Sensors, Signals and Noise

COURSE OUTLINE

- Introduction
- Signals and Noise
- Filtering: Band-Pass Filters 4 – BPF4
- Sensors and associated electronics

Band-Pass Filtering 4

- **From principles to real LIA instruments**

- Overall gain, Postamplifier and Preamplifier
- Linearity and Dynamic Reserve; Gain distribution before and after the Phase-Sensitive Detector (PSD)
- Multiplier: dynamic range and linearity issues
- Switched amplifiers instead of multipliers

- **Lock-in Amplifier with Squarewave Reference**

- Weighting function; action on sinusoidal signal; action on noise; action on squarewave signal; fundamental-only response

- **Pre-filtering complementary to the LIA Filtering**

- Pre-filters complementary to LIA. Issues in prefilter design and evaluation.
- Issues in case of variable signal frequency.
- Notch filters

- **Reference phase adjustment and waveform conditioning**

- **Lock-in Amplifiers with Digital Signal Processing (DSP)**

- **APPENDIX 1** – LIA with any waveform of the reference and of the signal

- **APPENDIX 2** – LIA: websites of interest

From principles to real LIA instruments

From principles to real LIA instruments

In principle:

a LIA consisting simply in a **Phase-Sensitive Detector** (PSD with the two basic stages Multiplier and LPF) provides a flexible and effective band-pass filtering that can achieve very narrow bandwidth. It is thus able to recover for measurement with good precision even very small modulated signals buried in much higher noise, down to an ideal limit value S/N<<1 that can be computed from the LIA principles.

But in practice:

the non-ideal features of the actual circuits of the PSD set to the recovery of small signals buried in high noise an actual limit much more stringent than the ideal one.

However:

by introducing in the LIA structure modifications and further stages, the hindering features can be counteracted and the actual detection limit can be improved towards the ideal limit. For instance, in real cases nanoVolt signals can be extracted from wideband noise with 1000 times (60dB) greater rms value. We shall discuss the limiting features and the available remedies and outline the structure of real LIAs.

From principles to real LIA instruments

High gain for the signal

The modulated input signal is converted by the LIA in a slow demodulated signal, with components from DC to a fairly low frequency limit. This signal must be supplied to a meter circuit that measures its amplitude, i.e. nowadays ordinarily an ADC. The LIA output signal must have scale adequate for the ADC (typically 10V full scale), whereas the LIA input signal is very small: therefore, the LIA **must provide high overall gain for the signal.**

Post-Amplifier (after the PSD)

A high-gain amplifier after the PSD (denoted here Post-Amplifier or Output Amplifier) is employed to raise the demodulated signal to a scale suitable for the ADC.
Notice that the post-amplifier:

1. must be a **DC-coupled** amplifier with upper bandlimit adequate to the demodulated signal
2. receives a signal accompanied by low noise, since it operates after the PSD filtering
3. It has drift of the baseline offset and low-frequency noise, which affect the measurement since they occur **after the PSD** and are **not filtered**

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Pre-Amplifier (before the PSD)

If the demodulated signal is very small, comparable or lower than the baseline drift and noise of the post-amplifier referred to its input, the measurement will be spoiled. A **preamplifier before the PSD** is necessary in order to avoid or reduce this drawback. Notice that the pre-amplifier:

1. processes the modulated input signals, hence it is an **AC coupled** amplifier, either **wide-band** type including the modulation frequency f_m **or narrow-band tuned to** $\boldsymbol{f_m}$
2. receives a signal accompanied by high noise, because it operates before the PSD
3. may have baseline drift and low-frequency noise, but their role is minor because they are filtered by the PSD (and by the AC-coupled amplifier itself).

WARNING: Signal and Noise MUST stay within the Linear Dynamic Range

In order to obtain the foreseen improvement of S/N, the processing of signal and noise in the LIA must be accurately linear. Deviations from linearity produce detrimental effects (self-modulation of the noise, generation of spurious harmonics, etc.), which irrevocably alter the measure and degrade the LIA performance. The signal and noise must remain well within the linear dynamic range in every stage involved, particularly in the multiplier (and in the preamplifier).

From principles to real LIA instruments

Wide-Band Preamplifiers and Tuned Preamplifiers

When a **wide-band preamplifier** is employed to raise the level of a very small input signal, a problem arises with very small input S/N<<1. The gain required for the signal works on a noise which is much higher than the signal, hence it brings this amplified noise out of the linear dynamic range of the multiplier.

In such cases, for exploiting the required gain it is necessary to reduce the noise received by the preamplifier with a **pre-filter**. Adequate reduction of the LIA input noise is obtained in many cases with prefilter passband much wider than that of the LIA. Such a prefiltering would be a useless nonsense in an ideal apparatus, but in real cases it is a necessary feature for avoiding nonlinearity in intermediate stages. On the other hand, we will see that a very narrow-band prefilter is not advisable.

Preamplifiers that incorporate prefiltering are currently available from LIA manufacturers; they are called **tuned preamplifiers or selective preamplifiers**.

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Linearity limits and problems

- The **multiplier dynamic range** (linear behavior range) does not depend on the gain setting of the preamplifier, it is set just by the multiplier circuit.
- The **preamplifier output dynamic range** is constant, independent of gain setting.
- Therefore, there is a **maximum acceptable input signal** that must not be exceeded for maintaining linear behavior of preamplifier and multiplier; increasing the preamplifier gain by a factor decreases this limit by the same factor
- There is also a **maximum input rms noise** that can be applied maintaining linear behavior; an increase of preamp gain decreases also this limit.
- Also the **post-amplifier** has limited linear range, but problems met are much less severe. In fact, the post-amp receives low-level noise (filtered noise), whereas preamp and multiplier process high-level noise (not filtered or just prefiltered)
- Each setting of preamplifier and post-amplifier gains determines an **input full-scale signal**, i.e an input signal level that produces full-scale LIA output signal.
  Note, however, that a given value of input full-scale signal can be obtained with different combinations of preamp gain and post-amp gain

From principles to real LIA instruments

Linearity limits and Dynamic Reserve

- For any given setting of preamp and postamp gain the LIA will have
  - b) a full-scale input signal
  - b) a maximum tolerable rms input noise
- It is defined **Dynamic Reserve (DR)** the ratio

$$\frac{\textbf{maximum tolerable rms noise input}}{\textbf{full-scale signal input}}$$

- The **Dynamic Reserve** characterizes quantitatively the real capability of the LIA of correctly extracting a signal from a higher noise with that gain setting
- A given full-scale input signal can be obtained with different combinations of preamp gain and post-amp gain and the maximum DR is obtained by the configuration that employs the lowest preamp gain
- The maximum DR attained by a LIA is a significant figure and is customarily specified by the manufacturer. The maximum DR can attain 1000 (60dB) with moderate values of full-scale signals, which exploit moderate preamplifier gain (typically full-scale value down to about 1μV). The maximum DR decreases with very low full-scale input signal values, which require higher sensitivity and therefore higher preamplifier gain.

From principles to real LIA instruments

Linearity limits and Dynamic Reserve (DR)

- The LIA user is warned: the DR figure specified by the manufacturer is not an absolute qualification index of the LIA ; for evaluations and comparisons it is necessary to know exactly how that DR was defined. The problem is that the various manufacturers **refer to different kinds of input noise** and **give different meanings to «tolerable»**; there is not a precisely defined and generally agreed standard for DR definition. In fact:
  1) «Noise» may mean wide-band or narrow-band; it can be within the LIA operating frequency range or extend beyond it; etc.
  2) «Tolerable» may either mean noise level attained just before that overloading occurs (saturated output) or a level where the measured signal is just slightly distorted.
- In the most significant and most widely accepted **DR definition**:
  1) **noise** is considered to be distributed over the **operating frequency range of the LIA** (ordinarily up to 100kHz). The noise at higher frequencies far away from the signal can be blocked by other filters; the advantage of the LIA is that it is able to cut noise at frequencies near to the frequency of the modulated signal;
  2) **tolerable** means that it causes only a **minor deterioration of the measurement**, typically an alteration of the signal smaller than a few percent of the full-scale value.
  Such a definition really tells to the user the limits within which the LIA can actually recover signals buried in high noise.

From principles to real LIA instruments

Gain distribution between PreAmplifier and Post-Amplifier

The **overall** gain of the LIA depends only on the desired **sensitivity** (i.e. the amplitude of the input signal that produces full-scale output). The **distribution** of gain between Pre-amplifier and Post-amplifier depends also **on the input** ***$(S/N)_{in}$*** , since it has to satisfy two conflicting requirements:

1) Ensure Dynamic Reserve sufficient to deal with the actual $(S/N)_{in}$ at the LIA input
2) Minimize the detrimental effect on the measurement due to baseline drift and low-frequency noise of the Post-Amplifier

Since increasing the preamplifier gain by a factor decreases the DR by the same factor, the first requirement limits the pre-amplifier gain to a value that ensures

$$DR \geq \left(N/S \right)_{in}$$

In cases with $(S/N)_{in}$ <<1 the limited preamp gain allowed by the requirement (1) may not fulfill the requirement (2), but the requirement (1) is prioritary because it is essential for correct operation of the LIA.

