Sensors, Signals and Noise

COURSE OUTLINE

- Introduction
- Signals and Noise
- Filtering
- Sensors: PD1 – PhotoDetector Fundamentals

Photons and photodetector principles

- Photons and Spectral ranges
- Reflection and Absorption of Photons in materials
- Thermal Photodetector Principles
- Quantum Photodetector Principles
- Photon Statistics and Noise
- Current Signals of Quantum Photodetectors
- Appendix: S-R theorem

Photons and Spectral ranges

Photons

- Light = electromagnetic waves with frequency ν and wavelength λ propagation speed (in vacuum) $c = 2{,}998 \cdot 10^8\ m/s$

$$\boxed{c = \lambda \nu}$$

- Spectral ranges:

$\lambda < 400nm$	Ultraviolet (UV)
$400nm < \lambda < 750nm$	Visible (VIS)
$750nm < \lambda < 3\ \mu m$	Near-infrared (NIR)
$3\ \mu m < \lambda < 30\ \mu m$	Mid-infrared (MIR)
$30\ \mu m < \lambda$	Far-infrared (FIR)

- Photon: quantum of electromagnetic energy

$E_p = h\nu$ quantum energy (Planck's constant $h = 7{,}6 \cdot 10^{-34}\ J{\cdot}s$)

Rather than E_p in Joules, the electron-voltage V_p is employed:

$E_p = q\ V_p$ (electron charge $q = 1{,}602\ 10^{-19}\ C$)

V_p in Volts (or electron-Volts eV)

Photon Energy and Momentum

Photon Energy E_p

inversely proportional to the wavelength $E_p = h\nu = \frac{hc}{\lambda}$

from $E_p = qV_p$ we get $$V_p = \frac{hc}{q}\frac{1}{\lambda}$$

universal constant *hc/q = 1,2398 ·10* $^{-6}$ *m·V ≈ 1,24 μm·V*

$$\boxed{V_p = \frac{1,24}{\lambda}}$$

with V_p in Volts and λ in μm

400nm < λ *< 750nm*	VIS range	*3,10 eV >* V_p *> 1,65 eV*
750nm < λ *< 3μm*	NIR range	*1,65 eV >* V_p *> 0,41 eV*

