t}{T_f}}$

Step-response $u(t) = 1(t)\, e^{-\frac{t}{T_f}}$

Transfer function

$$H(f) = \frac{j\,2\pi f T_f}{1 + j\,2\pi f T_f}$$

$$|H(f)|^2 = \frac{(2\pi f T_f)^2}{1 + (2\pi f T_f)^2}$$



3 dB frequency (Pole) $f_p = 1/2\pi T_f$

A view of High-Pass Filtering

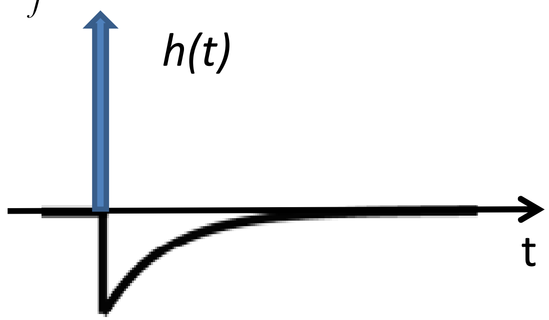
The intuitive view

«High-Pass Filter = All-Pass - Low-Pass Filter»

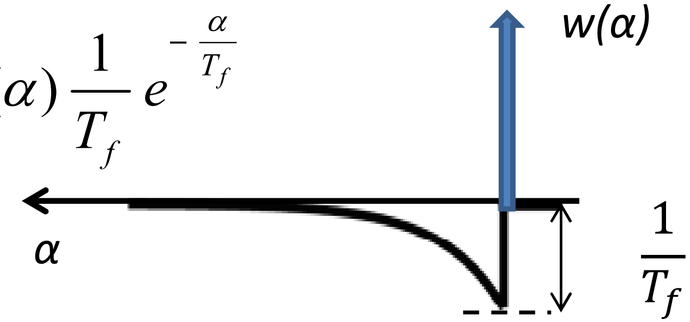
is confirmed by

Transfer function $$H(f) = 1 - \frac{1}{1 + j\,2\pi f T_f} = \frac{j\,2\pi f T_f}{1 + j\,2\pi f T_f}$$

δ-response $$h(t) = \delta(t) - 1(t)\,\frac{1}{T_f}\,e^{-\frac{t}{T_f}}$$



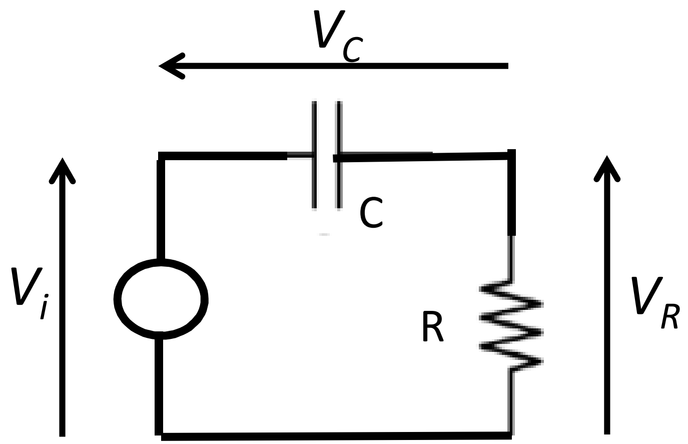
Weighting function $$w(\alpha) = \delta(\alpha) - 1(\alpha)\,\frac{1}{T_f}\,e^{-\frac{\alpha}{T_f}}$$



A view of High-Pass Filtering

The circuit mesh structure itself confirms that

«High-Pass Filter = All-Pass - Low-Pass Filter»



V_i = input voltage

V_C = low-pass filtered V_i

V_R = high-pass filtered V_i

Kirchoff's mesh voltage law

$$V_i = V_C + V_R$$

Therefore $V_R = V_i - V_C$

that is

High-pass filtered V_i = resistor voltage =

= input voltage V_i – capacitor voltage =

= input voltage V_i – Low-pass filtered V_i

Band-limit of CR differentiator

High-pass band-limit for White noise

Premise: with only a high-pass CR filter the white noise power $\overline{n_B^2}$ *is divergent, therefore we consider here also a low-pass filter with band-limit* f_s *>> 1/RC.*

The high-pass band-limit f_i of the CR filter with weighting function *W(f)* is defined by

$$\overline{n_B^2} = S_B \int_0^{f_S} \left|W(f)\right|^2 df = S_B \int_0^{f_S} \frac{\left(f/f_p\right)^2}{1+\left(f/f_p\right)^2}\, df = S_B(f_S - f_i)$$

The computation of the integral can be avoided by recalling that

CR high pass filter = all-pass – RC low-pass filter

and therefore

high-pass band-limit f_i of the CR filter = low-pass band-limit f_h of the RC filter

$$f_{i\,CR} = f_{h\,RC} = \frac{1}{4RC}$$

Band-limit of CR differentiator

High-pass band-limit for 1/f noise

Premise: with only a high-pass CR filter the 1/f noise power $\overline{n_f^2}$ *is divergent, therefore we consider here also a low-pass filter with a high band-limit* $f_s >> 1/RC$.

The high-pass band-limit f_{if} of the CR filter is defined by

$$\overline{n_f^2} = S_B f_c \int_0^{f_S} \frac{\left(f/f_p\right)^2}{1+\left(f/f_p\right)^2} \frac{df}{f} = S_B f_c \int_{f_{if}}^{f_S} \frac{df}{f} = S_B f_c \ln\left(\frac{f_S}{f_{if}}\right)$$

In this case the first integral is fairly easily computed* and shows that

$$f_{if} = \frac{f_p}{\sqrt{1+\left(f_p/f_s\right)^2}}$$

that is, for $f_s >> f_p$

$$f_{if} \approx f_p = \frac{1}{2\pi RC}$$

** See the appendix. Since the high-pass limit of the CR differentiator is also the low-pass limit of the RC integrator, we can avoid the computation for the latter (which is fairly difficult).*

About band-limits and noise power

- The upper frequency limit f_S :

  - is necessary for limiting the white noise power

  - is useful also for limiting the 1/f noise power

  - the level of f_S is dictated by the pulse signal to be measured

- The lower frequency limit f_i :

  - is necessary for limiting the *1/f* noise power,

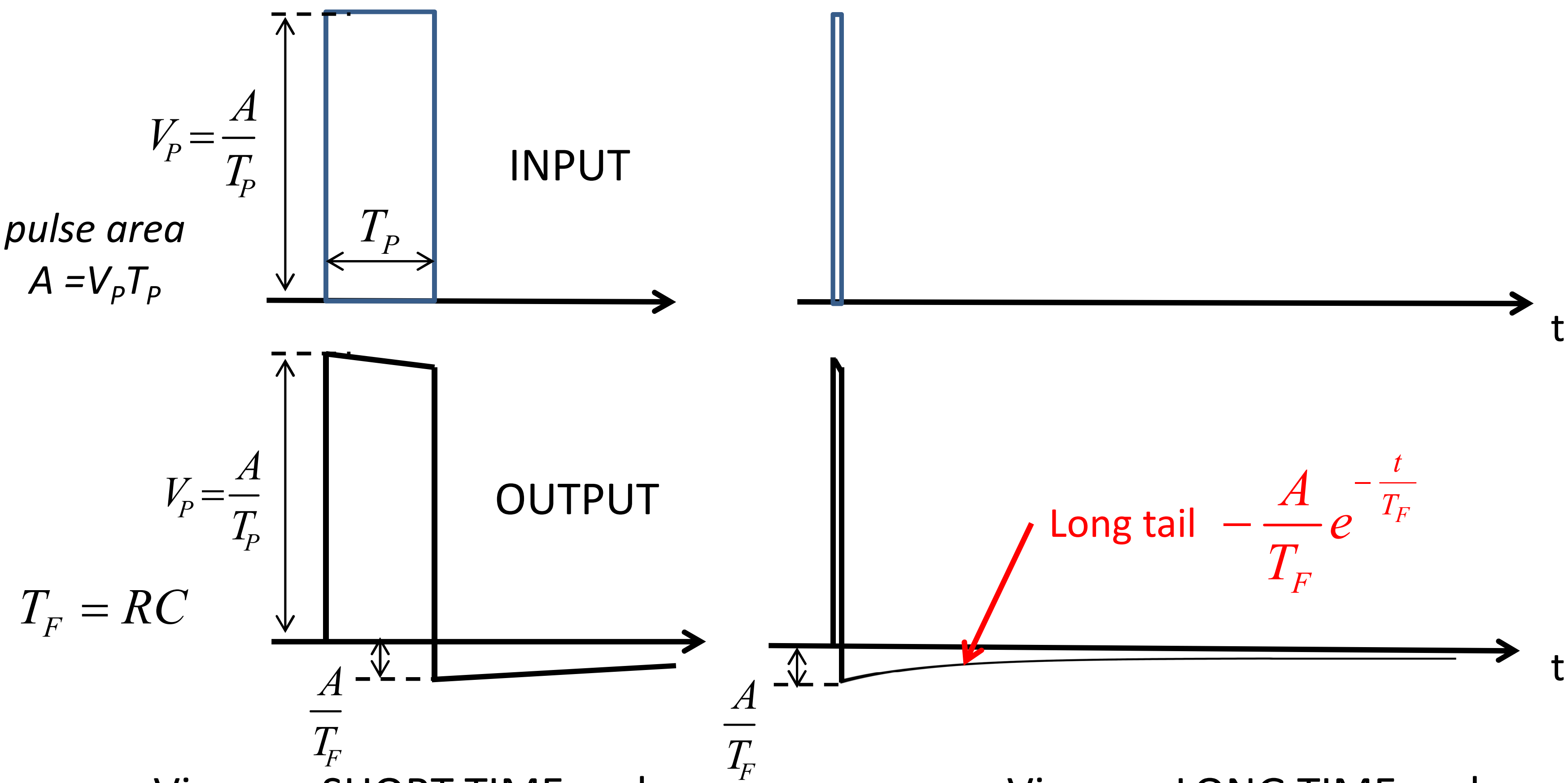
  - the selected level of f_i is conditioned by the pulse signal, it cannot be arbitrary

  - however, the reduction of *1/f* noise is significant even with fairly low f_i , that is, with f_S / f_i values that are high, but anyway finite.