Therefore, in cases with $(S/N)_{in}$ <<1 if it is not possible to reduce adequately the input noise by prefiltering, the preamp gain must be limited and some worsening of the measurement due to drift and low-frequency noise of the post-amplifier is unavoidable.

From principles to real LIA instruments

Multiplier Circuit issues about Linearity and Dynamic Range

For a multiplier circuit, linear behavior means that the output has a linear dependance separately from each one of the two inputs.

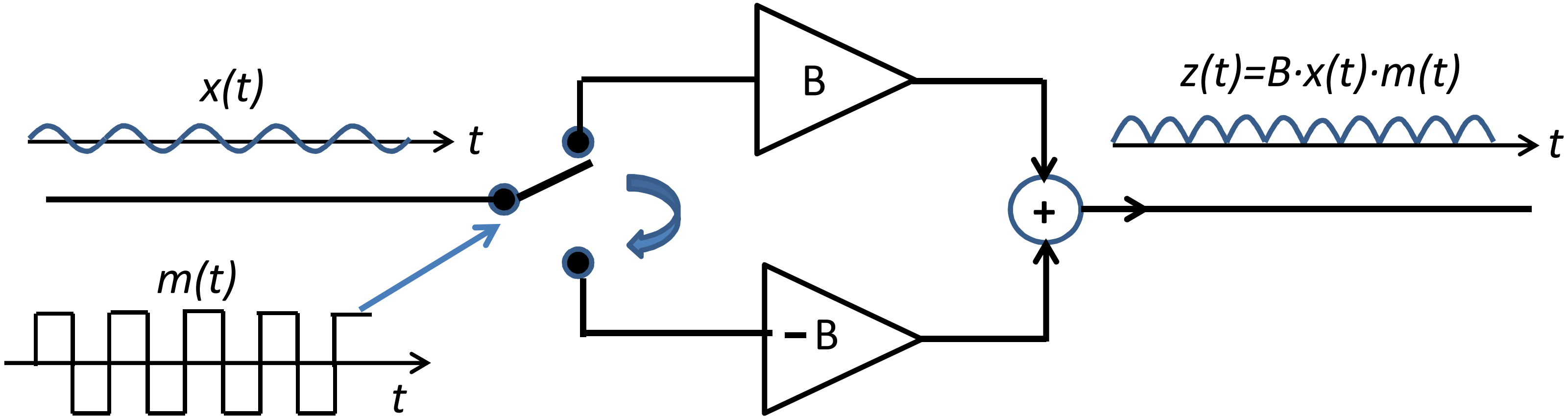
In the earlier years of LIA development (’40s and 50’s) analog multipliers were quite rudimentary (they were devices with nonlinear I-V characteristic – mostly diodes – which was exploited in a parabolic approximation over a limited range); they had small linear dynamic range and were a main weak point of LIAs.

In the following decades remarkable progress was achieved, driven by new ideas in the circuit design (Gilbert cell multipliers), supported by steadily improved integrated circuit technologies and with the assistance of specialist technologies (such as laser trimming of parameters) similarly to modern high performance amplifiers (instrumentation amplifiers). Modern analog multiplier circuits are similar to operational amplifiers, but they are far **more susceptible to noise and offset-voltage** related problems (these errors may become multiplied) and have significantly **lower dynamic range**. When dealing with high frequency signals, phase-related problems may be quite complex. In conclusion, manufacturing high-performance analog multipliers is far more difficult and the cost is higher. Therefore, in general they are used only for circuits where they are indispensable.

From principles to real LIA instruments

Switched Amplifier Circuits instead of Analog Multipliers

We have seen that modulation with squarewave reference *m(t)* can be implemented with circuits based simply on switches and amplifiers, avoiding recourse to analog multipliers



The noise referred to the input, the linearity and the dynamic range of these circuits are remarkably **better than those of analog multiplier circuits** (even high-performance types) because they are limited just by the performance of amplifiers and switch-devices.

Therefore, switched linear circuit configurations are often employed as demodulator stage in LIAs in order to avoid the limitations of analog multipliers.

Lock-in Amplifier with Squarewave Reference

LIA with non-sinusoidal Reference

The weighting function $w_L(\alpha)$ of a LIA is the multiplication of reference waveform $m(\alpha)$ (periodic at frequency f_m) and weighting function $w_F(\alpha)$ of the LPF

$$\boxed{w_L(\alpha) = m(\alpha) \cdot w_F(\alpha)}$$

In frequency domain this corresponds to the convolution of the F-transforms

$$\boxed{W_L(f) = M(f) * W_F(f)}$$

Since:

a) the transform $M(f)$ of a periodic $m(\alpha)$ is composed by lines at f_m (fundamental) and integer multiple frequencies (harmonics)

b) $W_F(f)$ of the LPF has bandwidth much smaller than f_m

the result of the convolution of $W_F(f)$ by any line of $M(f)$ does not overlap the result by any other line (with very good approximation). We conclude that:

the $W_L(f)$ is a set of **replicas of $W_F(f)$** centered on each line of $M(f)$, **multiplied by the line-weight** and **phase-shifted by the line-phase**.

With very good approximation, the module diagram can thus be obtained simply as

$$\boxed{|W_L(f)| \cong |M| * |W_F|}$$

LIA with non-sinusoidal Reference

For computing the noise output of a LIA we need the spectrum $|W_L(f)|^2$ in the frequency domain, or the autocorrelation $k_{LL}(\alpha)$ of the weighting function in time domain. The considerations made for $W_L(f)$ directly lead to conclude that:

> $|W_L(f)|^2$ is a set of **replicas of $|W_F(f)|^2$** centered on each line of $M(f)$ and **multiplied by the square of the line-weight**

as summarized in the equation

$$\boxed{\left|W_L(f)\right|^2 \cong \left|M\right|^2 * \left|W_F\right|^2}$$

which in the time-domain corresponds to

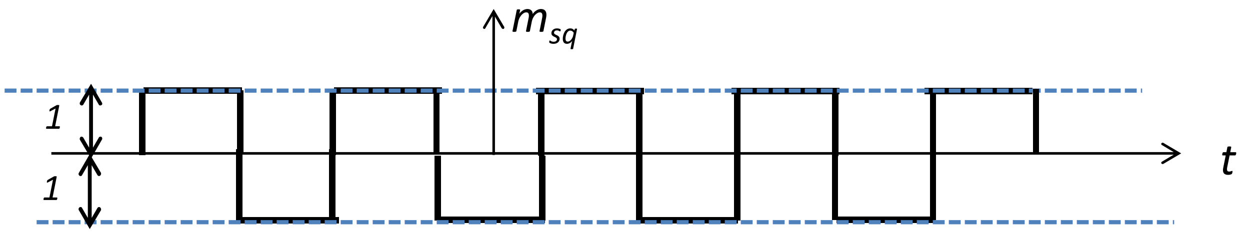
$$\boxed{k_{LL}(\alpha) \cong k_{mm}(\alpha) \cdot k_{FF}(\alpha)}$$

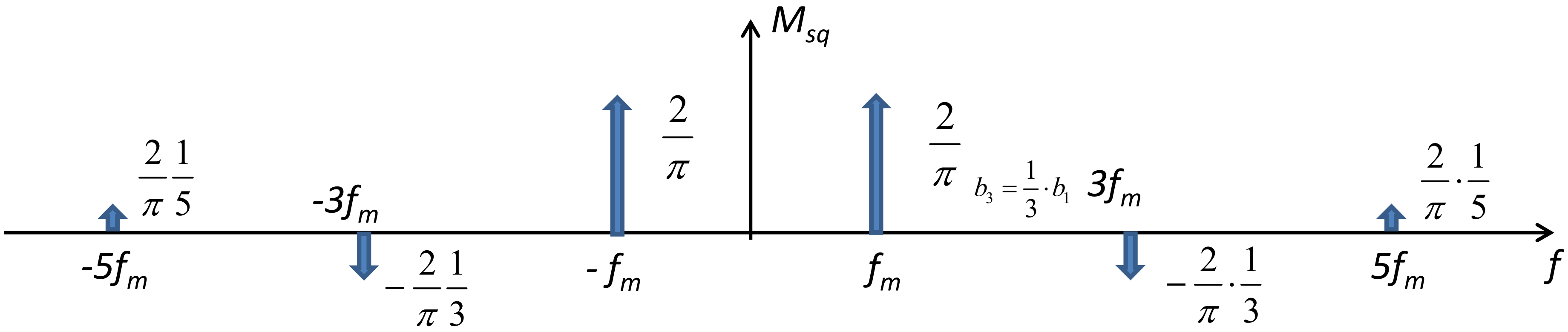
In summary, cases with non-sinusoidal periodic reference waveform can be analyzed by considering the various Fourier components of the reference and by exploiting for each one the conclusions reached for a sinusoidal reference and by superposing the effects of all components.

