Sensors, Signals and Noise

COURSE OUTLINE

- Introduction
- Signals and Noise
- Filtering: Band-Pass Filters 2 – BPF2
- Sensors and associated electronics

Band-Pass Filters 2

- Band-pass filtering with High-pass plus Low-Pass filters (CR - RC and LR – RC)
- LCR parallel Resonant Filter
- Pro's and Con's of real tuned filters
- Appendix: LCR series Resonant Filter

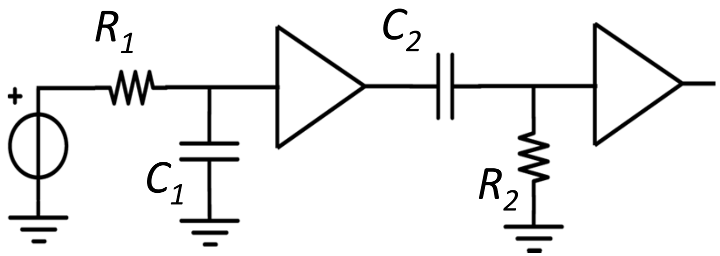
Band-pass filtering with High-pass plus Low-Pass filters (CR - RC and LR – RC)

RC lowpass plus CR highpass = bandpass

Cascaded two-cell filter:

low-pass $T_1 = R_1C_1$ $f_{p1} = 1/2\pi T_1$

high-pass $T_2 = R_2C_2$ $f_{p2} = 1/2\pi T_2$



$$H = \frac{V_{out}}{V_{in}} = \frac{1}{1 + jf/f_{p1}} \cdot \frac{jf/f_{p2}}{1 + jf/f_{p2}}$$

$$|H|^2 = \frac{1}{1 + \left(f/f_{p1}\right)^2} \cdot \frac{\left(f/f_{p2}\right)^2}{1 + \left(f/f_{p2}\right)^2}$$

With equal poles $T_1 = T_2 = T$ and $f_{p1} = f_{p1} = f_p$

$$H = \frac{jf/f_p}{\left(1 + jf/f_p\right)^2}$$

$$|H| = \frac{f/f_p}{1 + \left(f/f_p\right)^2}$$

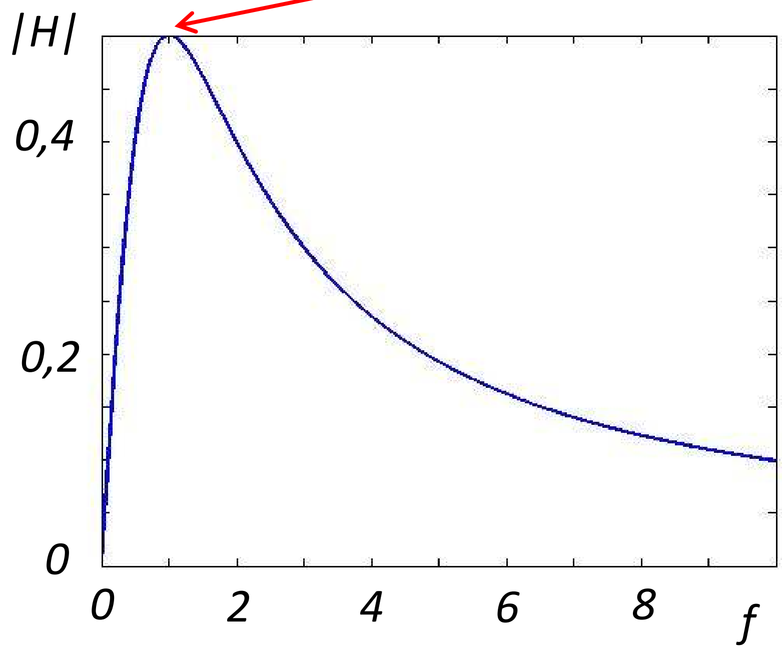
at band center $f = f_p$ peak value and phase zero

$$\begin{cases} \left|H\left(f_p\right)\right| = \dfrac{1}{2} \\ \arg H\left(f_p\right) = 0 \end{cases}$$

RC lowpass plus CR highpass = bandpass

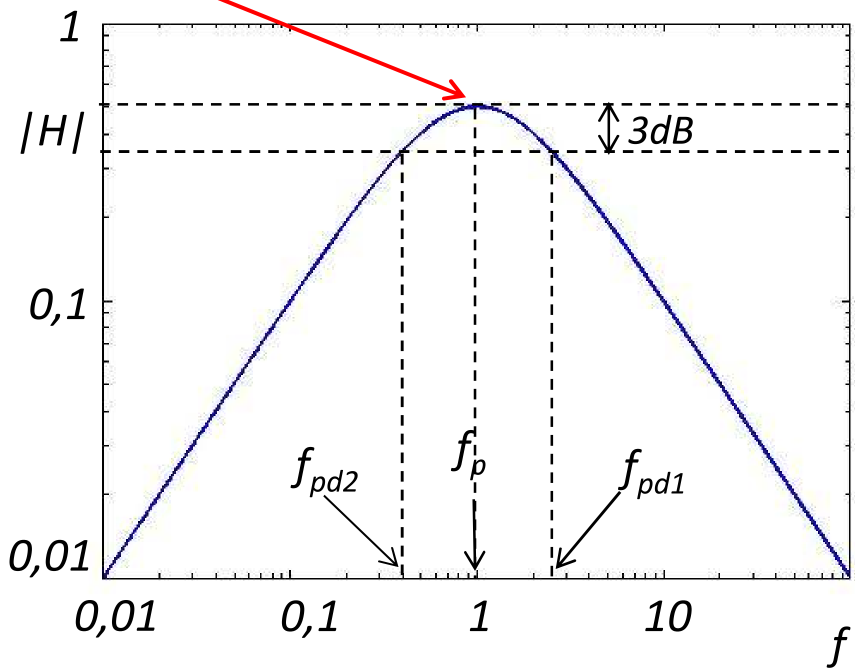
Plots of |H(f)| **with f_p = 1**

NB: peak is 0,5



Linear – linear plot

|H(f)| vs. f



Log – Log plot (Bode plot)

|H(f)| vs. f

3dB-down Bandwidth of CR-RC

$$|H| = \frac{f/f_p}{1+\left(f/f_p\right)^2}$$ peak value at $f=f_p$ $$\left|H\left(f_p\right)\right| = \frac{1}{2}$$