Constant-Parameter High-Pass Filters in measurements of pulses in sequence

CR filter and pulse sequence

Let's look in detail the effect of a high-pass filter (RC = T_F) on a pulse signal

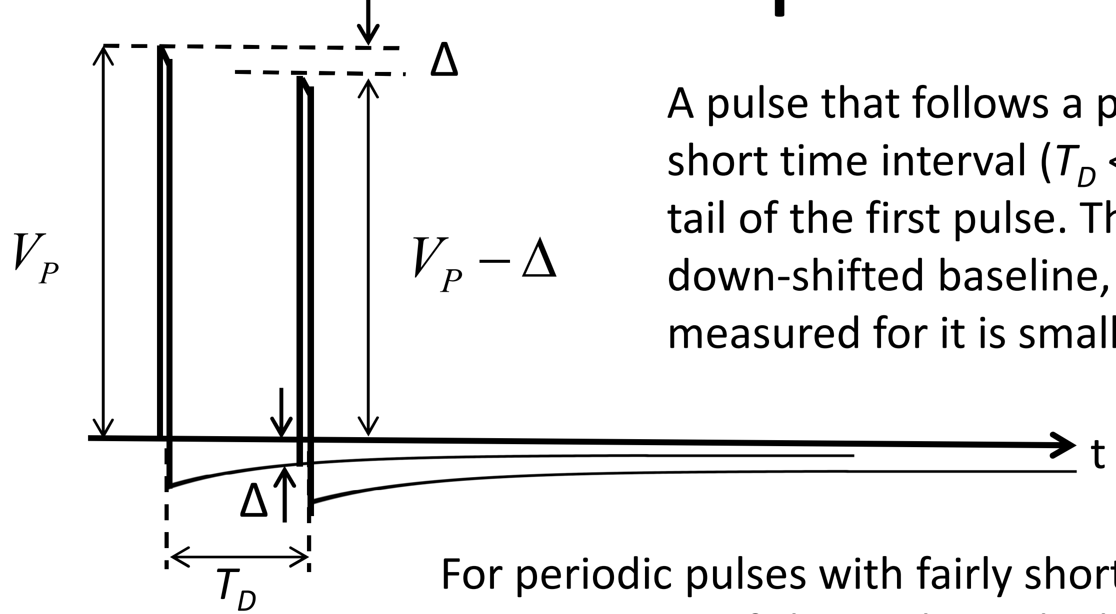


View on SHORT TIME scale

View on LONG TIME scale

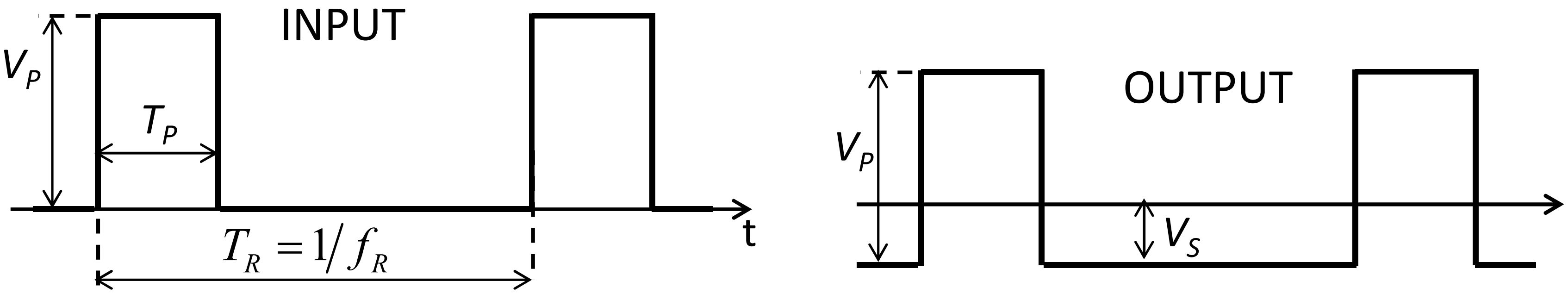
NB: DC transfer of CR is zero → net area of the output signal is zero

CR filter and pulse sequence



A pulse that follows a previous one within a fairly short time interval ($T_D < 5\ T_F$) steps on the slow tail of the first pulse. Therefore, it starts from a down-shifted baseline, so that the amplitude measured for it is smaller than the true one.

For periodic pulses with fairly short repetition period $T_R << T_F$, the superposition of slow pulse-tails shifts down the baseline by a V_S that makes zero the net area of the output signal



Repetition-rate-dependent baseline-shift $V_S = V_P \dfrac{T_P}{T_R} = A\ f_R$

Drawbacks of the CR differentiator filter

The high-pass filtering (differentiator action) of the CR filter has **MIXED effects.**

- The effect **on noise is ADVANTAGEOUS:** by cutting off the the low frequencies it markedly decreases the 1/f noise power (and mildly reduces the white noise power)
- The effect **on the signal is DISADVANTAGEOUS**:

➢ it decreases the signal amplitude by cutting off the low frequencies of the signal , hence f_i must be kept low ($f_i << f_S$ of the pulse) in order to limit the signal loss. However, this limits also the reduction of 1/f noise

➢ it generates slow tails after the pulses, which shift down the baseline and thus cause an error in the measured amplitude of a following pulse

➢ With a **periodic** sequence of equal pulses, all pulses find the same baseline shift. The amplitude error is constant, sistematically dependent on the repetition rate.

➢ With **random-repetition** pulses (e.g. pulses from ionizing radiation detectors) the pulses occur randomly in time. Hence the random superposition of tails produces a randomly fluctuating baseline shift. The resulting amplitude error is random:
in this case the effect is equivalent to that of an additional noise source.

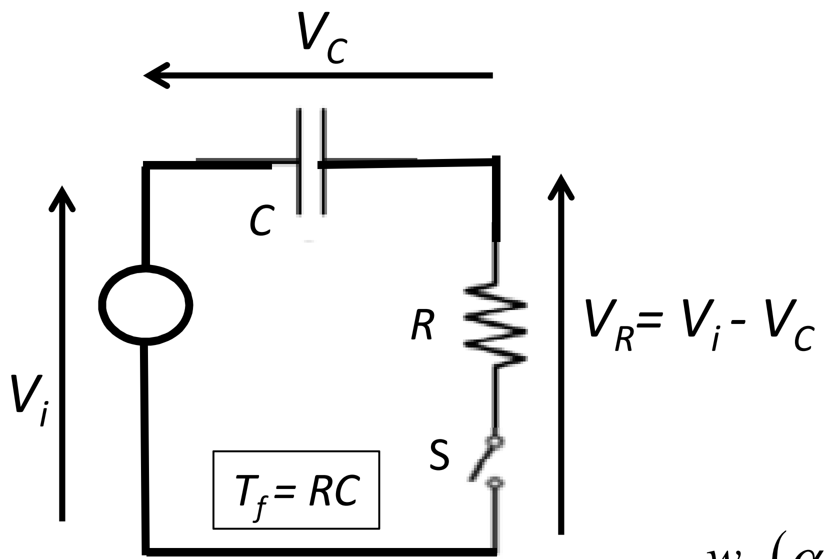
CONCLUSION: a differentiator action is **desirable on noise**, but **NOT on the signal.**