This approach can be employed in both the approaches seen, i.e. a) to employ the LIA weighting function or b) to follow stage-by-stage the processing of signal and noise

F-transform of a Squarewave

$m_{sq}(t)$ = symmetrical squarewave (from +1 to-1) at frequency f_m

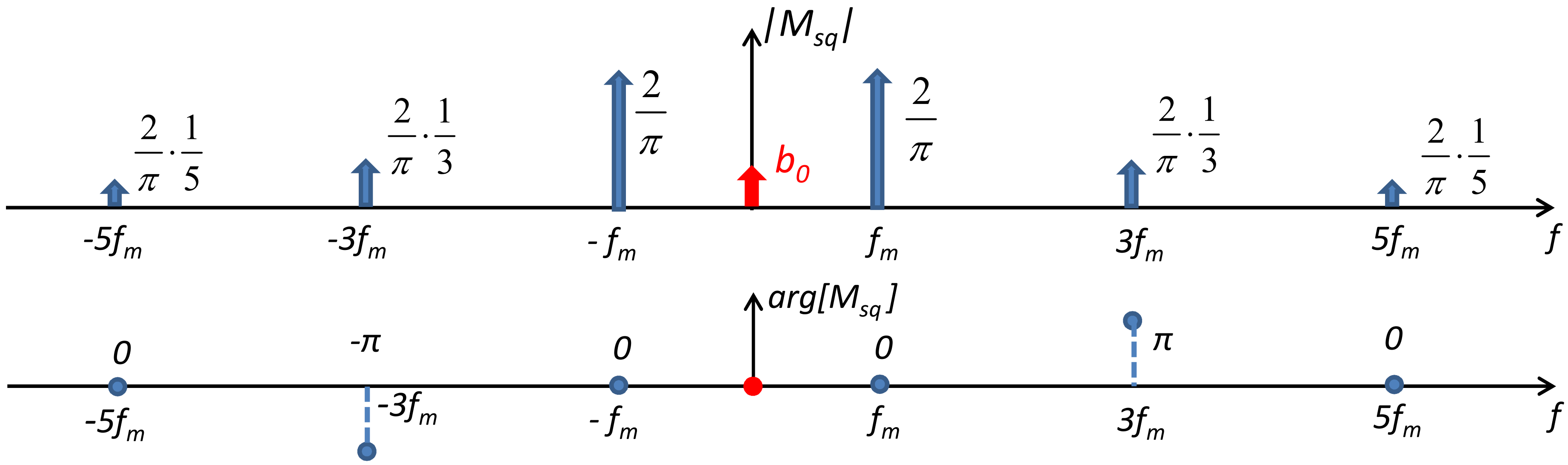




- Only odd-harmonic sinusoidal components
- Components with alternately positive and negative sign, i.e. alternately *0* and π phase
- Component amplitude decreasing as the reciprocal order:

$b_1 = \frac{4}{\pi}$ at fundamental f_m ; $b_3 = \frac{1}{3} \cdot b_1$ at 3 f_m ; $b_3 = \frac{1}{5} \cdot b_1$ at 5 f_m ; ......

F-transform of a real Squarewave



$$\boxed{M_{sq}(f) = \sum_{k=0}^{\infty} \frac{b_{2k+1}}{2}\left[\delta(f - f_{2k+1}) + \delta(f + f_{2k+1})\right]}$$