3dB down points f_{pd1} and f_{pd2} $$\left|H\left(f_{pd1}\right)\right| = \left|H\left(f_{pd2}\right)\right| = \frac{\left|H\left(f_p\right)\right|}{\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

$$x = f/f_p \longrightarrow \frac{x}{1+x^2} = \frac{1}{2\sqrt{2}} \longrightarrow x_{1,2} = \sqrt{2} \pm 1$$

$$f_{pd1,pd2} = \left(\sqrt{2} \pm 1\right) f_p$$

3dB down pass-band $$\boxed{\Delta f_p = f_{pd1} - f_{pd2} = 2 f_p}$$

NOT narrow-band !! $$\boxed{\frac{\Delta f_p}{f_p} = 2}$$

White noise bandwidth of CR-RC

From the definition of white noise bandwidth Δf_n (with unilateral S_B)

$$\overline{n_B^2} = S_B \cdot \left| H\left(f_p \right) \right|^2 \cdot \Delta f_n = S_B \cdot \frac{1}{4} \cdot \Delta f_n$$

by comparison with the computed* output power

$$\overline{n_B^2} = S_B \cdot \int_0^\infty \left| H\left(f \right) \right|^2 df_n = S_B \cdot f_p \cdot \frac{\pi}{4}$$

we get

$$\boxed{\Delta f_n = \pi f_p = \frac{1}{2T} = \frac{\pi}{2} \Delta f_p}$$

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* Noise computation

$$\overline{n_B^2} = S_B \cdot \int_0^\infty \frac{\left(f/f_p \right)^2}{\left[1 + \left(f/f_p \right)^2 \right]^2} df = S_B f_p \cdot \int_0^\infty \frac{x^2}{\left[1 + x^2 \right]^2} dx = S_B f_p \cdot \int_0^\infty \frac{2x}{\left[1 + x^2 \right]^2} \cdot \frac{x}{2} dx$$

$$= S_B f_p \cdot \left\{ \left| -\frac{x}{2} \frac{1}{1 + x^2} \right|_0^\infty + \frac{1}{2} \int_0^\infty \frac{1}{1 + x^2} dx \right\} = S_B f_p \cdot \frac{1}{2} \left| arctgx \right|_0^\infty = S_B f_p \cdot \frac{\pi}{4}$$

δ-response of the CR-RC

$$H(s)=\frac{sT}{(1+sT)^2}=\frac{1}{1+sT}-\frac{1}{(1+sT)^2}$$

$$h(t)=\frac{1}{T}e^{-\frac{t}{T}}-\frac{t}{T^2}e^{-\frac{t}{T}} \quad \Longrightarrow \quad k_{hh}(0)=\int_0^\infty h^2(\alpha)d\alpha=\frac{1}{4T}$$

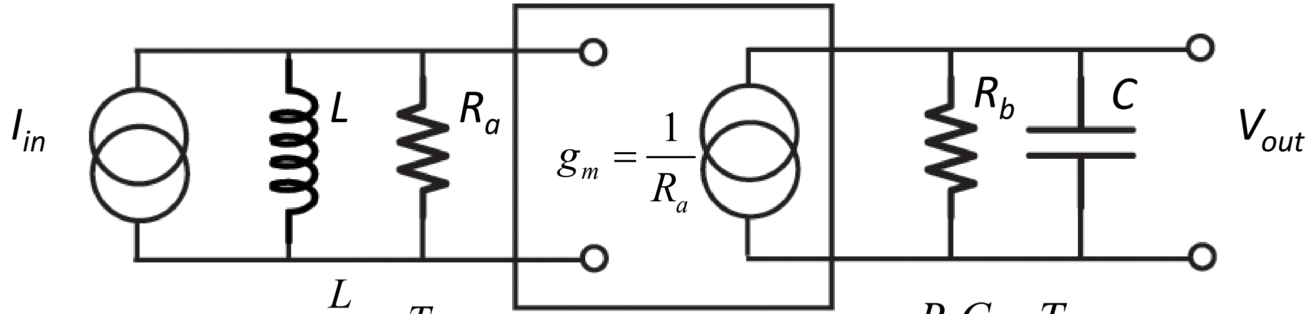
By the Fourier transfom properties ($|H|^2$ area in f = k_{hh} value in τ = 0) we get

$$k_{hh}(0)=\left|H\left(f_p\right)\right|^2 2\Delta f_n=\frac{1}{2}\Delta f_n$$

which confirms that

$$\boxed{\Delta f_n=\frac{1}{2T}=\frac{\pi}{2}\Delta f_p}$$

LR parallel high pass plus RC parallel low-pass



$$\frac{L}{R_a} = T_a \qquad R_b C = T_b$$

$$Z_a = R_a \frac{j\omega T_p}{1 + j\omega T_p} \qquad Z_a = R_b \frac{1}{1 + j\omega T_p}$$

$$\frac{V_{out}}{I_{in}} = Z_a \frac{1}{R_a} Z_b = R_b \frac{j\omega T_p}{\left(1 + j\omega T_p\right)^2} = R_b \cdot H\left(j\omega\right)$$

In order to have equal poles $T_a = T_b = T_p$ it must be $R_a R_b = L/C$

Let us take equal resistances $R_a = R_b$, so that the high-pass filter at high frequency (well in its passband, far above the pole f_p) has impedance equal to the low-pass filter at low frequency (well in its passband, far below the pole f_p)

$$R_a = R_b = R_o = \sqrt{\frac{L}{C}} \qquad T_p = R_o C = \frac{L}{R_o} = \sqrt{LC} \qquad \omega_p = \frac{1}{T_p} = \frac{1}{\sqrt{LC}}$$

LR parallel high pass plus RC parallel low-pass

$$\frac{V_{out}}{I_{in}} = R_o \frac{j\omega T_p}{\left(1 + j\omega T_p\right)^2} = R_o \cdot H\left(j\omega\right)$$

$$\frac{I_{out}}{I_{in}} = \frac{V_{out}}{R_o I_{in}} = \frac{j\omega T_p}{\left(1 + j\omega T_p\right)^2} = H\left(j\omega\right)$$

H(jω) is the adimensional transfer, i.e. the «gain» from input current to output current in the resistor $R_b = R_o$. It is confirmed that at the band-center f_p

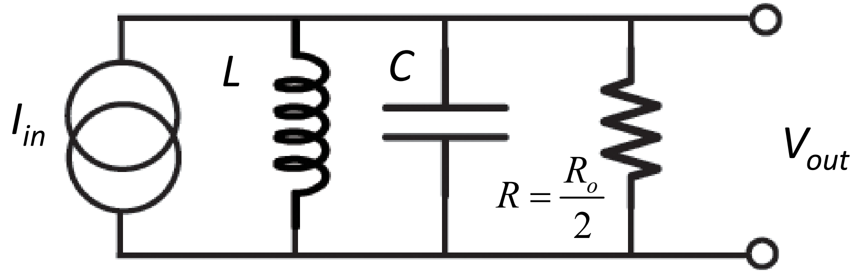
$$\frac{I_{out}\left(f_p\right)}{I_{in}\left(f_p\right)} = H\left(f_p\right) = \frac{1}{2} \qquad \text{and} \qquad \frac{P_{out}\left(f_p\right)}{P_{in}\left(f_p\right)} = \left|H\left(f_p\right)\right|^2 = \frac{1}{4}$$

This strong attenuation occurs because at $f=f_p$ only a fraction $1/\sqrt{2}$ of the input current flows in the resistor R_a of the input cell and is transferred to the output cell, where again only a fraction $1/\sqrt{2}$ flows in the resistor R_b .
This issue is overcome by employing a resonant circuit instead of a LR high-pass plus a decoupled RC low-pass