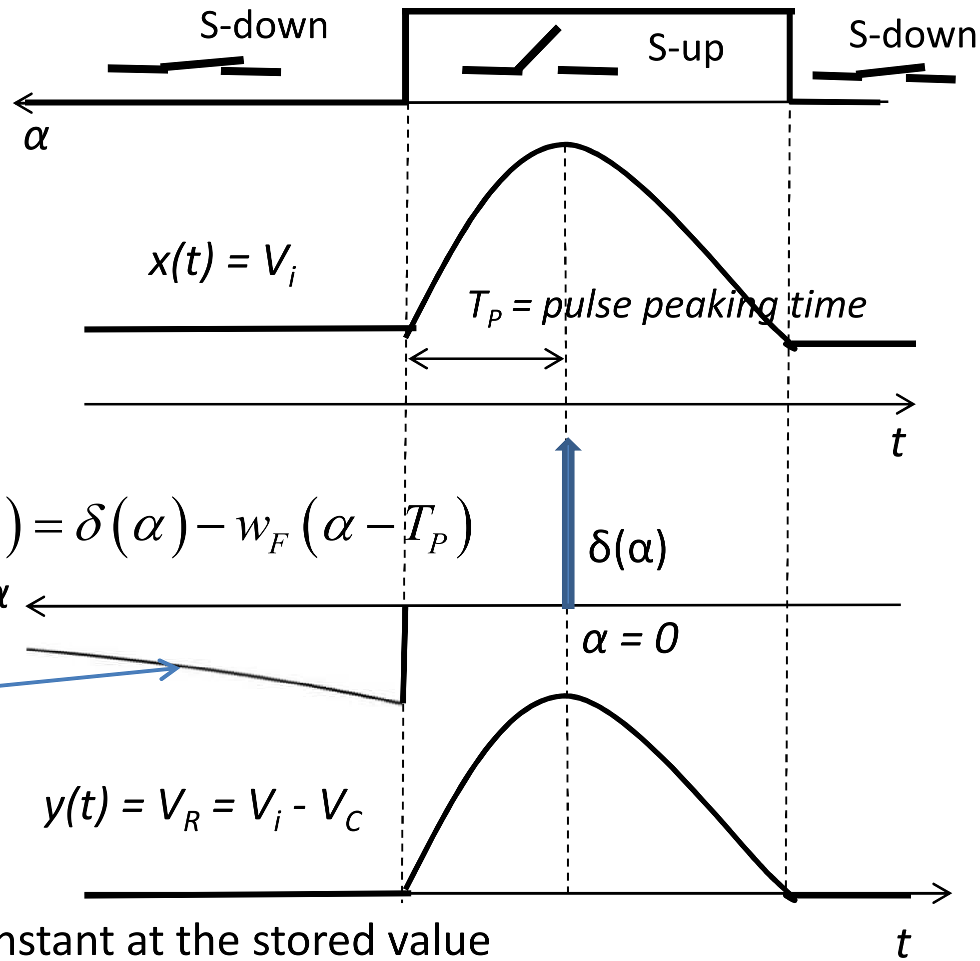
WANTED: not a constant-parameter differentiator, but a true **Base-Line Restorer (BLR)**

Switched-Parameter High-Pass Filter: the Baseline Restorer

Baseline Restorer (BLR) principle: switched CR

High-pass filtering action **on the noise** and **NOT on the signal: switched-parameter** CR filter with CR → ∞ when signal is present, finite CR = T_F when no pulse is present



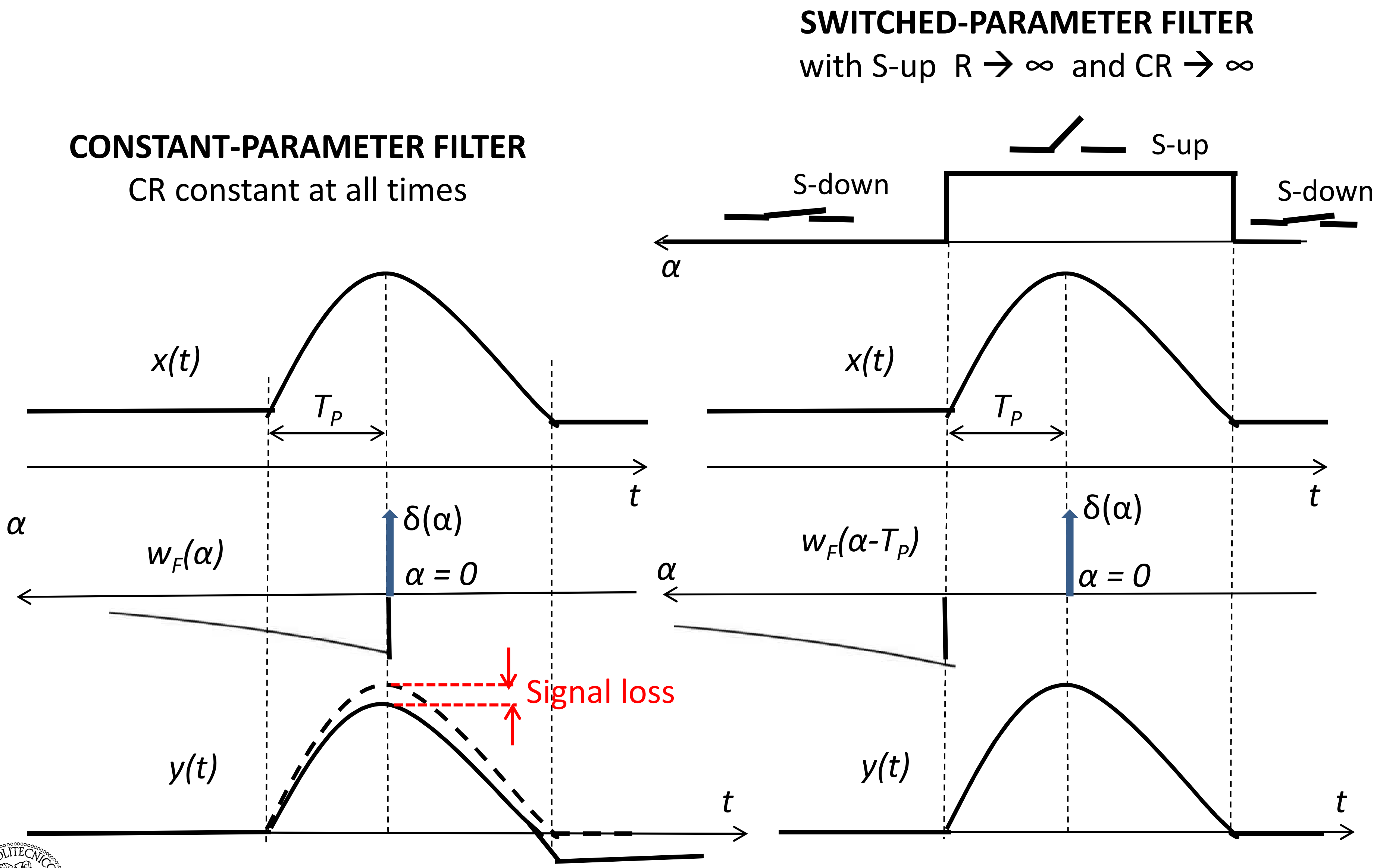


$$w_B(\alpha) = \delta(\alpha) - w_F(\alpha - T_P)$$

$$w_F(\alpha - T_P) = 1(\alpha - T_P)\frac{1}{T_f}e^{-\frac{\alpha - T_P}{T_f}}$$

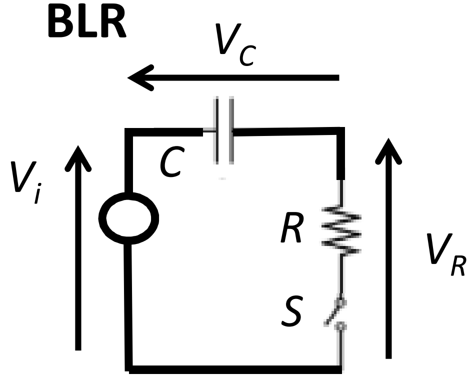
As S is open at the pulse onset (at $\alpha = T_P$), charging of C stops and voltage V_C stays constant at the stored value

Comparing constant CR filter and BLR



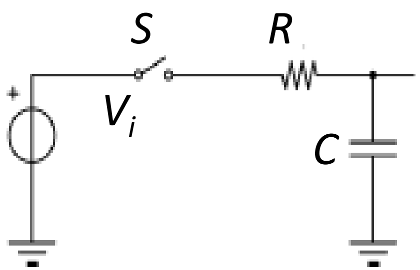
BLR = All Pass - Low-Pass Boxcar Integrator