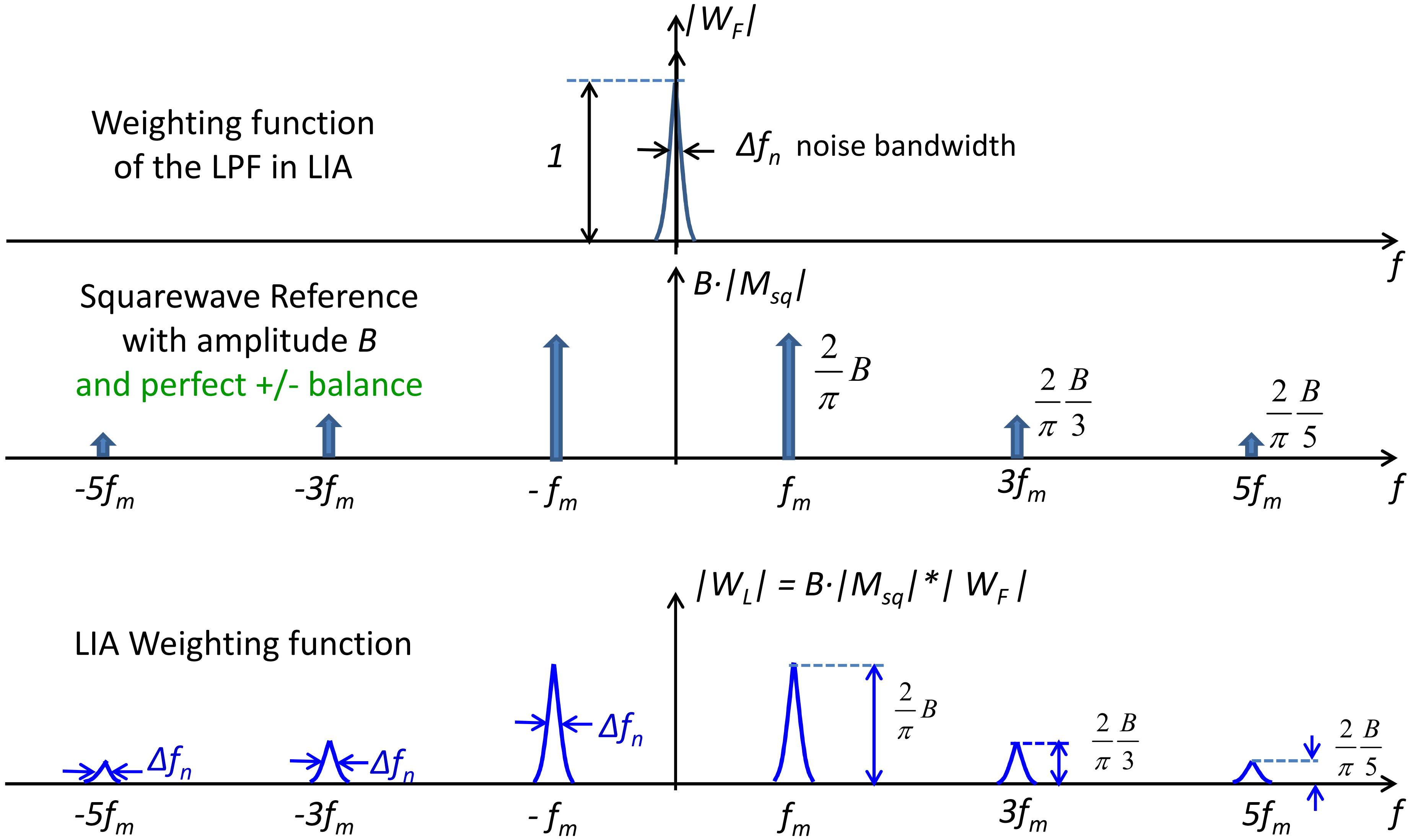
with

$$\boxed{b_{2k+1} = \frac{(-1)^k}{(2k+1)} \cdot \frac{2}{\pi}}$$

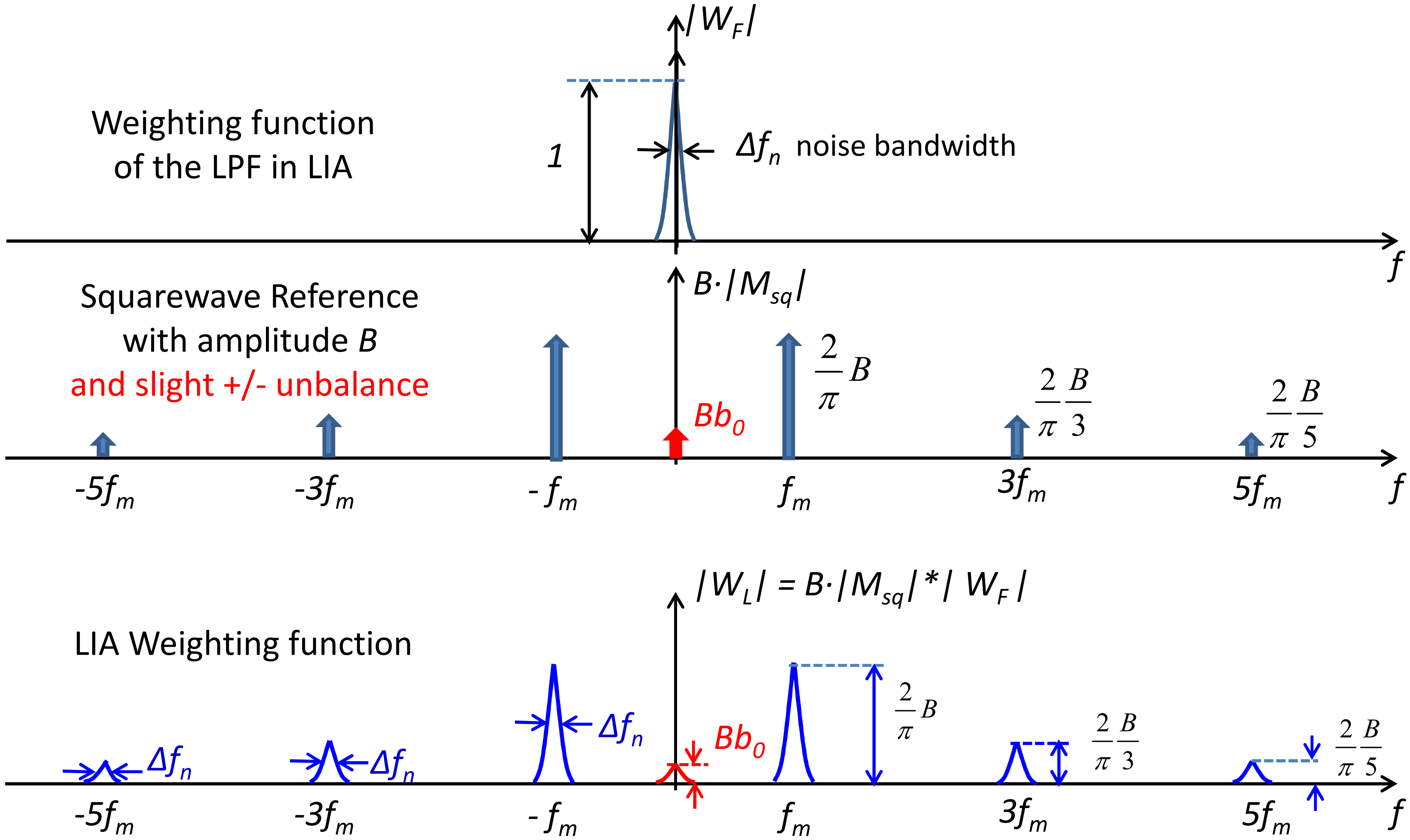
k = 0, 1, 2, 3 ..... and $f_{2k+1} = (2k+1) f_m$

BEWARE: In cases where the squarewave has non-zero mean value (e.g. slight asymmetry in amplitude and/or duration of positive and negative lobes) it has also a DC component with amplitude b_0 given by the ratio of the mean value to the peak amplitude (i.e. by the relative unbalance of positive and negative area)

LIA Weighting function with Squarewave Reference



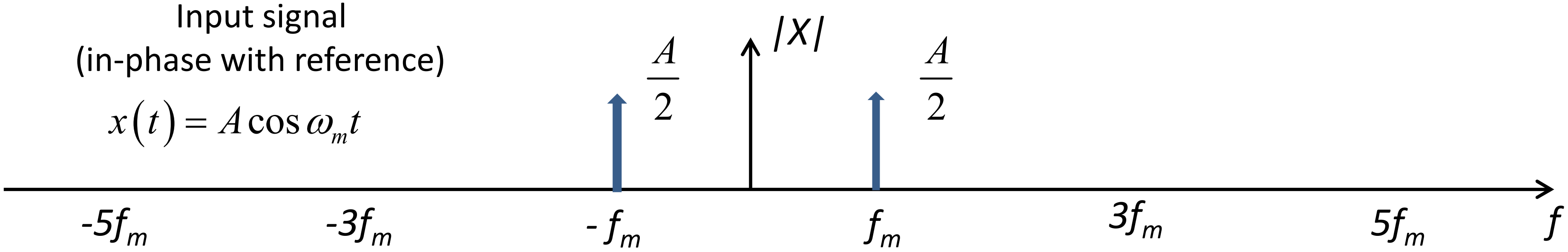
LIA Weighting function with Squarewave Reference



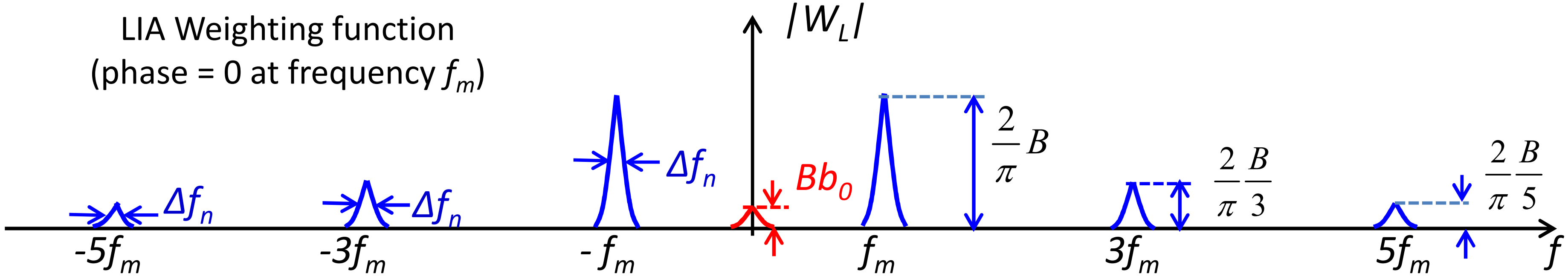
Sinusoidal signal through LIA with Squarewave Reference

Input signal
(in-phase with reference)

$$x(t) = A\cos\omega_m t$$



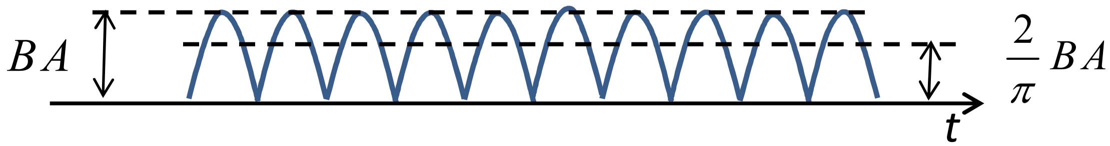
LIA Weighting function
(phase = 0 at frequency f_m)



Output signal

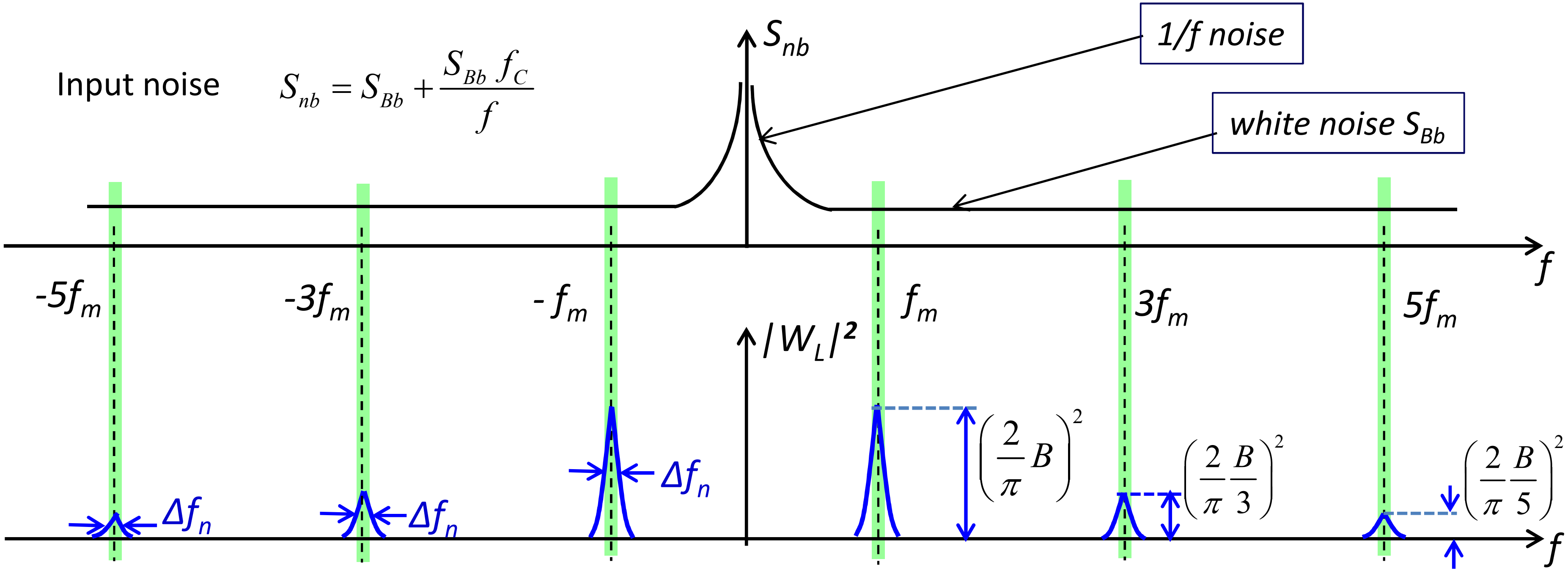
$$s_y = \int_{-\infty}^{\infty} X(f)\, W_L(-f)\, df = 2\cdot\frac{A}{2}\cdot\frac{2}{\pi}B = \frac{2}{\pi}B\cdot A$$