LCR parallel Resonant Filter

From LR-RC to damped resonant circuit

Taking out the buffer (the transconductance amplifier) we have



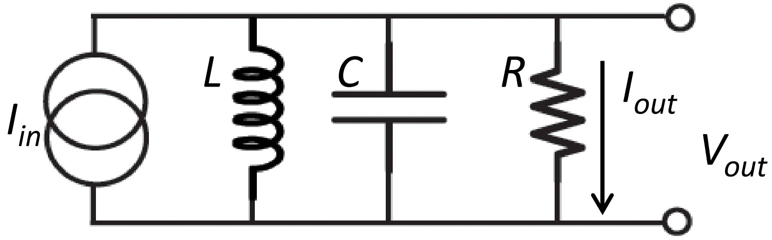
We will show that this filter:

- is resonant at $\omega_o = \frac{1}{\sqrt{LC}}$ with characteristic resistance $R_o = \sqrt{\frac{L}{C}}$
- is critically damped (i.e. it has two real and coincident poles)
- at band center f_o the L and C reactance compensate each other, so that the impedance is purely resistive $|Z(f_o)| = R$ and all the input current flows in R. That is, at $f = f_o$ there is no attenuation $|H(f_o)| = \mathbf{1}$

and we will deal with the question:

- can the bandpass filtering be improved by reducing the damping, that is, by using higher resistance R > R_o/2 ?

LRC parallel resonant filter



$$Z = \frac{V_{out}}{I_{in}} = \frac{1}{j\omega C + \frac{1}{j\omega L} + \frac{1}{R}} = R\frac{j\omega \frac{L}{R}}{\left(1-\omega^2 LC\right) + j\omega \frac{L}{R}}$$

Denoting by $H\left(\omega\right) = \frac{I_{out}}{I_{in}} = \frac{1}{R}\frac{V_{out}}{I_{in}} = \frac{j\omega \frac{1}{RC}}{\left(\frac{1}{LC} - \omega^2\right) + j\omega \frac{1}{RC}}$ we have $\boxed{Z = R\cdot H\left(\omega\right)}$

At the **resonance frequency** $\boxed{\omega_o = \frac{1}{\sqrt{LC}}}$ the reactive impedances cancel each other

so that the impedance is purely resistive $Z\left(\omega_o\right) = R$

that is $H\left(\omega_o\right) = 1 \quad and \quad \arg H\left(\omega_o\right) = 0$

Another basic parameter is the **characteristic resistance *R_o*** , which for the oscillation at $\omega=\omega_o$ represents the ratio

(amplitude of voltage on C) / (amplitude of current in L)

$$\boxed{R_o = \sqrt{\frac{L}{C}}}$$

LRC parallel resonant filter

$$H(\omega) = \frac{j\omega \frac{1}{RC}}{\left(\frac{1}{LC} - \omega^2\right) + j\omega \frac{1}{RC}}$$

With characteristic parameters $\boxed{\omega_o = \frac{1}{\sqrt{LC}}}$ $\boxed{R_o = \sqrt{\frac{L}{C}}}$

The poles of H(s) are

$$s_{1,2} = -\frac{1}{2RC} \pm \frac{1}{2RC}\sqrt{1 - 4\frac{RC}{L/R}} = -\frac{1}{2RC} \pm \frac{1}{2RC}\sqrt{1 - \frac{R^2}{\left(\frac{R_o}{2}\right)^2}} == -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

By defining $\boxed{\alpha_o = \frac{1}{2RC}}$

we can write $\boxed{s_{1,2} = -\alpha_o \pm \sqrt{\alpha_o^2 - \omega_o^2}}$

LRC parallel resonant filter

$$s_{1,2} = -\alpha_o \pm \sqrt{\alpha_o^2 - \omega_o^2}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$ $$\alpha_o = \frac{1}{2RC}$$

The δ-response h(t) is:

- damped (real poles) if $\omega_o^2 < \alpha_o^2$, which implies $R^2 < \left(\frac{R_o}{2}\right)^2$
- critically damped (coincident real poles) if $\omega_o^2 = \alpha_o^2$, that is if $R^2 = \left(\frac{R_o}{2}\right)^2$
- oscillatory (complex poles) if $\omega_o^2 > \alpha_o^2$, that is with $R^2 > \left(\frac{R_o}{2}\right)^2$

  In the oscillatory cases

  $s_{1,2} = -\alpha_o \pm j\omega_p$ with $$\omega_p = \omega_o\sqrt{1-\frac{\alpha_o^2}{\omega_o^2}} = \omega_o\sqrt{1-\frac{R_o^2}{(R/2)^2}}$$

- The **higher is R** with respect to R_o , the **lower is the dissipation**

  and the **slower the damping** of the oscillation

If $R^2 \gg \left(\frac{R_o}{2}\right)^2$ the dissipation is low, so that $\omega_o^2 \gg \alpha_o^2$ and the pole $\omega_p \rightarrow \omega_o$

$$s_{1,2} \cong -\alpha_o \pm j\omega_o$$

Resonator Quality Factor Q

The energy *E* stored in the circuit oscillates from C to L and back while it decays exponentially due to dissipation in R. At the V maxima the energy is all in C, hence the decay is traced by the envelope of the V^2 oscillation, so that $E(t) = E(0)e^{-2\alpha_o t}$

The relative loss rate **in time** is

$$\frac{1}{E}\frac{dE}{dt} = -2\alpha_o$$

It is more significant, however, the loss rate referred to the progress of the oscillation, that is, referred to the oscillation phase $\vartheta = \omega_o t$ rather than time *t*

By setting $t = \frac{\vartheta}{\omega_o}$ we get $E(\vartheta) \propto e^{-\frac{2\alpha_o}{\omega_o}\vartheta}$ and

$$\frac{1}{E}\frac{dE}{d\vartheta} = -\frac{2\alpha_o}{\omega_o}$$

The lower is this loss rate, the higher is the resonator quality.
The reciprocal of this relative loss rate is defined **Quality Factor Q** of the resonator