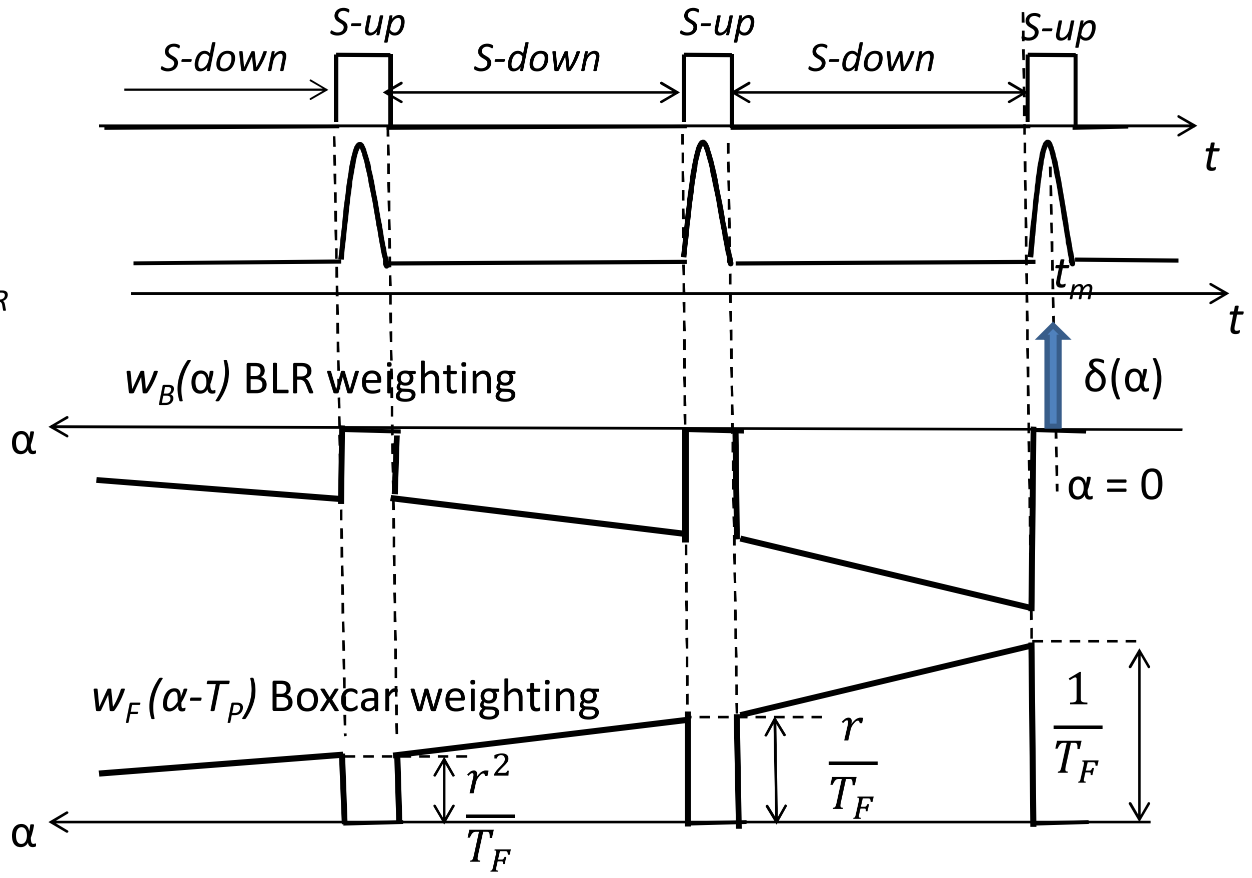
BLR



$V_R = V_i - V_C$

BOXCAR

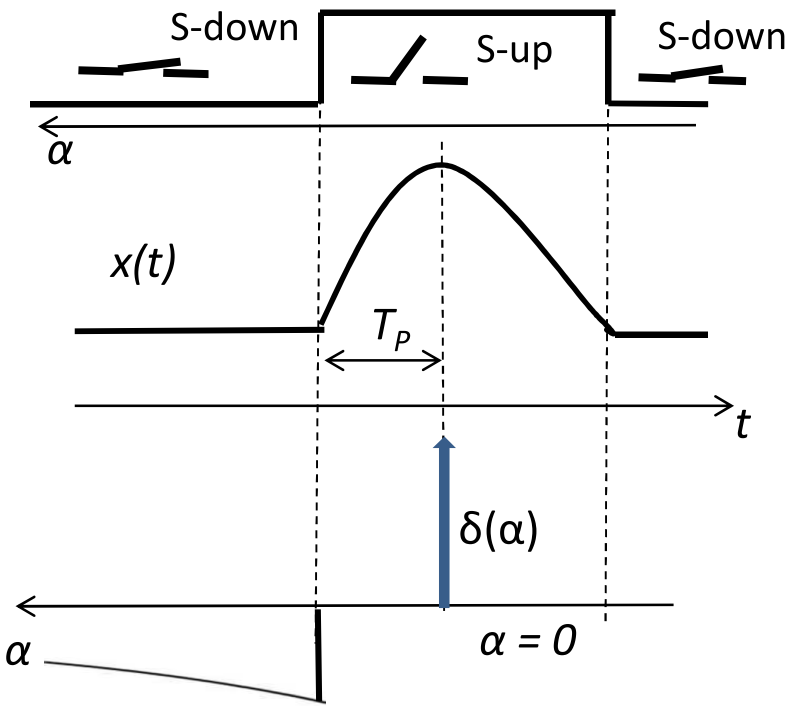




BLR weighting = All Pass – Low-pass Boxcar weighting

$$w_B(\alpha) = \delta(\alpha) - w_F(\alpha - T_P)$$

BLR weighting in frequency



BLR principle is alike filtered zero-setting, but with a **basic advantage:**
much **shorter T_P**
much **higher band-limit f_{if} (high-pass)**

(the BLR switch is electronically controlled, the interval T_P can be very short)

BLR weighting = All Pass – Low-pass

$$w_B(\alpha) = \delta(\alpha) - w_F(\alpha - T_P)$$

Low-pass weighting in frequency:

$$W_F(\omega) = F[w_F(\alpha)] = R_F(\omega) + i\, I_F(\omega)$$

BLR weighting in frequency:

$$W_B(\omega) = 1 - e^{j\omega T_P} W_F(\omega) = 1 - [\cos\omega T_P - j\sin\omega T_P]\cdot[R_F + jI_F] =$$

$$= [1 - R_F\cos\omega T_P - I_F\sin\omega T_P] - j[I_F\cos\omega T_P - R_F\sin\omega T_P]$$

BLR weighting in frequency

BLR weighting for noise:

$$\left|W_B(\omega)\right|^2 = [1 - R_F \cos\omega T_P - I_F \sin\omega T_P]^2 + [I_F \cos\omega T_P - R_F \sin\omega T_P]^2 =$$

$$= 1 + R_F^2 + I_F^2 - 2R_F \cos\omega T_P + 2I_F \sin\omega T_P =$$

$$= 1 + \left|W_F\right|^2 - 2R_F \cos\omega T_P - 2I_F \sin\omega T_P$$

Let's consider just cases where the interval between pulses is much longer than T_F so that

$$w_F(\alpha) = 1(\alpha)\,\frac{1}{T_f}\,e^{-\frac{\alpha}{T_f}} \qquad \text{and} \qquad W_F(\omega) = \frac{1}{1 + j\omega T_F}$$

and therefore

$$\left|W_B(\omega)\right|^2 = 1 + \frac{1}{1 + \omega^2 T_F^2} - 2\frac{1}{1 + \omega^2 T_F^2}\cos\omega T_P + 2\omega T_F \cdot \frac{1}{1 + \omega^2 T_F^2}\sin\omega T_P$$

BLR cutoff

In the low-frequency region $\omega \ll \frac{1}{T_P}$ with the approximations

$$\sin \omega T_P \approx \omega T_P \qquad \cos \omega T_P = 1 - \frac{\omega^2 {T_P}^2}{2}$$

we get

$$\left|W_B(\omega)\right|^2 \approx 1 + \frac{1}{1+\omega^2 T_F^2} - \frac{2}{1+\omega^2 T_F^2} + \frac{\omega^2 T_P^2}{1+\omega^2 T_F^2} + 2\frac{\omega^2 T_P T_F}{1+\omega^2 T_F^2} =$$