NB This is easily verified in time, since: LIA output *y(t)* = time average of *z(t)=x(t)·B m(t)* =
= mean of the in-phase component of the sinusoidal input rectified and multiplied by B



Noise through LIA with perfect Squarewave Reference

Input noise $S_{nb} = S_{Bb} + \dfrac{S_{Bb} f_C}{f}$



Output Noise $\overline{n_{yL}^2} = \int_{-\infty}^{\infty} S_{nb}(f)\left|W_L(f)\right|^2 df = 2 \cdot S_{Bb}\Delta f_n \left(\frac{2}{\pi}B\right)^2 \left[1 + \frac{1}{3^2} + \frac{1}{5^2} + ...\right]$

The factor $\left[1 + (1/3)^2 + (1/5)^2 + ...\right] = \pi^2/8 \cong (1{,}11)^2$ represents the enhanced noise due also to the higher passbands at the harmonic frequencies

$$\overline{n_{yL}^2} = 2 S_{Bb}\Delta f_n \left(\frac{2}{\pi}\right)^2 B^2 \frac{\pi^2}{8} = B^2 S_{Bb}\Delta f_n$$

No signal is collected in these passbands. Therefore, the S/N is reduced with respect to the case of sinusoidal reference, but the reduction is moderate.

S/N with Sinusoidal Signal and perfect Squarewave Reference

Output Signal $s_y = \frac{2}{\pi} B \cdot A$ (for sinusoidal input signal in phase)

Output Noise $\overline{n_{yL}^2} = S_{Bb} \Delta f_n B^2$

so that
$$\left(\frac{S}{N}\right)_{L,sqw} = \frac{s_y}{\sqrt{\overline{n_{yL}^2}}} = \frac{A}{\frac{\pi}{2}\sqrt{S_{Bb}\Delta f_n}}$$

which in comparison to the result obtained with sinusoidal reference

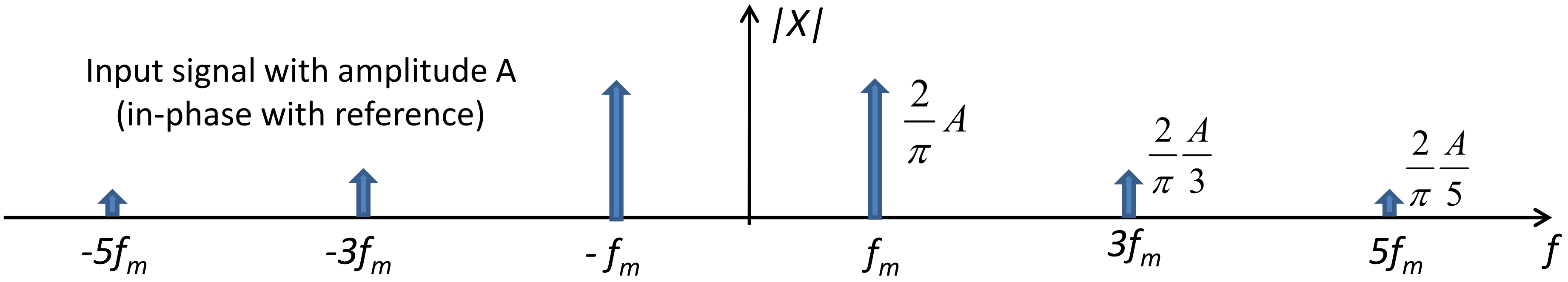
$$\left(\frac{S}{N}\right)_{L,\sin} = \frac{A}{\sqrt{2 S_{Bb}\Delta f_n}}$$

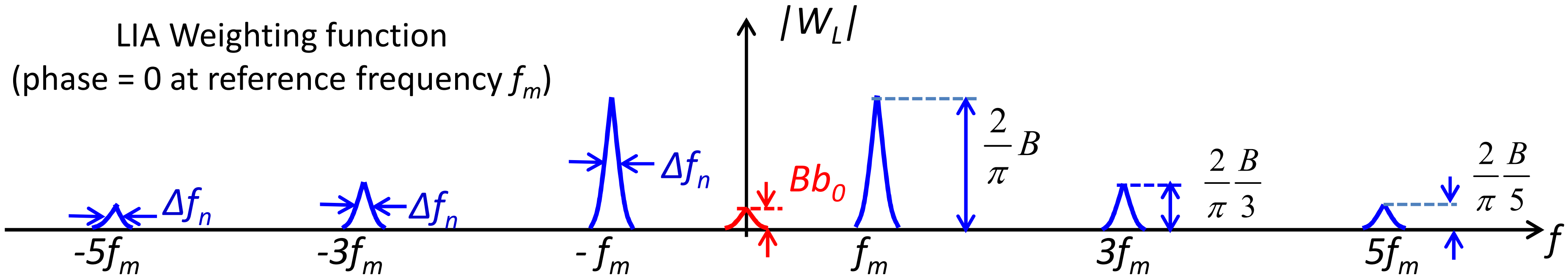
is just moderately lower

$$\left(\frac{S}{N}\right)_{L,sqw} = \frac{\sqrt{2}}{\frac{\pi}{2}}\left(\frac{S}{N}\right)_{L,\sin} \cong \frac{1}{1{,}11}\left(\frac{S}{N}\right)_{L,\sin}$$

We will now deal with another case often met in practice: the signal to be measured is a squarewave in phase with the squarewave reference

Squarewave signal through LIA with Squarewave Reference

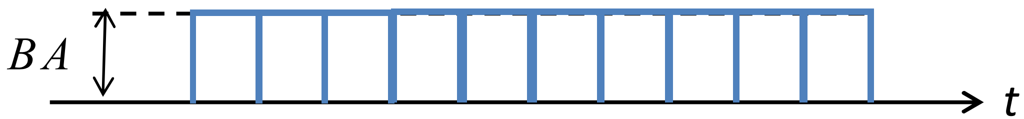




Output signal

$$s_y = \int_{-\infty}^{\infty} X(f)\, W_L(-f)\, df = 2\cdot\left[\frac{2}{\pi}A\cdot\frac{2}{\pi}B\right]\cdot\left[1+\frac{1}{3^2}+\frac{1}{5^2}+.....\right] = 2\cdot\left[\frac{2}{\pi}A\cdot\frac{2}{\pi}B\right]\frac{\pi^2}{8} = AB$$

NB1 This is easily verified in time, since: LIA output *y(t)* = time average of *z(t)=x(t)·B m(t)*



NB2 As concerns the **output noise**, it has already been discussed and illustrated in slide 21

S/N with Squarewave Signal and perfect Squarewave Reference

Output Signal $s_y = B \cdot A$ for squarewave input signal in phase

Output Noise $\overline{n_{yL}^2} = S_{Bb} \Delta f_n B^2$

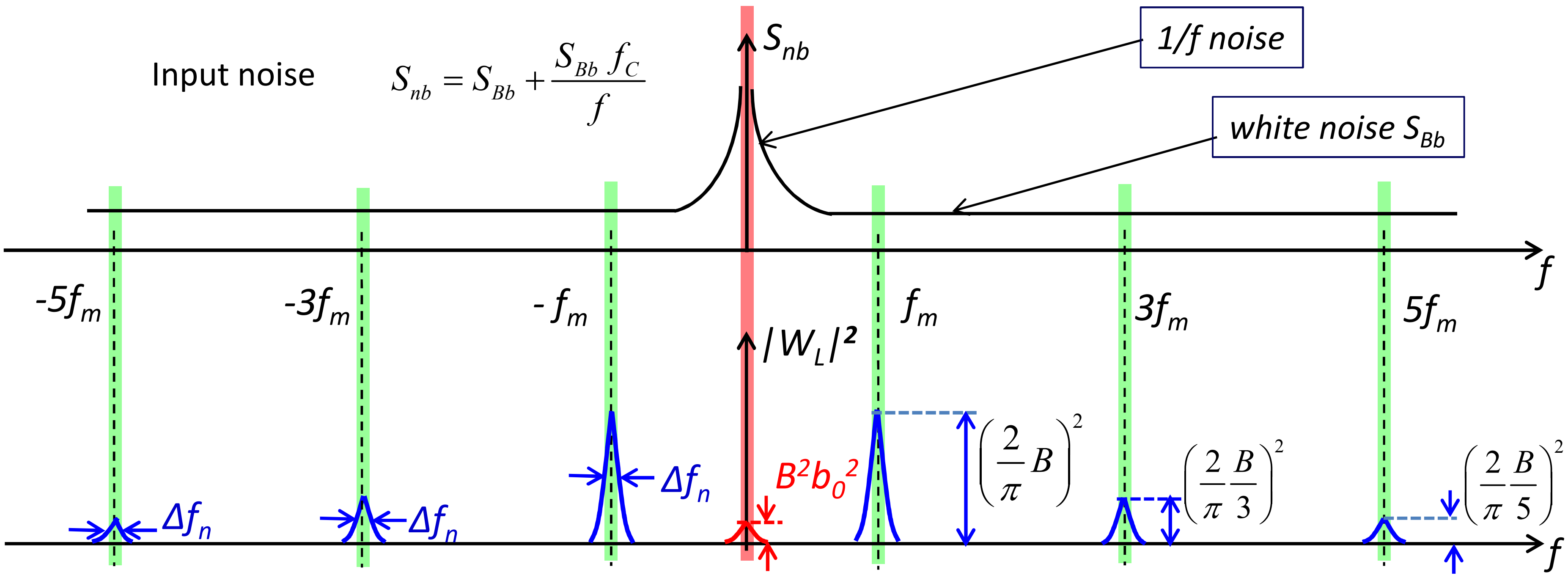
so that
$$\left(\frac{S}{N}\right)_{L,sqw} = \frac{s_y}{\sqrt{\overline{n_{yL}^2}}} = \frac{A}{\sqrt{S_{Bb}\Delta f_n}}$$

in power terms
$$\left(\frac{S}{N}\right)_{L,sqw}^2 = \frac{A^2}{S_{Bb}\Delta f_n}$$