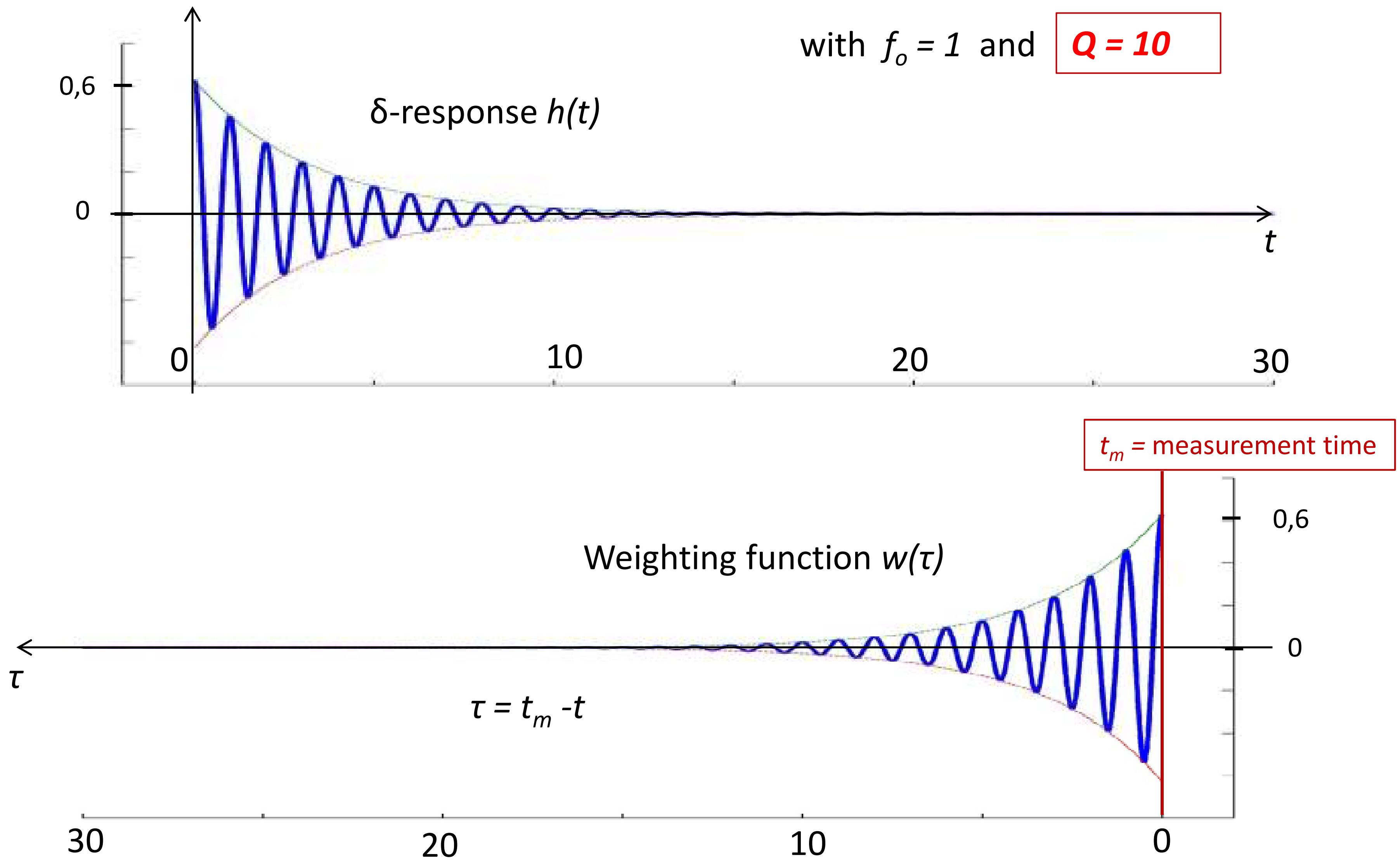
$$-\frac{1}{E}\frac{dE}{d\vartheta} = \frac{1}{Q}$$ that is $$Q = \frac{\omega_o}{2\alpha_o} = \frac{R}{R_o}$$

The **higher** is R >> R_o the lower is the dissipation ($Q \rightarrow \infty$ for $R \rightarrow \infty$)

NB: with high Q the relative energy loss in 1 oscillation period is small and can be evaluated by a linear approximation $\frac{\Delta E}{E} \cong \frac{2\pi}{Q}$

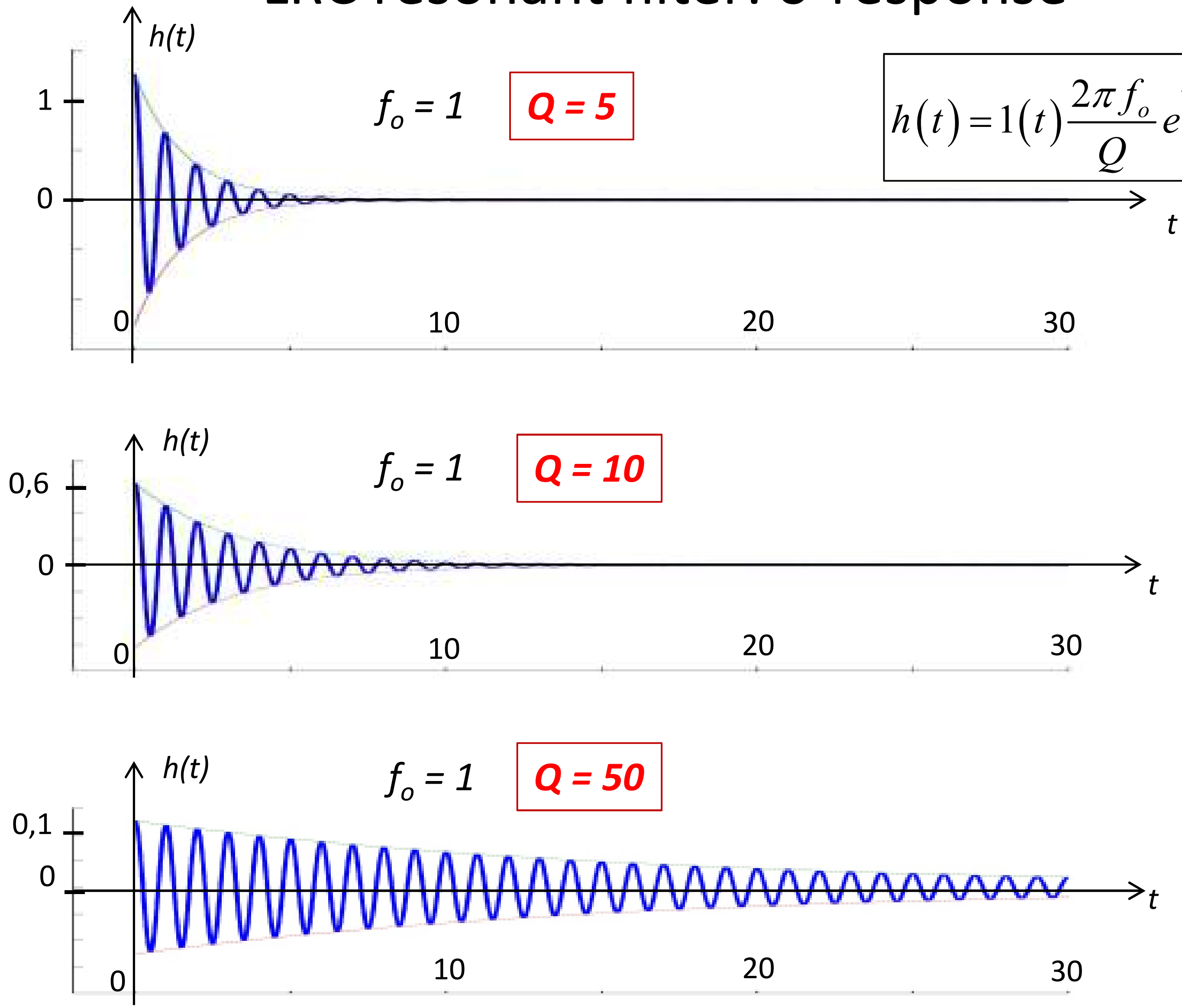
LRC resonant filter: δ-response and weighting function

with $f_o = 1$ and **$Q = 10$**



LRC resonant filter: δ-response

$$h(t) = 1(t)\frac{2\pi f_o}{Q} e^{-\frac{\pi f_o}{Q}t} \cos\omega_o t$$



LRC resonant filter:
δ-response and weighting function

In cases with low dissipation, i.e. with $R^2 \gg \left(\frac{R_o}{2}\right)^2$ and $\omega_o^2 \gg \alpha_o^2$

the poles are $\boxed{s_{1,2} \cong -\alpha_o \pm j\omega_o}$

δ-response
$$h(t) = 1(t)\frac{1}{RC}e^{-\alpha_o t}\cos\omega_o t = 1(t)\frac{\omega_o}{Q}e^{-\frac{\omega_o}{2Q}t}\cos\omega_o t$$

weighting function
(of measure at time t_m)
$$w(\tau) = h(t_m - \tau) = 1(t_m - \tau)\frac{1}{RC}e^{-\alpha_o(t_m - \tau)}\cos\omega_o\tau$$

voltage on C
δ-response
$$v(t) = R \cdot h(t) = 1(t)\frac{1}{C}e^{-\alpha_o t}\cos\omega_o t$$