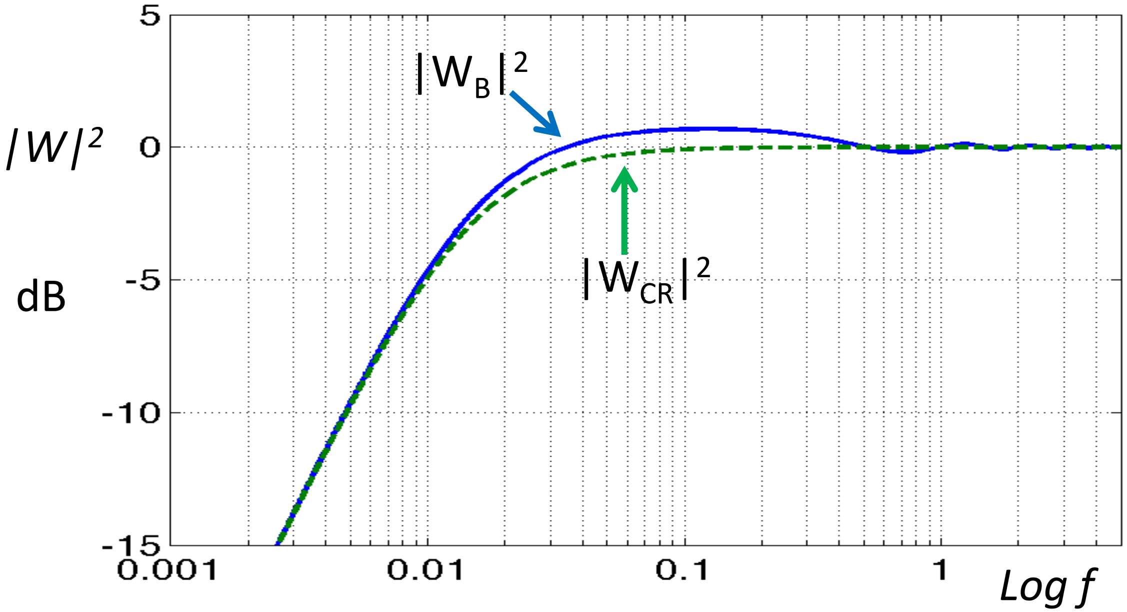
$$= \frac{\omega^2 \left(T_P + T_F\right)^2}{1+\omega^2 T_F^2} = \frac{\omega^2 T_F^2}{1+\omega^2 T_F^2}\left(1 + \frac{T_P}{T_F}\right)^2$$

and in the lower region $\omega \ll \frac{1}{T_F} \ll \frac{1}{T_P}$

$$\left|W_B(\omega)\right|^2 \approx \omega^2 \left(T_P + T_F\right)^2$$

That is, the BLR has a cutoff equivalent to a CR high-pass with RC= T_P+T_F

BLR vs. CR High-Pass Filter: Cut-Off



BODE DIAGRAM
highlights
the low-freq cutoff

Example:
BLR with $T_P = 1$ and $T_F = 10$
CR filter with $RC = T_P + T_F$

$f << 1/T_F$
(i.e. $f << 0{,}1$ in the example)

$$\left|W_B(\omega)\right|^2 \approx \omega^2 \left(T_P + T_F\right)^2$$

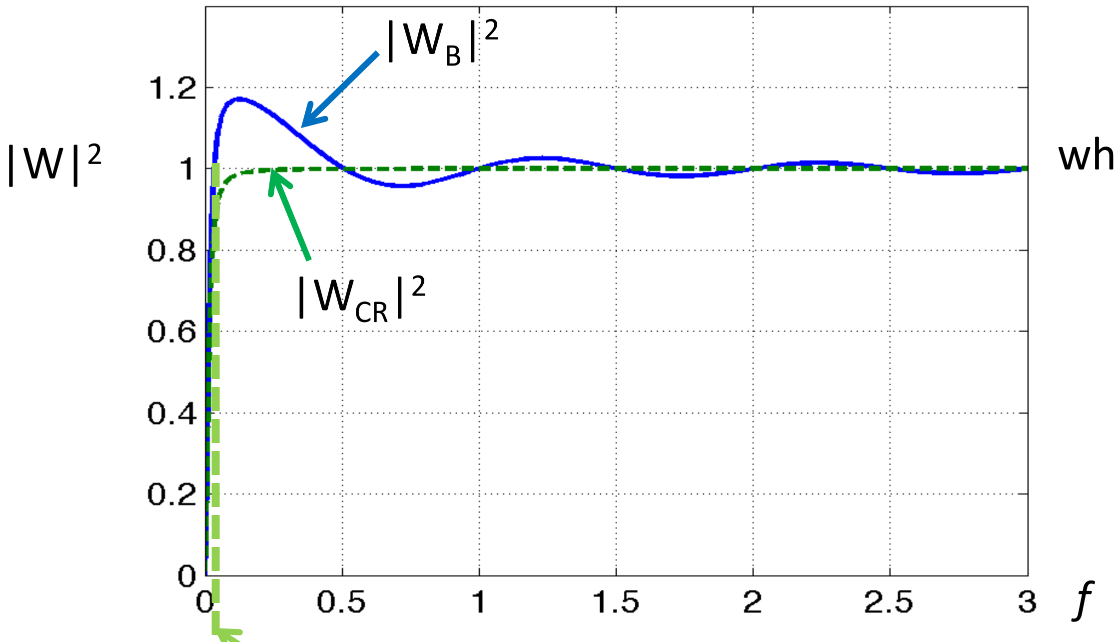
$$\left|W_{CR}(\omega)\right|^2 \approx \omega^2 R^2 C^2$$

$1/T_F < f << 1/T_P$
(i.e. $f << 1$ in the example)

$$\left|W_B(\omega)\right|^2 \approx \frac{\omega^2 \left(T_P + T_F\right)^2}{1 + \omega^2 T_F^2}$$

$$\left|W_{CR}(\omega)\right|^2 = \frac{\omega^2 R^2 C^2}{1 + \omega^2 R^2 C^2}$$

BLR vs. CR High-Pass Filter: White Noise



$$f_{ni} \approx \frac{\pi}{2} f_p = \frac{1}{4RC} = \frac{1}{4\left(T_F + T_P\right)} \approx \frac{1}{4T_F}$$

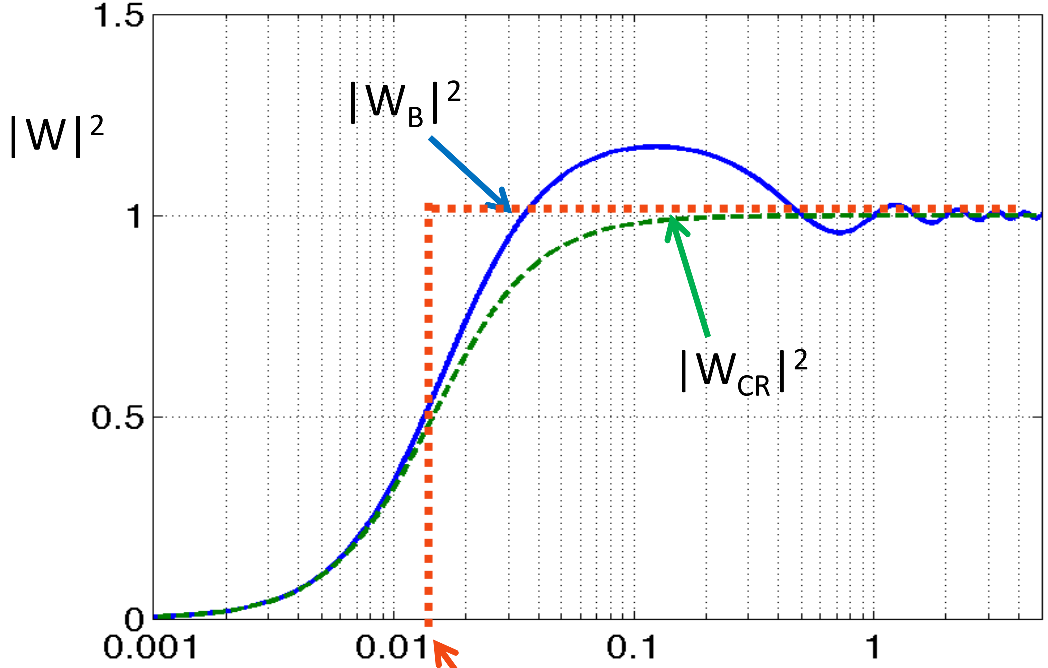
LIN –LIN DIAGRAM
highlights
white noise power ∝ area of $|W|^2$

Example:
BLR with T_P = 1 and T_F = 10
CR filter with RC = $T_P + T_F$

f_{ni} = BLR high-pass band-limit for white noise. Note that:

- f_{ni} is equal to that of the equivalent CR High-pass filter
- f_{ni} is equal to bandlimit of the low-pass section in the BLR circuit

BLR vs. CR High-Pass Filter: 1/f Noise



LIN –LOG DIAGRAM
highlights
1/f noise power ∝ area of $|W|^2$

Example:
BLR with $T_P = 1$ and $T_F = 10$
CR filter with $RC = T_P + T_F$

$$f_{fi} \approx f_p = \frac{1}{2\pi RC} = \frac{1}{2\pi\left(T_F + T_P\right)} \approx \frac{1}{2\pi T_F}$$

f_{fi} = BLR high-pass band-limit for 1/f noise. Note that:

- f_{fi} is equal to that of the equivalent CR High-pass filter
- f_{fi} is equal to bandlimit of the low-pass section in the BLR circuit

Selection of the BLR parameters

The BLR filtering is ruled by:

1. ***T_P*** **time delay** from switch opening to pulse-amplitude measurement.
There is **no choice:** T_P is equal to the rise time from pulse onset to peak.
In fact, T_P can't be shorter than the rise of the pulse signal and should be as short as possible for filtering effectively of the 1/f noise.