Notice that
$$\boxed{\left(\frac{S}{N}\right)_{L.sqw}^2 = \frac{in - phase\ signal\ \ power}{half\ \ power\ of\ white\ noise\ in\ the\ band\ \ \Delta f_n}}$$

- the latter statement is the same as for the case of sinusoidal signal and sinusoidal reference, that is, the other case seen with equal waveforms of signal and reference
- However, for equal amplitude *A* the squarewave signal has double power and correspondingly higher S/N

Noise through LIA with imperfect Squarewave Reference



A squarewave with non-zero-mean generates a spurious band at f=0 with additional noise

$$\overline{n_{yL0}^2} = \widehat{S_{nb}} \Delta f_n B^2 b_0^2$$

Because of the 1/f noise, the mean density $\widehat{S_{nb}}$ in the band can be very high $\widehat{S_{nb}} \gg S_{Bb}$ so that even with small spurious band $b_0 << 1$ the added noise $\overline{n_{yL0}^2}$ can be comparable to the basic term $\overline{n_{yL}^2}$ or even larger

$$\boxed{\frac{\overline{n_{yL0}^2}}{\overline{n_{yL}^2}} = \frac{\widehat{S_{nb}}}{S_{Bb}} b_0^2}$$

Summary and comparison of LIA results

	SINUSOIDAL Reference	SQUAREWAVE Reference
SINUSOIDAL Signal amplitude A power $P = \frac{A^2}{2}$ A_{min} minimum measurable amplitude (at S/N=1)	$\frac{S}{N} = \frac{A}{\sqrt{2}\sqrt{S_{Bb}\Delta f_n}} = \frac{\sqrt{P}}{\sqrt{S_{Bb}\Delta f_n}}$	$\frac{S}{N} = \frac{A}{\frac{\pi}{2}\sqrt{S_{Bb}\Delta f_n}}$ $= \frac{\sqrt{P}}{\frac{\pi}{2\sqrt{2}}\sqrt{S_{Bb}\Delta f_n}}$
	$A_{\min} = \sqrt{2}\sqrt{S_{Bb}\Delta f_n} = 1{,}41\sqrt{S_{Bb}\Delta f_n}$	$A_{\min} = \frac{\pi}{2}\sqrt{S_{Bb}\Delta f_n}$ $= 1{,}57\sqrt{S_{Bb}\Delta f_n}$
SQUAREWAVE Signal amplitude A power $P = A^2$ A_{min} minimum measurable amplitude (at S/N=1)	$\frac{S}{N} = \frac{A}{\frac{\pi}{2\sqrt{2}}\sqrt{S_{Bb}\Delta f_n}} = \frac{\sqrt{P}}{\frac{\pi}{2\sqrt{2}}\sqrt{S_{Bb}\Delta f_n}}$	$\frac{S}{N} = \frac{A}{\sqrt{S_{Bb}\Delta f_n}} = \frac{\sqrt{P}}{\sqrt{S_{Bb}\Delta f_n}}$
	$A_{\min} = \frac{\pi}{2\sqrt{2}}\sqrt{S_{Bb}\Delta f_n} = 1{,}11\sqrt{S_{Bb}\Delta f_n}$	$A_{\min} = \sqrt{S_{Bb}\Delta f_n}$

Pre-filtering complementary to the LIA Filtering

Pre-filtering complementary to LIA Filtering

We have seen that constant-parameter filters are currently employed before the LIA in order to avoid to exceed the dynamic range of the LIA with a high input noise. Constant-parameter filters can be employed also to complement the filtering by the LIA or to correct some drawback, for instance:

a) In cases of imperfect reference waveform that causes a spurious band at f=0, the 1/f noise collected in this band can be reduced by a high-pass or bandpass pre-filter

b) In cases where a LIA with squarewave reference is employed and a «fundamental-only» response is required (i.e. only the fundamental component of a signal has to be measured), a low-pass or band-pass pre-filter can be employed to reduce the contribution of the signal components at the 3^d harmonic (and higher odd harmonics).

However, the design of a prefilter and the evaluation of its effect must carefully take into account the actual cutoff in frequency of the real filters, which is remarkably different from an ideal sharp cut.

About design and evaluation of Pre-filters

The design of the prefilter and the evaluation of its effect must carefully take into account the actual cutoff in frequency, in particular the finite slope of the attenuation vs frequency. This makes difficult to obtain high attenuation at unwanted frequencies multiples of the fundamental f_m and at the same time avoid attenuation at the fundamental frequency f_m .

- First-order filters (single-pole transfer function) have slope ≤20dB/decade. Therefore, the attenuation obtained for the 3^{d}harmonic is moderate (see previous case b) and in case of low f_m the attenuation of the 1/f noise obtained in the spurious band may be insufficient (see previous case a).
- The filters also give to the signal a phase shift that strongly varies with the frequency near to the pole. We shall see that this causes problems in the frequent cases where the signal frequency f_m is not perfectly stable. Note that current set-ups for AC measurements actually do not employ ultra-stable oscillators; for instance, this is the case for the voltage supply of Wheatstone bridges; for the voltage commands that control switches; etc.

About design and evaluation of Pre-filters

- Higher-order filters (multi-pole transfer function) have sharper cutoff with higher slope, but they give to the signal a greater phase shift with steeper variation with frequency, which enhances the problems met in cases where the signal frequency is not very stable.
- In many cases the signal to be measured is accompanied by high disturbing signals at a well defined frequency. Most frequently, interference from electrical power lines generates disturbances at line frequency and its multiples. Nonlinear loads on the power line generate 2° harmonic signals; power supplies and power regulators based on switching devices generate even higher harmonics.
- These narrow-band disturbances can be strongly reduced by **notch-filters**, that is, prefilters that provide a stop-band centered on the disturbing frequency. In fact, LIA manufacturers currently include in the LIA input stage notch-filters at the line frequency (NB: this means 50 Hz in Europe and 60 Hz in the US !!) and at its double.

Pre-filtering issues with variable signal frequency

Let us consider cases where a LIA is employed to measure a signal with fundamental frequency $\boldsymbol{f_m}$ **that slowly varies in time** and a correct reference is available, which tracks the frequency f_m . What are the consequences?

- **No problem for the frequency selection** by the LIA, it remains correct.
  Since the reference tracks the actual f_m and brings this information to the LIA, the center of the LIA admission band is locked anyway to the signal frequency.
- There is a **problem for the phase selection** by the LIA. Since the phase-shift due to the prefilter varies with the frequency, the signal that reaches the LIA suffers a phase variation and is no more in-phase with the reference. The phase difference produces a variation of the output signal amplitude, which is incorrectly ascribed to a variation of the input signal amplitude.
- However, it is possible to **compensate the phase shif**t that affects the signal by passing the reference through a filter identical to the one acting on the signal. Signal and reference are thus subjected to the same phase shift and the phase difference between them is not altered.

Reference:
phase adjustment and waveform conditioning

Reference: phase adjustment

In order to be useful as **reference** for measuring with a LIA a given periodic signal, the **essential necessary features** of an auxiliary signal are:

1) fundamental **frequency identical** to the signal
2) **constant phase difference φ** with respect to the signal.
   (NB: not necessarily $\varphi=0$, but it is necessary that φ = constant !)

If the auxiliary signal has high and constant amplitude, negligible noise and clean waveform (free from harmonics), it can be directly adjusted to $\varphi=0$ with a phase-shifter filter and supplied to the multiplier as reference waveform.