LRC resonant filter transfer function

$$H(\omega) = \frac{I_{out}}{I_{in}} = \frac{j\omega \frac{1}{RC}}{\left(\frac{1}{LC} - \omega^2\right) + j\omega \frac{1}{RC}}$$

Taking into account

$$\omega_o = \frac{1}{\sqrt{LC}} \qquad R_o = \sqrt{\frac{L}{C}} \qquad \alpha_o = \frac{1}{2RC} \qquad Q = \frac{\omega_o}{2\alpha_o} = \frac{R}{R_o}$$

the transfer can be expressed in terms of resonance frequency ω_o and quality factor Q

$$H(\omega) = \frac{j\omega \cdot \frac{\omega_o}{Q}}{\left(\omega_o^2 - \omega^2\right) + j\omega \cdot \frac{\omega_o}{Q}}$$

LRC resonant filter transfer function: phase

$$H(\omega) = \frac{j\omega \frac{1}{RC}}{\left(\frac{1}{LC} - \omega^2\right) + j\omega \frac{1}{RC}} = \frac{j\omega \cdot \frac{\omega_o}{Q}}{\left(\omega_o^2 - \omega^2\right) + j\omega \cdot \frac{\omega_o}{Q}}$$

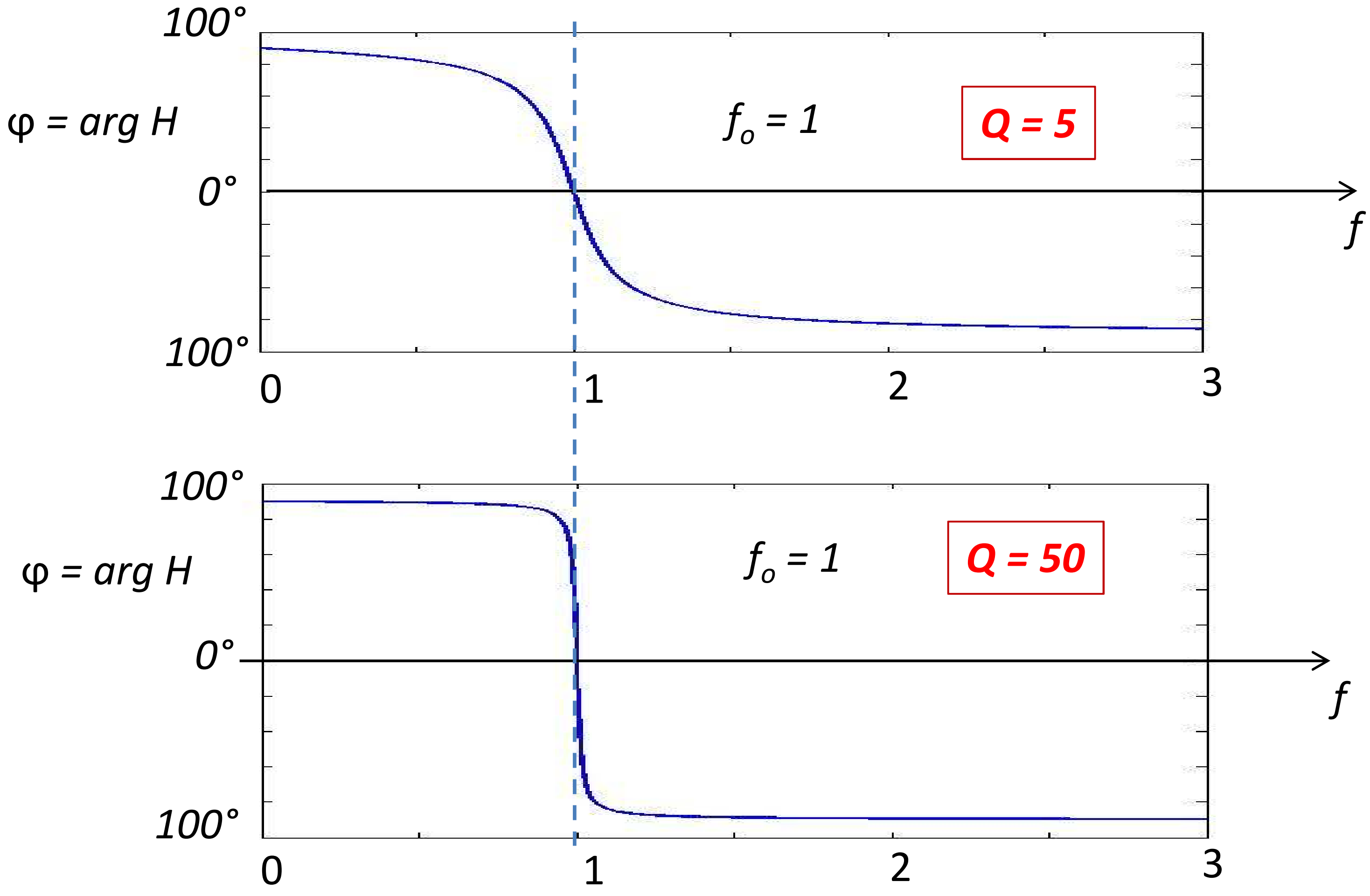
$$\varphi = \arg H(\omega) = arctg\left[\frac{Q}{\omega\omega_o}(\omega_o + \omega)(\omega_o - \omega)\right]$$

For $\omega \rightarrow +\infty$ $|H| \rightarrow 0$ $\varphi = \arg H(\omega_o) \rightarrow -\pi/2$ (- 90°)
For $\omega \rightarrow -\infty$ $|H| \rightarrow 0$ $\varphi = \arg H(\omega_o) \rightarrow +\pi/2$ (90°)

For $\omega = \omega_o$ $H(\omega_o) = 1$ $\varphi = \arg H(\omega_o) = 0$ and $\left(\frac{d\varphi}{d\omega}\right)_{\omega=\omega_o} = -2\frac{Q}{\omega_o}$

The phase impressed by the filter is exactly zero at exactly the band center,
but rapidly increases as ω is shifted.
Note that the higher is Q the steeper is the increase $\left(\frac{d\varphi}{d\omega}\right) \propto Q$