2. ***$T_F = RC$*** **differentiation time constant: to be selected** for optimizing the overall filtering of noise. The question is: how should T_F be selected for

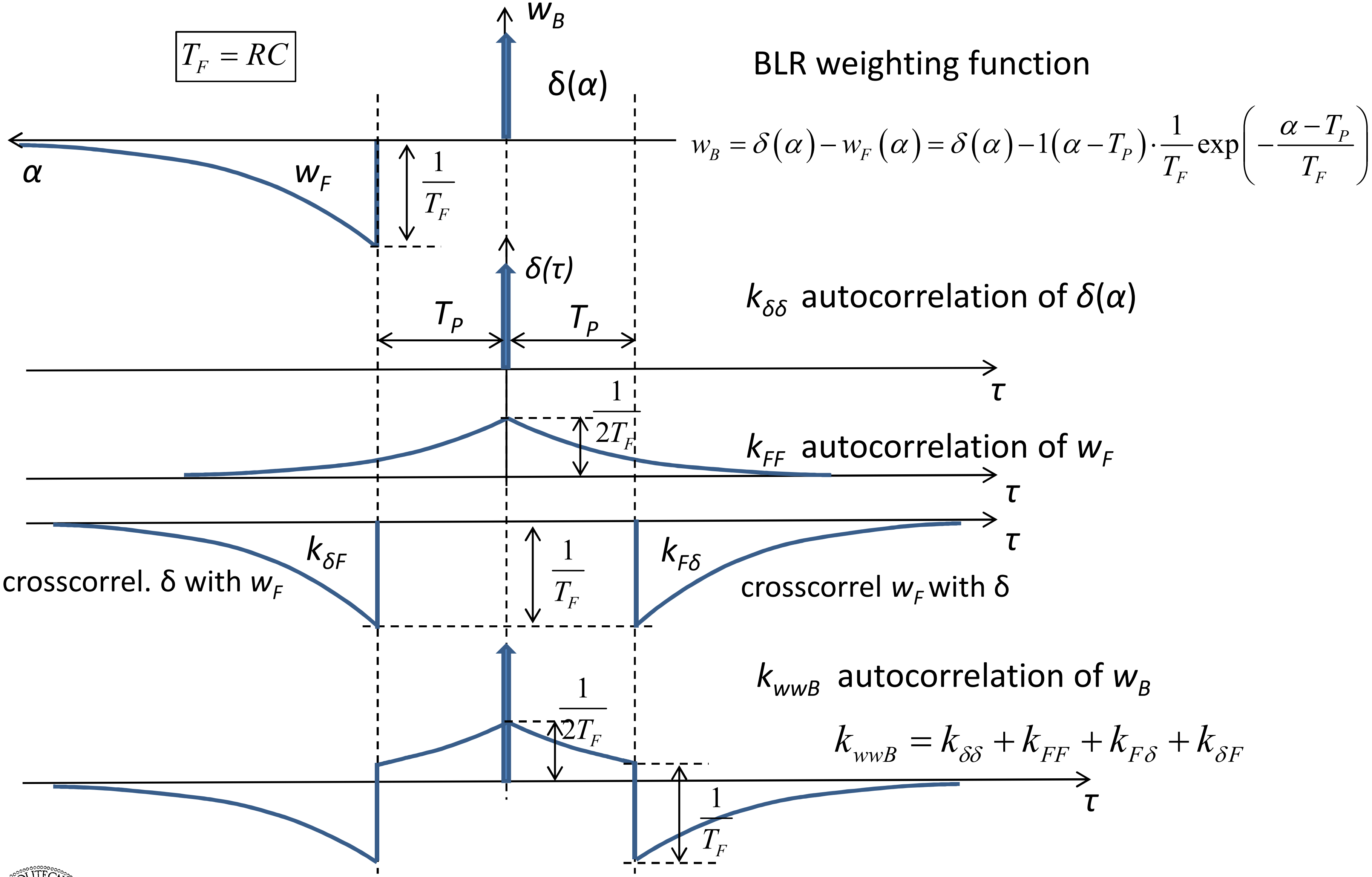
a) providing a good reduction of the 1/f noise power and

b) avoiding to enhance significantly the white noise power

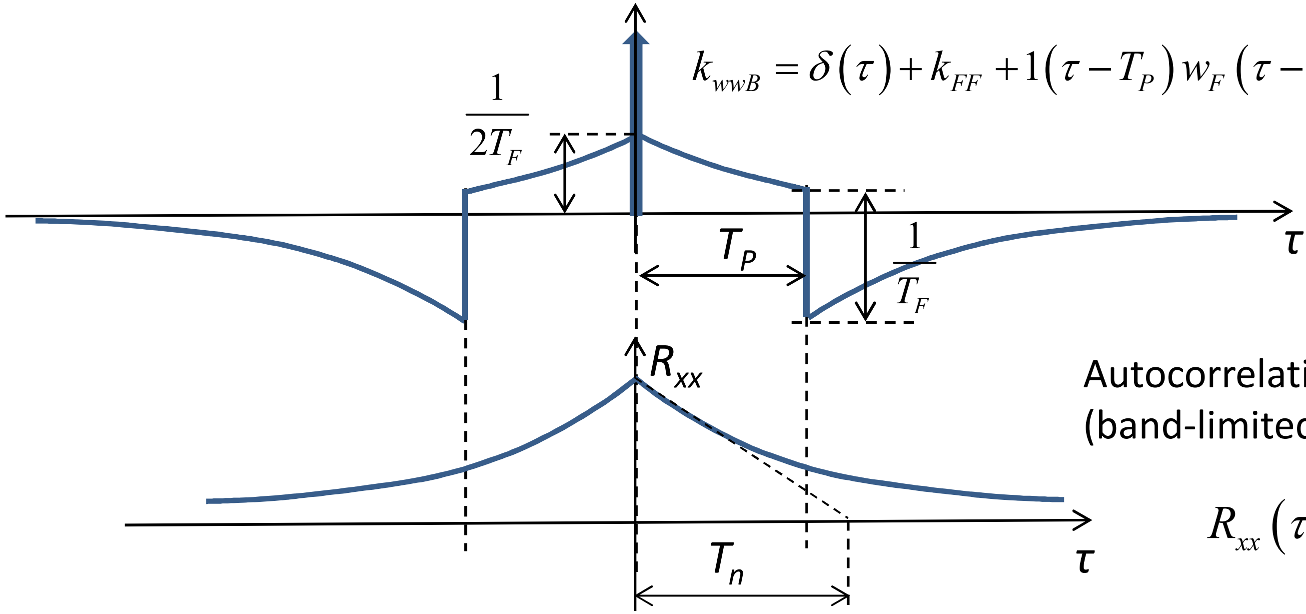
Since the BLR cutoff is set by $1/(T_P + T_F)$, a very short T_F might look advisable, but it is not: a BLR with $T_F \ll T_P$ operates like a CDS, hence it doubles the white noise and remarkably enhances also the 1/f noise above the cutoff frequency.

In the following discussion about the T_F selection, for focusing the ideas we will refer to a specific case: signals from a high impedance sensor processed by an approximately optimum filter, namely a CR-RC filter. The output corresponding to the input wide-band noise is a white spectrum band-limited by a simple pole. Such a situation is met in practice also in many other cases.
A better insight in the issue is gained with a **time-domain analysis of BLR filtering**

BLR Filtering of Noise: time-domain analysis



BLR Filtering of Band-Limited White Noise



$$k_{wwB} = \delta(\tau) + k_{FF} + 1(\tau - T_P)\, w_F(\tau - T_P) + 1(-\tau + T_P)\, w_F(-\tau + T_P)$$

$$w_F(\tau) = \frac{1}{T_F}\exp\left(-\frac{\tau}{T_F}\right)$$

Autocorrelation of white noise (band-limited by a single pole)

$$R_{xx}(\tau) = \overline{n_x^2}\, e^{-\frac{\tau}{T_n}}$$

$$\overline{n_B^2} = \int_{-\infty}^{\infty} R_{xx}(\tau)\, k_{wwB}(\tau)\, d\tau = R_{xx}(0) + 2\int_0^{\infty} R_{xx}(\tau)\frac{1}{2} w_F(\tau)\, d\tau - 2\int_T^{\infty} R_{xx}(\tau)\, w_F(\tau - T_P)\, d\tau =$$

$$= R_{xx}(0) + \int_{-\infty}^{\infty} R_{xx}(\tau)\frac{1}{2} w_F(\tau)\, d\tau - 2\int_0^{\infty} R_{xx}(\beta + T_P)\, w_F(\beta)\, d\beta$$

Denoting

$$r_{xx}(\tau) = \frac{R_{xx}(\tau)}{R_{xx}(0)} = \frac{R_{xx}(\tau)}{\overline{n_x^2}}$$

We have

$$\overline{n_B^2} = \overline{n_x^2}\left\{1 + \int_{-\infty}^{\infty} r_{xx}(\tau)\frac{1}{T_F} e^{-\frac{\tau}{T_F}}\, d\tau - 2\int_0^{\infty} r_{xx}(\beta + T_P)\frac{1}{T_F} e^{-\frac{\beta}{T_F}}\, d\beta\right\}$$