- An adjustable phase-shifter is currently included in LIAs for re-phasing the reference. The phase adjustment can be controlled manually by observing the output signal amplitude, which is maximum when $\varphi=0$.
- Many LIA's besides the adjustable phase shifter include an additional filter, which gives phase shift φ_a switchable from $\varphi_a=\pi/2$ to $\varphi_a=0$. Setting $\varphi_a=\pi/2$, when $\varphi=0$ is reached the signal is in quadrature and the output is zero. Notice that observing the output signal while φ is varied it is easier to identify when it reaches zero rather than when it reaches the maximum. After the adjustment to $\varphi=0$, the additional filter is switched back to $\varphi_a=0$ and the LIA is ready to operate.

Reference: waveform conditioning

In many cases an auxiliary signal is available that has the correct essential features (identical frequency and constant phase difference φ with respect to the signal), but is not adequate as reference because of some drawback: it is small and plagued by high noise; it has variable amplitude; it includes unwanted harmonics; etc. In such cases the waveform must be first improved and then phase-adjusted.

- A well-designed LIA receives the external reference and generates an internal reference signal with frequency and phase locked to the external reference, but with waveform independent from it, clean and with constant amplitude. This is usually denoted as «**reference conditioning**»
- Nowadays the solution of choice for the reference conditioning is to employ a Phase Locked Loop (PLL). PLLs with very good performance are currently available (nowadays even in integrated circuit form) and make possible to lock also on small and noisy signals.
- It is interesting to note that a PLL exploits essentially the same basic principles of the LIA for comparing a signal internally generated (by a voltage-controlled oscillator) with the external reference and lock to it.

LIA with Digital Signal Processing (DSP)

Digital Signal Processing (DSP) in LIA

- The limitations of the **analog multiplier** (small dynamic range of linear operation, etc.) are the main cause of limitation to the LIA performance (limited Dynamic Reserve; necessity of a high-gain DC-coupled post-amplifier with inherent baseline drift; etc.)
- These drawbacks can be avoided and the LIA performance can be greatly improved with Digital Signal Processing (DSP) techniques, i.e. by carrying out **with digital electronics the multiplication** of the digitized signals and noise and the **low-pass filtering** that completes the PSD
- However, in order to have an adequate conversion of the input analog waveform (signal plus noise) in a stream of digital data, the **ADC must fulfill severe requirements**. Wide dynamic range and high linearity are necessary, together with high resolution in amplitude (high number of ADC bits or quantization levels) and time (sampling rate >200kHz, i.e. AD conversion time of a few microseconds).
- In the years 70's pioneering work produced the first DSP-LIAs. They exploited an ADC scheme that had high amplitude resolution, linearity and dynamic range, but sampling rate limited to a few kHz. It operated with signals modulated at up to 1kHz and attained unprecedented performance, that practically opened to LIA applications the low frequency range, where the performance of analog LIAs was poor.
- In the years 90's the progress in ADC design and technology made possible to design general purpose DSP-LIAs for operation in the usual frequency range up to 100kHz.

Dynamic Reserve: DSP-LIAs vs analog LIAs

The LIA Dynamic reserve (DR) is the ratio

$$\frac{\text{a) maximum tolerable input rms noise}}{\text{b) input signal for full-scale output}}$$

For **analog LIAs**:

a) the maximum noise is limited by the **dynamic range of the analog multiplier**
b) the full-scale signal should be high enough to avoid measurement errors due to the **baseline drift of the post-amplifier**

The DR can attain ≈ 60dB (i.e. ≈1000) in favourable instances, but it is often lower.

For **DSP-LIAs**:

a) the maximum noise is limited by the **dynamic range of the ADC**
b) the full-scale signal should be high enough to avoid insufficient resolution and errors due to the **amplitude quantization by the ADC**.

The DR can attain ≈ 100dB (i.e. ≈ 100000) in current instances

DSP-LIAs offer such a remarkable advantage essentially because

- the dynamic range of current ADCs is much higher than that of analog multipliers
- the quantization interval of current ADCs is small and produces small quantization errors, which can be further reduced by the action of the LPF in the PSD

ADCs for DSP Lock-in Amplifiers

Typical features of currently available high-linearity ADCs suitable for DSP-LIAs are:

- input dynamic range 10 V (-5v to +5V)
- quantization at 16 bits (i.e. with 65.536 quantization levels)
- sampling frequency >250 kHz (i.e. AD conversion time *< 4 μs*)

At first sight, the amplitude resolution may appear insufficient to obtain high DR.
With noise extended over all the 16bits (65.536 levels) the signal is limited to:

6 bits (≈65 levels)	for DR = 60dB (i.e. ≈1000)
≈2,5bits (≈6 levels)	for DR = 80dB (i.e. ≈10000)
< 1bit (<1 level)	for DR = 100dB (i.e. ≈100000)

Nevertheless, adequate resolution is obtained because each value at the LIA output is not a single digitized sample, but a weighted **average of a high number of quantized samples** (of signal plus noise). It is well known and intuitive that in the average the ADC resolution is enhanced by increasing the number of samples. With long averaging time of the digital LPF, the resolution can attain a very small fraction of the quantization interval and be adequate to work with DR up to > 100dB.

However, **high DR is at the expense of the LIA response time**, ruled by the LPF averaging time. The averaging time necessary for attaining very high DR may indeed be longer than that required for adequately filtering the noise.

Signal input channel in DSP-LIA

The analog input channel brings signal and noise to the PSD input in analog LIAs and to the ADC input in DSP-LIAs. The dynamic range of the ADCs is much wider than that of analog PSDs and relaxes the requirements set to the analog input electronics.

- It is no more necessary to include noise prefiltering for enhancing the DR. The dynamic range and resolution of the ADC are sufficient to ensure high DR
- It is no more necessary to include prefiltering for complementing or correcting the PSD action. Since the reference waveform is digitally sinthetized and the multiplication is carried out numerically, the PSD action is almost perfect and avoids spurious filtering passbands
- An AC-coupled preamplifier is still necessary for providing gain adequate to raise very small signals to level suitable for the ADC.
- Notch filters (at power line frequency and its double) are still useful and currently provided in order to block high-level disturbances
- An anti-aliasing filter is included for removing frequencies above the specified signal range, which would otherwise cause spurious results of the sampling.

Reference input channel in DSP-LIA

- The external reference must bring to the LIA the information of frequency and phase of the signal. No special requirements are set to the waveform: to be suitable to the purpose, it just has to show two well defined zero-crossings per period.
- In a DSP-LIA the external reference is supplied to a second DSP block, which operates as a digital Phase Locked Loop (PLL) and acquires the signal frequency and phase.
- This block then digitally generates a stream of values, which correspond to the digitized amplitudes of the reference waveform at the various instants where the input waveform is sampled. The digital waveform thus generated is practically almost perfect.
- The phase of the digital reference can be easily adjusted with high precision.
- There is inherently high flexibility in the digital generation and control of the reference. For instance: the shape of the reference waveform can be easily changed; a double-frequency reference can be easily obtained for measuring the 2° harmonic of the signal; etc.

Overview of DSP and Analog LIAs

- The capability of converting accurately analog waveforms of signal plus noise in «digital waveforms» (i.e. streams of digital data) and applying Digital Signal Processing electronics and techniques opened the way to develop LIAs with higher level of performance, versatility and flexibility.
- with respect to analog LIAs, DSP-LIAs offer superior quality filtering and are free from the post-amplifier problems (baseline drift etc.)
- DSP-LIA instruments are more costly than analog LIAs, but offer a better price/performance ratio. Furthermore, their development can benefit of the constant progress in DSP, driven by the requests from mass applications.

There are anyway various cases where analog LIAs are still preferable or are the only option. For instance, cases where:

- Signals modulated at high frequency (MHz and more) are to be measured.
- Signals at intermediate frequencies (< 200KHz) must be processed by a LIA with fairly fast response, that is, with short averaging time of the LPF.
- This requirement is typical in cases where the LIA is part of a control feedback loop. It is also met in case of signals doubly modulated at two widely different frequencies, which are demodulated in two steps. A first analog LIAs with fast response demodulates the higher frequency, thus producing an output a signal at the lower modulation frequency, which is then fed to the second LIA

Appendix 1 - LIA
Signal and Reference with any Waveform

Appendix 1 - LIA: Signal and Reference with any Waveform

Any **periodic reference *m(t)*** with fundamental frequency f_m and amplitude *B* is a sum of sinusoidal components

$$m(t) = B\sum_{k=1}^{\infty} b_k \cos\left(k\omega_m + \psi_k\right)$$

This is true also for any **periodic signal *x(t)*** with fundamental frequency f_m and amplitude A