LRC resonant filter transfer function: phase



LRC resonant filter transfer function: module

$$\left|H(\omega)\right|^2 = \frac{\omega^2 \cdot \dfrac{\omega_o^2}{Q^2}}{\left(\omega_o^2 - \omega^2\right)^2 + \omega^2 \cdot \dfrac{\omega_o^2}{Q^2}} = \frac{1}{1 + Q^2\left(\dfrac{\omega - \omega_o}{\omega_o}\right)^2\left(\dfrac{\omega + \omega_o}{\omega}\right)^2}$$

- «Lower wing» approximation valid for $\omega \ll \omega_o$

$\left|H(\omega)\right| \approx \left|H_L(\omega)\right| = \frac{\omega}{\omega_o}\frac{1}{Q}$ i.e. $\boxed{\left|H_L(\omega)\right| \propto \omega}$ (extrapolation at $\omega = \omega_o$ is $\left|H_L(\omega_o)\right| = \frac{1}{Q}$)

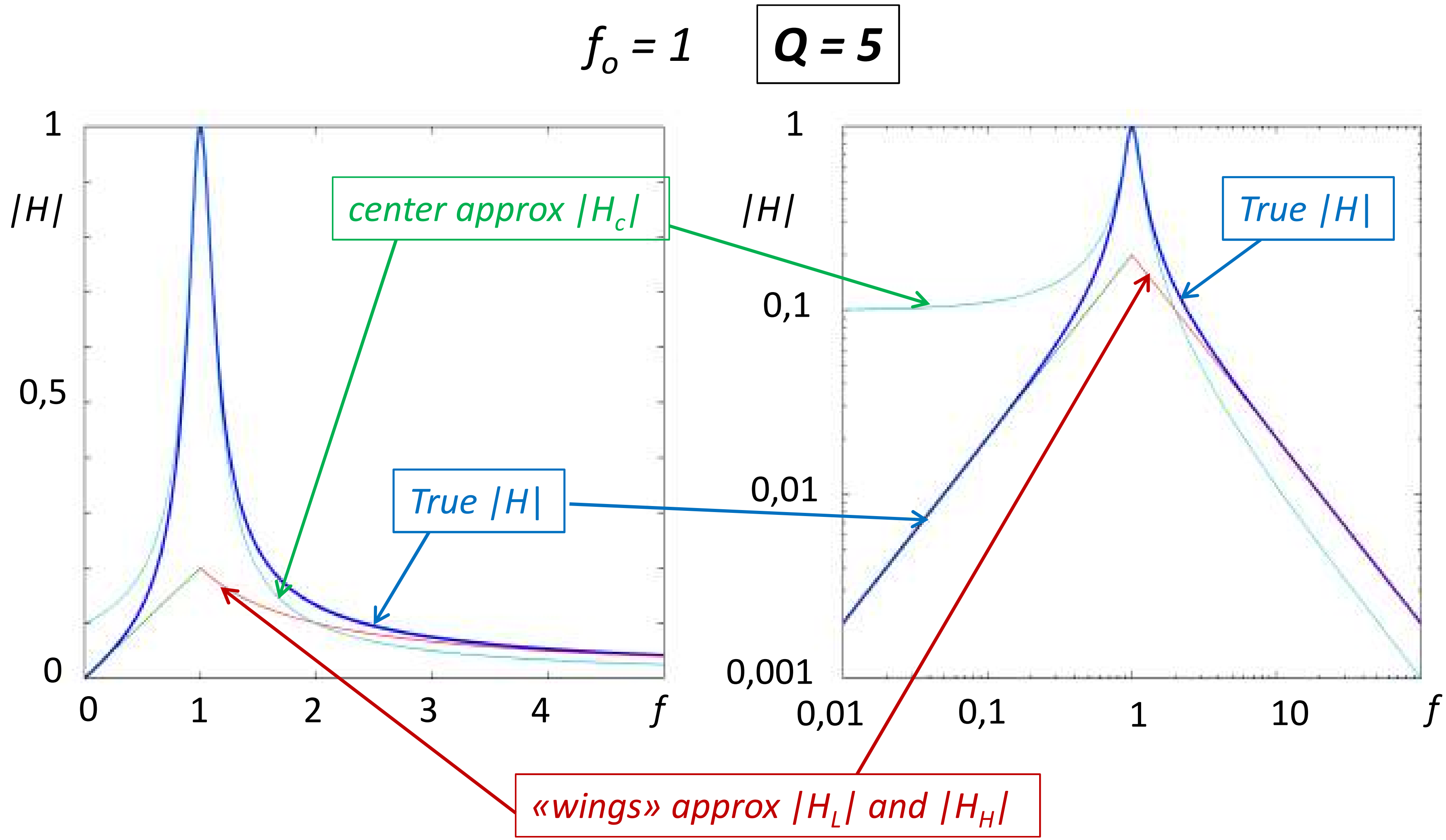
- «Higher wing» approximation valid for $\omega \gg \omega_o$

$\left|H(\omega)\right| \approx \left|H_H(\omega)\right| = \frac{\omega_o}{\omega}\frac{1}{Q}$ i.e. $\boxed{\left|H_H(\omega)\right| \propto \frac{1}{\omega}}$ (extrapolation at $\omega = \omega_o$ is $\left|H_H(\omega)\right| = \frac{1}{Q}$)

- «Central lobe» approximation valid for $|\omega - \omega_o| \ll \omega_o$, that is for $\omega \approx \omega_o$