BLR Filtering of Band-Limited White Noise

$$\overline{n_B^2} = \overline{n_x^2}\left\{1+\int_{-\infty}^{\infty} r_{xx}(\tau)\cdot\frac{1}{T_F}e^{-\frac{\tau}{T_F}}d\tau - 2\int_0^{\infty} r_{xx}(\beta+T_P)\cdot\frac{1}{T_F}e^{-\frac{\beta}{T_F}}d\beta\right\} =$$

$$= \overline{n_x^2}\left\{1+\frac{1}{T_F}\int_{-\infty}^{\infty} e^{-\tau\left(\frac{1}{T_F}+\frac{1}{T_n}\right)}d\tau - 2e^{-\frac{T_P}{T_n}}\frac{1}{T_F}\int_0^{\infty} e^{-\beta\left(\frac{1}{T_F}+\frac{1}{T_n}\right)}d\beta\right\} =$$

$$= \overline{n_x^2}\left[1+\frac{T_n}{T_n+T_F} - 2e^{-\frac{T_P}{T_n}}\frac{T_n}{T_n+T_F}\right]$$

and finally

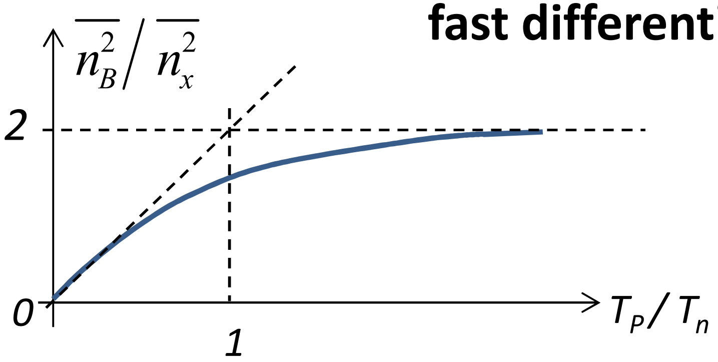
$$\boxed{\overline{n_B^2} = \overline{n_x^2}\left[1+\frac{T_n}{T_n+T_F}\left(1-2e^{-\frac{T_P}{T_n}}\right)\right]}$$

With **fast** **differentiation**, i.e. with $T_F \ll T_n$, it is quantitatively confirmed that the BLR acts like a CDS with $T=T_P$

$$\overline{n_B^2} \approx 2\overline{n_x^2}\cdot\left(1-e^{-\frac{T_P}{T_n}}\right)$$

BLR Filtering with fast differentiation

fast differentiation with $T_F << T_n$



$$\overline{n_B^2} \approx 2\overline{n_x^2} \cdot \left(1 - e^{-\frac{T_P}{T_n}}\right)$$

With $T_F << T_n$ the effect of BLR on **band-limited white noise** depends on how long is the correlation time T_n with respect to the delay T_P

- with **short correlation time** (wide band) the noise is **doubled**:

  with $T_n < T_P/5$ it is $\overline{n_B^2} \approx 2\overline{n_x^2}$

- with **moderate correlation time** (moderately wide band) the noise is enhanced:

  with $T_n \approx T_P/2$ it is $\overline{n_B^2} \approx 1,73\,\overline{n_x^2}$

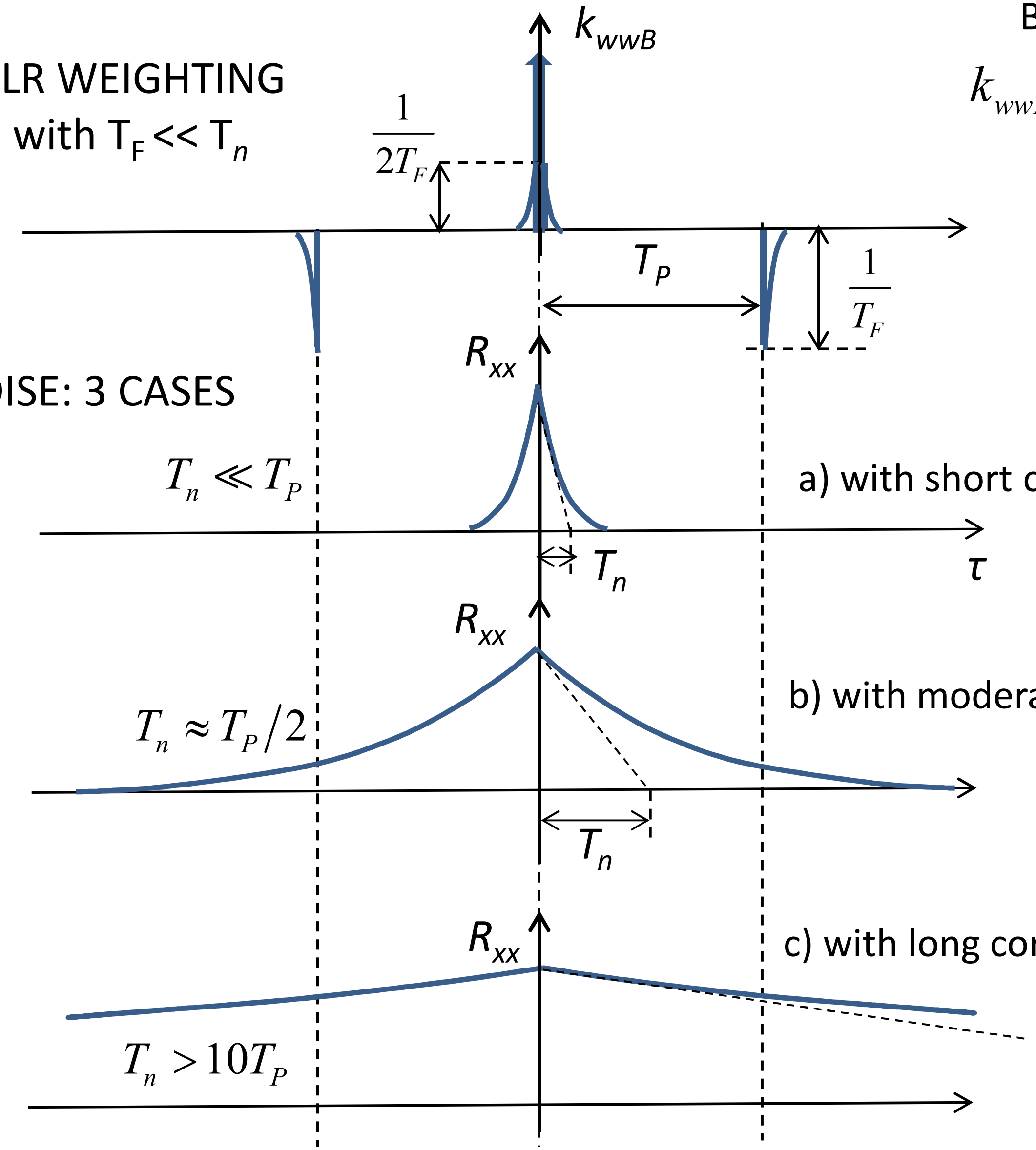
- only with **long correlation time** (low-frequency band) the noise is **attenuated*:**

  with $T_n > 10T_P$ it is $\overline{n_B^2} \approx \overline{n_x^2} \cdot 2\frac{T_P}{T_n} < 0,2\,\overline{n_x^2}$

** note that anyway the level is **double** of that given by a simple CR filter with equal cutoff, that is with $T_F = RC = T_P$*