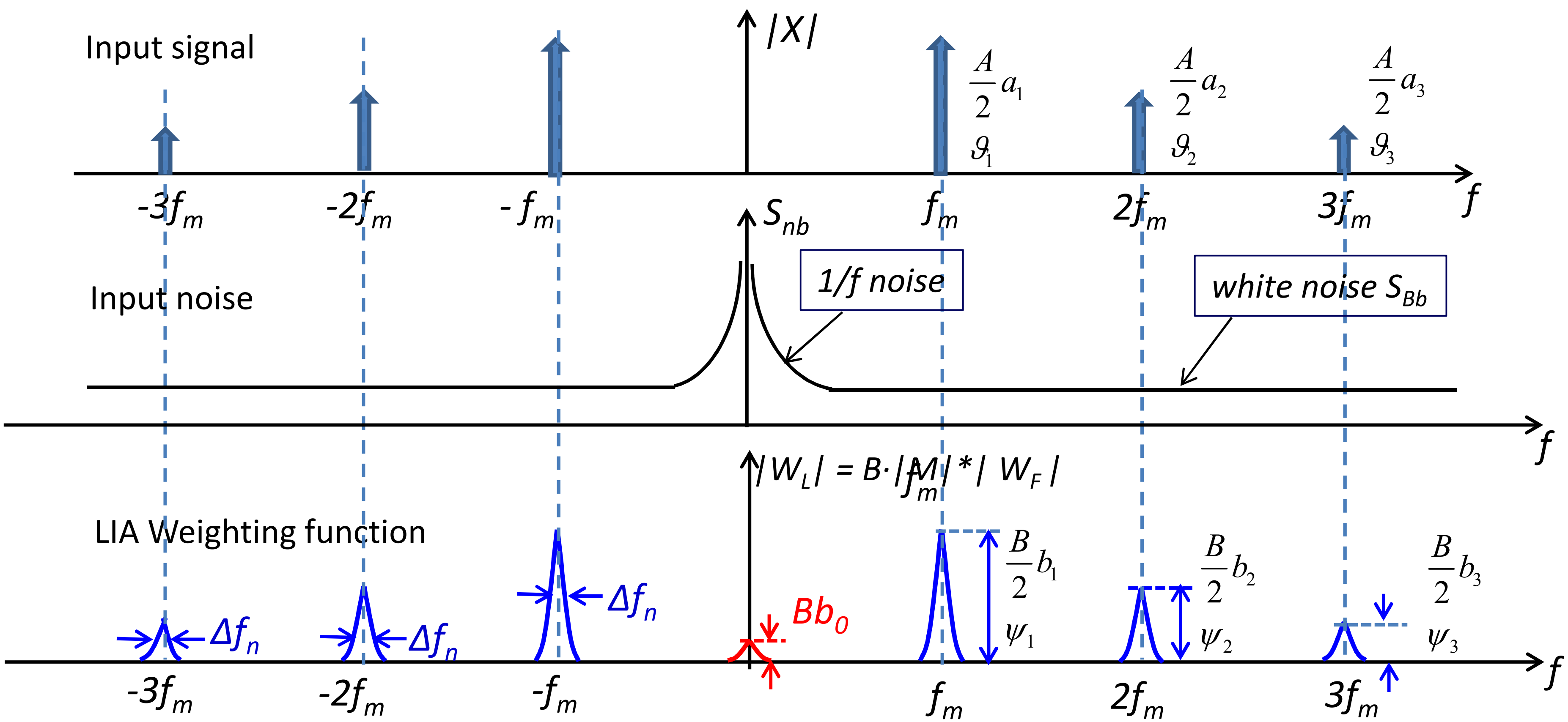
$$x(t) = A\sum_{k=1}^{\infty} a_k \cos\left(k\omega_m + \vartheta_k\right)$$

The components are **orthogonal** → Signal power = sum of component power

$$P_x = \frac{A^2}{2}\sum_{k=1}^{\infty} a_k^2$$

The signal and noise at the output of the LIA can be readily computed in the frequency domain by employing the LIA weighting function

Appendix 1 - LIA: Signal and Reference with any Waveform



LIA output

Signal $$s_y = \int_{-\infty}^{\infty} X(f)\, W_L(-f)\, df = B\frac{A}{2}\sum_{k=1}^{\infty} a_k b_k \cos\varphi_k$$ with $\varphi_k = \vartheta_k - \psi_k$

Noise $$\overline{n_{yL}^2} \cong B^2 b_0^2\, \widehat{S_{nb}}\, \Delta f_n + \frac{B^2}{2}\sum_{k=1}^{\infty} b_k^2 S_{nb}(k f_m)\,\Delta f_n = B^2 b_0^2\, \widehat{S_{nb}}\, \Delta f_n + \frac{B^2}{2} S_{nb}\,\Delta f_n \sum_{k=1}^{\infty} b_k^2$$

Appendix 1 – LIA: Signal and Reference with any Waveform

The DC component of the reference is an undesirable feature, because it collects 1/f noise and spoils the S/N.

With a reference *m(t)* free from DC component the S/N is

$$\left(\frac{S}{N}\right)^2 = \frac{s_y^2}{\overline{n_{yL}^2}} \cong \frac{B^2 A^2 \left(\frac{1}{2}\sum_{k=1}^{\infty} a_k b_k \cos\varphi_k\right)^2}{\frac{B^2}{2} S_{nb}\,\Delta f_n \sum_{k=1}^{\infty} b_k^2} = \frac{\frac{A^2}{2}\left(\sum_{k=1}^{\infty} a_k b_k \cos\varphi_k\right)^2}{S_{nb}\,\Delta f_n \sum_{k=1}^{\infty} b_k^2}$$

S/N is optimized with a reference waveform equal to the signal waveform:

with $b_k = a_k$ and $\varphi_k = 0$ for all k's

we obtain

$$\boxed{\left(\frac{S}{N}\right)_{opt}^2 = \frac{\frac{A^2}{2}\sum_{k=1}^{\infty} a_k^2}{S_{nb}\,\Delta f_n} = \frac{P_x}{S_{nb}\,\Delta f_n}}$$

This confirms in general the conclusion reached for sinusoidal and squarewave signals:

for a given signal, the **optimal S/N** is obtained **with reference waveform equal to the signal** and is the **ratio of signal power to half of the white noise power in the band *Δf***

Appendix 2 – LIA
Websites of interest

Wikipedia : http://en.wikipedia.org/wiki/Lock-in_amplifier

Stanford Research Systems : www.thinksrs.com

Signal Recovery : http://www.signalrecovery.com/

Summary and comparison of LIA results

	SINUSOIDAL **Reference**	SQUAREWAVE **Reference**
SINUSOIDAL **Signal** amplitude A power $P = \frac{A^2}{2}$	$\frac{S}{N} = \frac{A}{\sqrt{2}\sqrt{S_{Bb}\Delta f_n}} = \frac{\sqrt{P}}{\sqrt{S_{Bb}\Delta f_n}}$	$\frac{S}{N} = \frac{A}{\frac{\pi}{2}\sqrt{S_{Bb}\Delta f_n}} = \frac{\sqrt{P}}{\frac{\pi}{2\sqrt{2}}\sqrt{S_{Bb}\Delta f_n}}$
A_{min} minimum measurable amplitude (at $S/N=1$)	$A_{\min} = \sqrt{2}\sqrt{S_{Bb}\Delta f_n} = 1{,}41\sqrt{S_{Bb}\Delta f_n}$	$A_{\min} = \frac{\pi}{2}\sqrt{S_{Bb}\Delta f_n} = 1{,}57\sqrt{S_{Bb}\Delta f_n}$
SQUAREWAVE **Signal** amplitude A power $P = A^2$	$\frac{S}{N} = \frac{A}{\frac{\pi}{2\sqrt{2}}\sqrt{S_{Bb}\Delta f_n}} = \frac{\sqrt{P}}{\frac{\pi}{2\sqrt{2}}\sqrt{S_{Bb}\Delta f_n}}$	$\frac{S}{N} = \frac{A}{\sqrt{S_{Bb}\Delta f_n}} = \frac{\sqrt{P}}{\sqrt{S_{Bb}\Delta f_n}}$
A_{min} minimum measurable amplitude (at $S/N=1$)	$A_{\min} = \frac{\pi}{2\sqrt{2}}\sqrt{S_{Bb}\Delta f_n} = 1{,}11\sqrt{S_{Bb}\Delta f_n}$	$A_{\min} = \sqrt{S_{Bb}\Delta f_n}$

From principles to real LIA instruments

Linearity limits and Dynamic Reserve

- Since the dynamic range of the multiplier (and the output dynamic range of the preamplifier) do not change when the gain is changed, increasing the preamplifier gain decreases by the same factor the maximum rms noise tolerable at the preamplifier input (i.e. the limit set to the input noise for maintaining linear behavior)
- **For a given gain setting**, let us focus onto

  a) the maximum tolerable rms input noise

  b) the input signal corresponding to full-scale output signal of the LIA, that is, full-scale signal at the post-amplifier output.
- The characteristic ratio denoted **Dynamic Reserve (DR)**

$$\frac{\textbf{maximum tolerable input rms noise}}{\textbf{input signal for full-scale output}}$$

points out the limit within which the LIA can correctly extract a signal from a much higher noise (with that gain setting). It is a significant figure and is routinely specified by the manufacturer. It can attain 1000 (i.e. 60dB) with full-scale signals that require moderate preamplifier gain (typically with full-scale value down to about 1μV) but decreases with higher sensitivity setting.