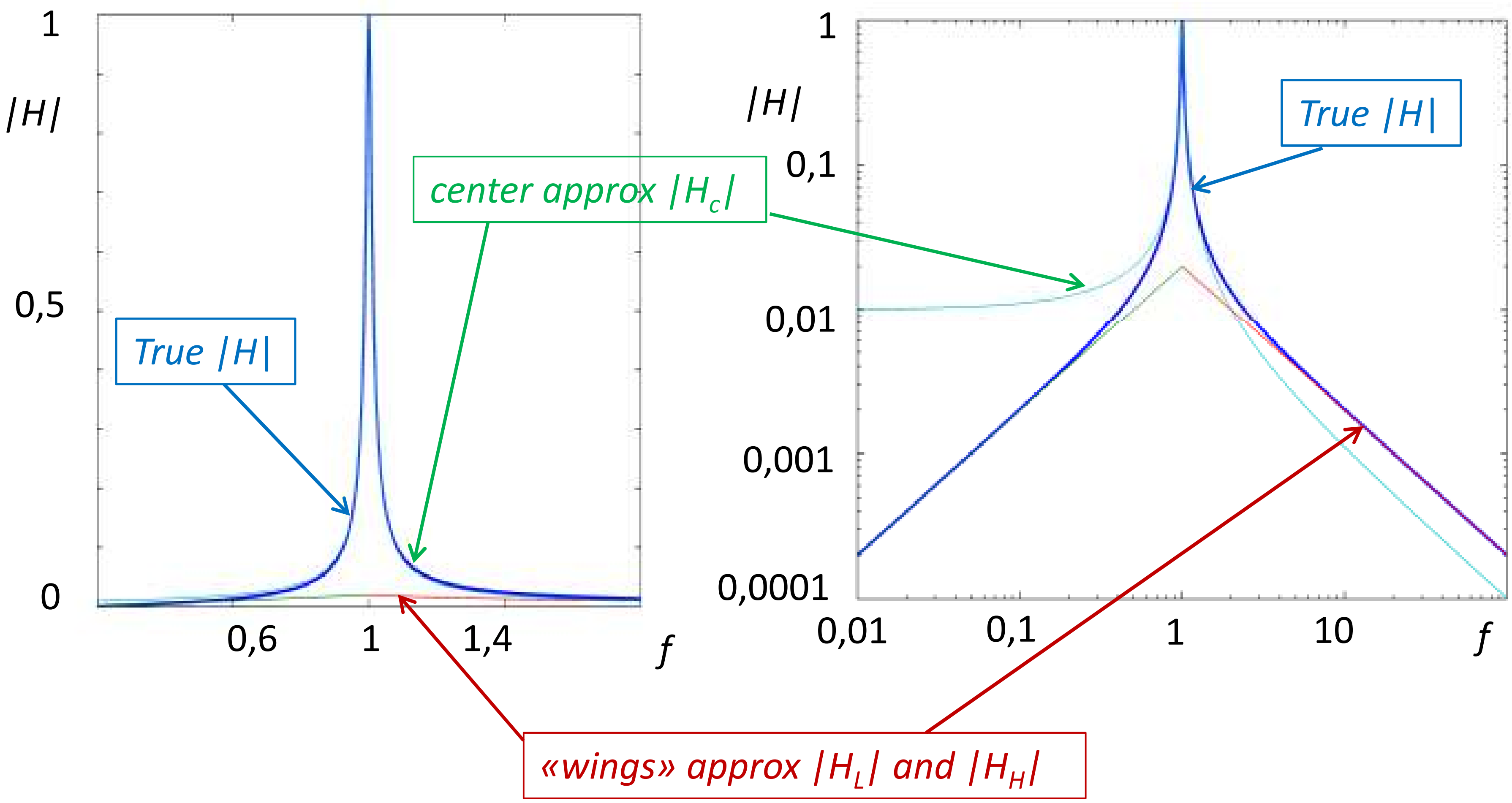
$$\boxed{\left|H(\omega)\right|^2 \approx \left|H_C(\omega)\right|^2 = \frac{1}{1 + 4Q^2\left(\dfrac{\omega - \omega_o}{\omega_o}\right)^2}}$$

LRC resonant filter transfer function: module



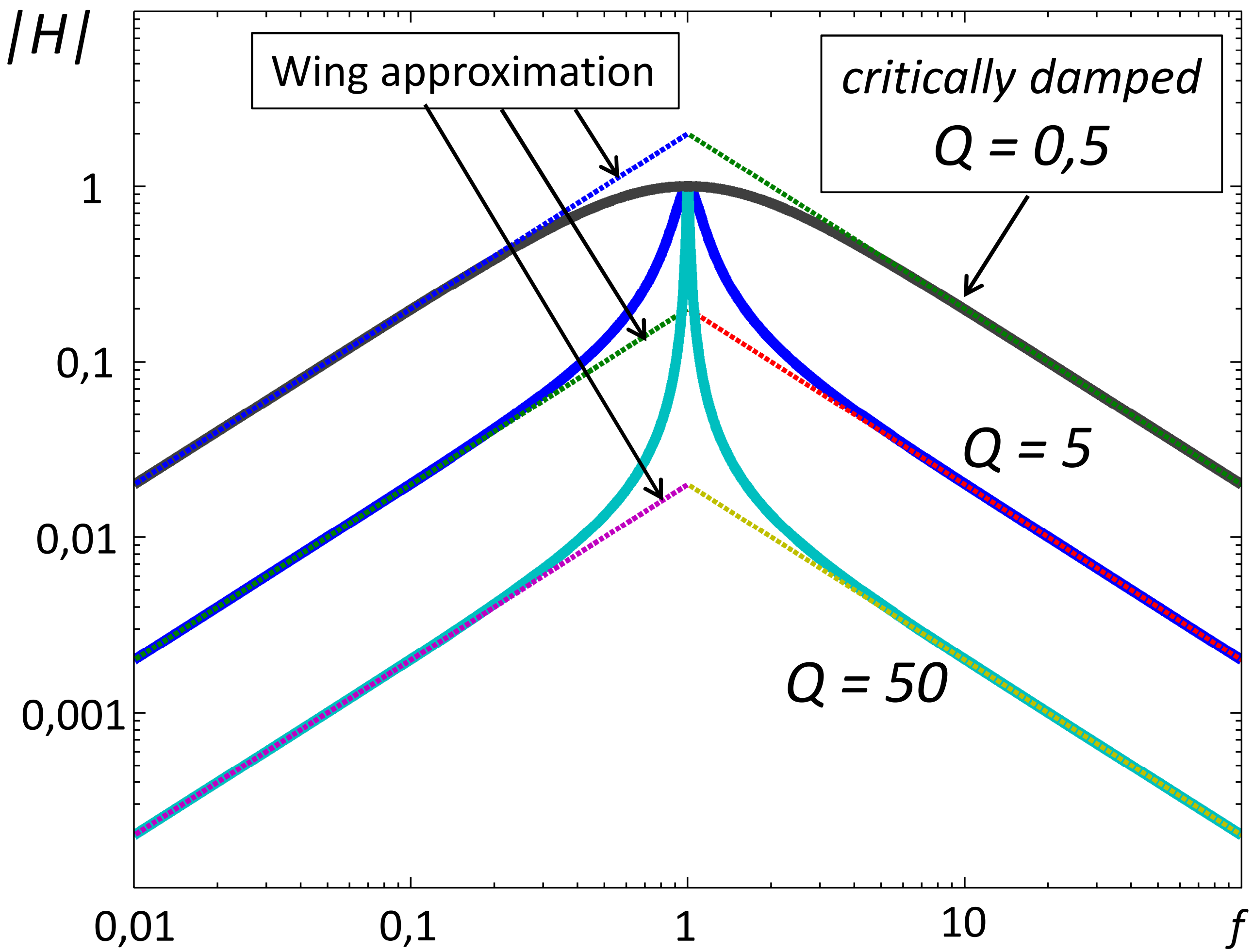
LRC resonant filter transfer function: module

$f_o = 1$ **$Q = 50$**



LRC resonant filter transfer function module

Comparison of filters with different Q



LRC resonant filter: signal bandpass

Bandwidth for signals: defined by the 3dB down points ω_{dL} and ω_{dH}
where $|H(\omega_{dL})|^2 = |H(\omega_{dH})|^2 = ½$

$$\Delta\omega_s = \omega_{dH} - \omega_{dL}$$

For cases with Q >> 1 we can use the central lobe approximation

$$\left|H_C(\omega_d)\right|^2 = \frac{1}{1+4Q^2\left(\dfrac{\omega_d-\omega_o}{\omega_o}\right)^2} = \frac{1}{2}$$

that is $\quad 4Q^2\left(\dfrac{\omega_d-\omega_o}{\omega_o}\right)^2 = 1$

and we find $\quad \omega_{dH} - \omega_o = \omega_o - \omega_{dL} = \dfrac{\omega_o}{2Q}$

The signal bandwidth thus is $\quad \boxed{\Delta\omega_s = \dfrac{\omega_o}{Q}} \qquad \boxed{\dfrac{\Delta\omega_s}{\omega_o} = \dfrac{\Delta f_s}{f_o} = \dfrac{1}{Q}}$