BLR Filtering with fast differentiation

BLR WEIGHTING with $T_F << T_n$



NOISE: 3 CASES

BLR weight autocorrelation

$$k_{wwB} = k_{\delta\delta} + k_{FF} + k_{F\delta} + k_{\delta F}$$

Noise Autocorrelation

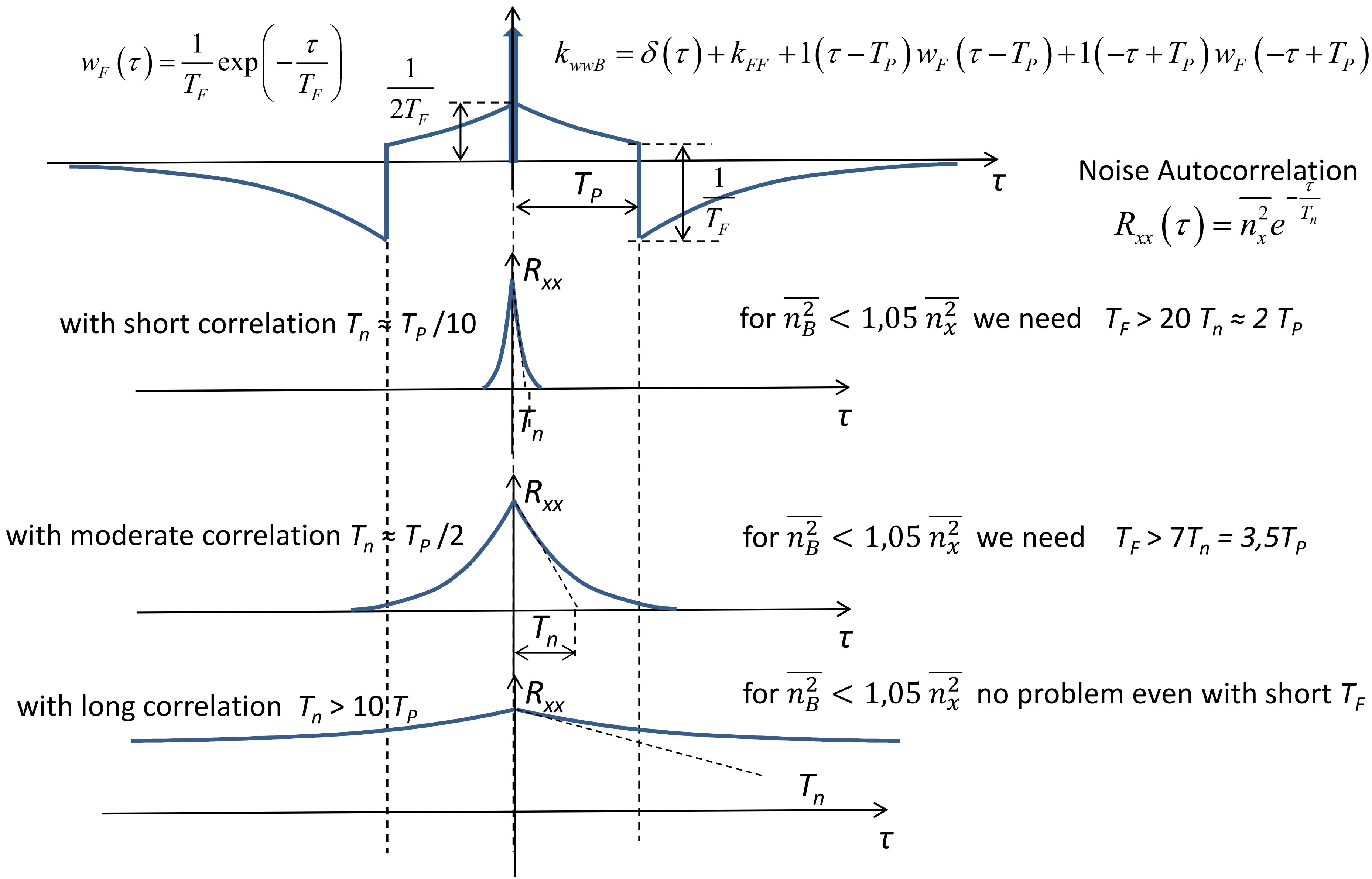
$$R_{xx}(\tau) = \overline{n_x^2} e^{-\frac{\tau}{T_n}}$$

a) with short correlation: $\overline{n_B^2} \approx 2\overline{n_x^2}$

b) with moderate correlation: $\overline{n_B^2} \approx 1,73\,\overline{n_x^2}$

c) with long correlation: $\overline{n_B^2} \approx \overline{n_x^2} \cdot 2\frac{T_P}{T_n} < 0,2\,\overline{n_x^2}$

BLR Filtering with slow differentiation



BLR Filtering with slow differentiation

With T_F **NOT negligible with respect to** T_n **,** the effect on white noise depends also **on the size of** T_F **compared to** T_n **and** T_P . A long T_F can limit the white noise enhancement

$$\overline{n_B^2} = \overline{n_x^2}\left[1+\frac{T_n}{T_n+T_F}\left(1-2e^{-\frac{T_P}{T_n}}\right)\right]$$

Let's evaluate how long must be T_F in the various cases of noise correlation

- with **short correlation time** $T_n \approx T_P/10$ it is

$$\overline{n_B^2} \approx \overline{n_x^2}\left(1+\frac{T_n}{T_n+T_F}\right)$$

for keeping $\overline{n_B^2} < 1{,}05\,\overline{n_x^2}$ we need $T_F > 20\,T_n \approx 2\,T_P$

- with **moderate correlation time** $T_n \approx T_P/2$ it is

$$\overline{n_B^2} \approx \overline{n_x^2}\left[1+\frac{T_n}{T_n+T_F}\left(1-\frac{2}{e^2}\right)\right] = \overline{n_x^2}\left[1+0{,}73\frac{T_n}{T_n+T_F}\right]$$

for keeping $\overline{n_B^2} < 1{,}05\,\overline{n_x^2}$ in this case we need $\boldsymbol{T_F > 7T_n = 3{,}5T_P}$

BLR Filtering with slow differentiation

- with **long correlation time** $T_n > 10\ T_P$ it is

$$\overline{n_B^2} \approx \overline{n_x^2}\left[1 - \frac{T_n}{T_n + T_F}\right] = \overline{n_x^2}\,\frac{T_F}{T_n + T_F}$$

  No problem with such a low-frequency noise: it is attenuted by the BLR just as by a CR constant-parameter filter (with equal time constant $T_F = RC$)

The most interesting case for us is noise with moderate T_n . In fact, when the BLR works on the output of an optimum (or approximate-optimum) filter for wideband noise, the correlation time T_n and delay T_p are comparable, since they are both closely related to the band-limit of the signal pulse.

- We conclude that for avoiding enhancement of the white noise it is necessary to select a fairly slow BLR differentiation, i.e. a fairly long T_F

$$\boxed{T_F \geq 5\,T_P}$$

- This approach is satisfactory also for filtering the 1/f noise, notwithstanding that making T_F longer than T_P shifts down the BLR cutoff frequency, hence reduces the attenuation of 1/f noise. This is counterbalanced by the fact that the enhancement of 1/f noise at frequencies above the cutoff is limited by the low-pass filtering in the baseline subtraction, whereas with short T_F it is remarkable.

BLR in summary

- The BLR is a high-pass filter that acts on noise and disturbances without affecting the pulse signal
- The BLR is a switched-parameter filter: the low-pass section within the high-pass filter structure is a boxcar integrator that acquires the baseline only in the intervals free from pulses
- The BLR can thus establish a high-pass band-limit at a high value (suitable for reducing efficiently the 1/f noise output power) without causing the signal loss suffered with a constant-parameter high-pass filter having the same band-limit
- The high-pass band-limit enforced by the BLR is given (with good approximation) by the low-pass bandlimit of the low-pass section in the BLR circuit structure
- The combination of: (1) optimum filter designed for the case of pulse signal in presence of wideband noise only (i.e. without 1/f noise) and (2) BLR specifically designed (for reducing the actual 1/f noise without worsening the wide-band noise) provides in most cases a quasi-optimum filtering solution.

Appendix: Band-limit of the high-pass CR differentiator for 1/f noise

Premise: with only a high-pass CR filter the 1/f noise power $\overline{n_f^2}$ *is divergent, therefore we consider here also a low-pass filter with very high band-limit* $f_s >> f_p = 1/2\pi RC$.

$$\overline{n_f^2} = S_B f_c \int_0^{f_S} \frac{\left(f/f_p\right)^2}{1+\left(f/f_p\right)^2} \frac{df}{f}$$ equivalent to $$\overline{n_f^2} = S_B f_c \int_{f_{if}}^{f_S} \frac{df}{f} = S_B f_c \ln\left(\frac{f_S}{f_{if}}\right)$$

By changing variable $\frac{f}{f_p} = x \qquad \frac{df}{f} = \frac{dx}{x} \qquad \frac{f_S}{f_p} = x_S$

We have $$\overline{n_f^2} = S_B f_c \int_0^{x_S} \frac{x^2}{1+x^2} \frac{dx}{x} = \frac{S_B f_c}{2} \int_0^{x_S} \frac{2x}{1+x^2} dx = \frac{S_B f_c}{2} \int_0^{x_S} \frac{1}{1+x^2} d\left(x^2\right)$$

and by setting $x^2 = y \qquad 2x dx = dy \qquad x_S^2 = y_S$

We obtain $$\overline{n_f^2} = \frac{S_B f_c}{2} \int_0^{y_S} \frac{1}{1+y} dy = \frac{S_B f_c}{2} \left|\ln(1+y)\right|_0^{y_S} = \frac{S_B f_c}{2} \ln(1+y_S)$$

Appendix: Band-limit of the high-pass CR differentiator for 1/f noise

$$\overline{n_f^2} = \frac{S_B f_c}{2} \ln(1+y_S) = S_B f_c \ln\sqrt{1+y_S} = S_B f_c \ln\sqrt{1+\left(\frac{f_S}{f_p}\right)^2}$$

Since the high-pass band-limit f_{if} is defined by

$$\overline{n_f^2} = S_B f_c \int_{f_{if}}^{f_S} \frac{df}{f} = S_B f_c \ln\left(\frac{f_S}{f_{if}}\right)$$

it is $$\frac{f_S}{f_{if}} = \sqrt{1+\left(\frac{f_S}{f_p}\right)^2}$$ and therefore $$f_{if} = \frac{f_p}{\sqrt{1+\left(f_p/f_s\right)^2}}$$

and taking into account that $f_s >> f_p$ we have finally

$$\boxed{f_{if} \approx f_p = \frac{1}{2\pi RC}}$$