Two basic advantages with respect to the CR-RC bandpass filter are quite evident:

- **No** signal attenuation at the center frequency
- **Narrow** filtering bandwidth even with moderately high Q values

LRC resonant filter: noise bandpass

The bandwidth for white noise is defined by

$$\Delta f_n = \int_0^\infty \left|H(f)\right|^2 df$$

In cases with Q >> 1 we can use for H(f) the central lobe approximation and take into account that $|H_c(f)|^2$ is with good approximation symmetrical with respect to the band center f_0 , thus obtaining

$$\int_0^\infty \left|H(f)\right|^2 df \approx 2\int_{f_o}^\infty \left|H_C(f)\right|^2$$

and therefore

$$\Delta f_n = 2\int_{f_o}^{\infty} \frac{1}{1+4Q^2\left(\dfrac{f-f_o}{f_o}\right)^2} df = \frac{f_o}{Q}\int_0^\infty \frac{1}{1+x^2} dx = \frac{\pi}{2}\frac{f_o}{Q}$$

$$\boxed{\Delta f_n = \frac{\pi}{2}\frac{f_o}{Q} = \frac{\pi}{2}\Delta f_s}$$

Pro’s and Con’s
of real tuned filters

Issues, Pro's and Con's of real tuned filters

- Real capacitors and inductors are not pure C and L. Their equivalent circuits include also finite resistances that model the internal sources of energy dissipation that inherently limit the Q of resonant circuits.
- In general, the dissipation is higher in components with higher value of L or C. Good quality capacitors with low dissipation are available from pF to about 1 μF. Inductors are more problematic than capacitors. Good quality components are available from nH to a few 100nH. Even components with fairly small L (typically a few 10 nH) have non negligible internal resistance.
- Stray reactances must not be overlooked. In discrete circuitry stray capacitances are in the order of pF and stray inductances are in the order of nH. In integrated circuits the values are much smaller, thanks to the very small physical size of the components.
- Since the resonance is at $f_o = 1 / \sqrt{2\pi LC}$, for obtaining a low frequency f_o high values of both L and C are required: in fact, with C=1 μF and L= 100 nH one gets f_o =1,26 MHz. Therefore, the Q values really obtained in the tuned filters progressively decrease as the desired resonant frequency decreases.

Issues, Pro’s and Con’s of real tuned filters

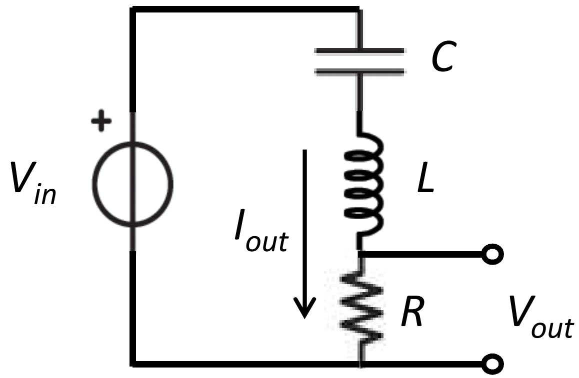
- For high frequencies f_o > 100MHz values of Q > 10 are currently obtained, up to almost Q ≈ 100 with clever design and high quality components.
- For intermediate frequencies 1MHz < f_o < 100MHz values up to Q ≈ 10 are obtained with careful design and implementation
- For f_o < 1MHz it becomes progressively more difficult to obtain high Q values as the frequency decreases. Anyway, even with moderate Q the performance of the tuned filters is remarkable and in many practical cases filters with Q ≈ 5 are really satisfactory.
- For a given Q, note that the noise bandwidth is reduced as the resonant frequency f_o is reduced: $\Delta f_n = \frac{\pi}{2}\frac{f_o}{Q}$.
- The analysis of actual tuned filters can be more complicate, the parasitic parameters may have a significant role, more sophisticated circuit configurations may be adopted, such as active filters that exploit the feedback for avoiding large inductors and employing only capacitors. However, the simple basic configuration studied here illustrates well the typical features and performance of constant-parameter narrow-band filters.
- Constant-parameter tuned filters are a simple and economical solution, widely employed in prefiltering stages and other simple situations, but their use in high-performance filtering is hindered by some intrinsic drawbacks.

Issues, Pro's and Con's of real tuned filters

- The accuracy and relative stability of f_o directly depends on that of the C and L values. Drift of f_o due to aging and temperature must be kept smaller than the filter bandwidth, in order to avoid uncontrolled variation of the output signal amplitude and phase. This may be really difficult in case of very narrow bandwidth. In particular, strong phase variations are caused by even small variations of f_o because of the strong dφ/df at band-center of filters with high Q
- Cascading simple filter stages for improving the cutoff characteristics is not practical for narrow-band filters, because they should have very accurately equal and stable f_o.
- The value of C influences both the center frequency f_o and the bandwidth Δf_S , so that it is not easy to design a filter with specified f_o and specified Δf_S .
- It is even more difficult to design a filter with adjustable f_o and constant bandwidth Δf_S , as it is required for measuring power spectra and for other applications.
- In cases where the frequency of a narrow-band signal is not very stable, a filter with very narrow bandwidth can be employed only if its center frequency can be adjusted to track that of the signal. As above outlined, this is not easy to obtain.

Appendix:
LCR series resonant filter

LRC series resonant filter



$$Y = \frac{I_{out}}{V_{in}} = \frac{1}{j\omega L + \frac{1}{j\omega C} + R} = \frac{1}{R}\frac{j\omega RC}{\left(1-\omega^2 LC\right) + j\omega RC} =$$

Denoting by $H\left(\omega\right) = \frac{V_{out}}{V_{in}} = \frac{j\omega \frac{R}{L}}{\left(\frac{1}{LC} - \omega^2\right) + j\omega \frac{R}{L}}$ we can write $Y\left(\omega\right) = \frac{1}{R} \cdot H\left(\omega\right)$

Resonance at $\omega_o = \frac{1}{\sqrt{LC}}$

characteristic resistance $R_o = \sqrt{\frac{L}{C}}$

LRC series resonant filter

The poles of H(ω) are

$$s_{1,2} = -\frac{1}{2L/R} \pm \sqrt{\left(\frac{1}{2L/R}\right)^2 - \frac{1}{LC}} = -\frac{1}{2L/R} \pm \frac{1}{2L/R}\sqrt{1 - \frac{R_o^2}{\left(\frac{R}{2}\right)^2}}$$

Denoting by $\alpha_o = \frac{1}{2L/R}$ we have $s_{1,2} = -\alpha_o \pm \sqrt{\alpha_o^2 - \omega_o^2}$

Quality factor

$$Q = \frac{\omega_o}{2\alpha_o} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{R_o}{R}$$

The **lower** is R << R_o
the lower is the dissipation
(Q → ∞ for R → 0)

Note that the transfer function expressed in terms of the resonance frequency ω_o and of the quality factor Q is equal to that of the parallel LCR resonator

$$H(\omega) = \frac{j\omega \cdot \frac{\omega_o}{Q}}{\left(\omega_o^2 - \omega^2\right) + j\omega \cdot \frac{\omega_o}{Q}}$$