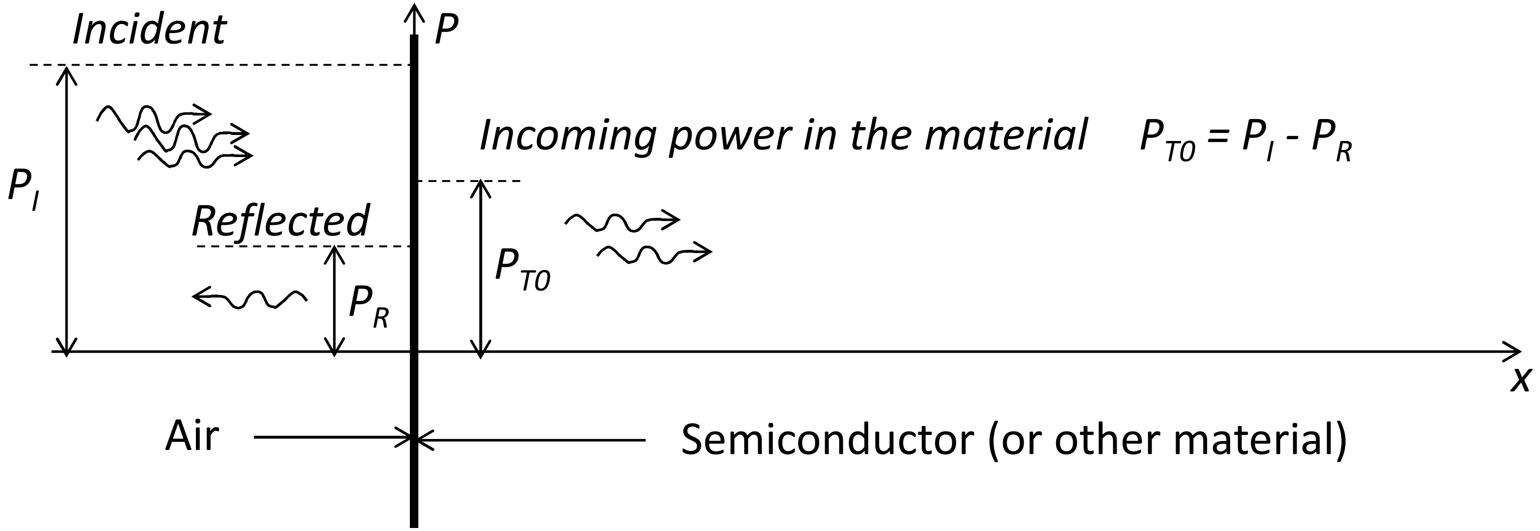
Photon Momentum p_p

$$p_p = \frac{E_p}{c} = \frac{h}{\lambda}$$

The photon momentum is **much smaller than the momentum of electrons**. In the electron transitions in solids due to photon absorption the electron momentum is almost constant while the electron energy is remarkably increased, so that the transition is almost vertical in the energy-momentum diagram

Reflection and Absorption of Photons

Reflection of Photons on the surface



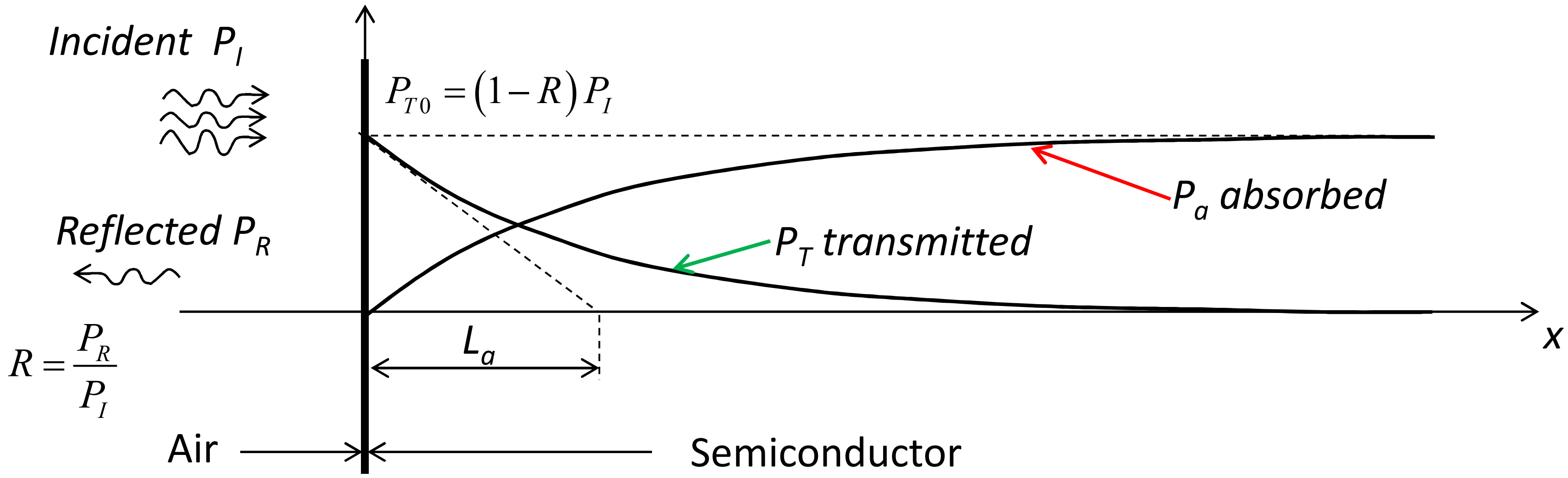
At the surface strong discontinuity of the refraction index *n*, from *n* = 1 for air to *n >1* for semiconductor: e.g. for silicon it is about n ≈ 3,4 and depends on the wavelength. This discontinuity gives a **high reflection coefficient *R***

$$R = \frac{P_R}{P_I}$$

(e.g. for silicon *R > 0,4* wavelength dependent)

Anti-reflection coating: deposition on the reflecting surface of a sequence of thin dielectric material layers with progressively decreasing *n* value. It provides a **gradual decrease** of the n value from semiconductor to air and such a smoother transition reduces the reflection

Absorption of Photons in materials



For moderate or low P_T the absorption in dx is proportional to P_T (linear optic effect)

$$-dP_T = \alpha P_T dx = P_T \frac{dx}{L_a}$$

α = optical absorption coefficient

$L_a = 1/\alpha$ = optical absorption depth

The optical power transmitted to position x is

$$P_T = P_{T0} \exp(-\alpha x) = P_{T0} \exp(-x/L_a)$$

The optical power absorbed from 0 to x is

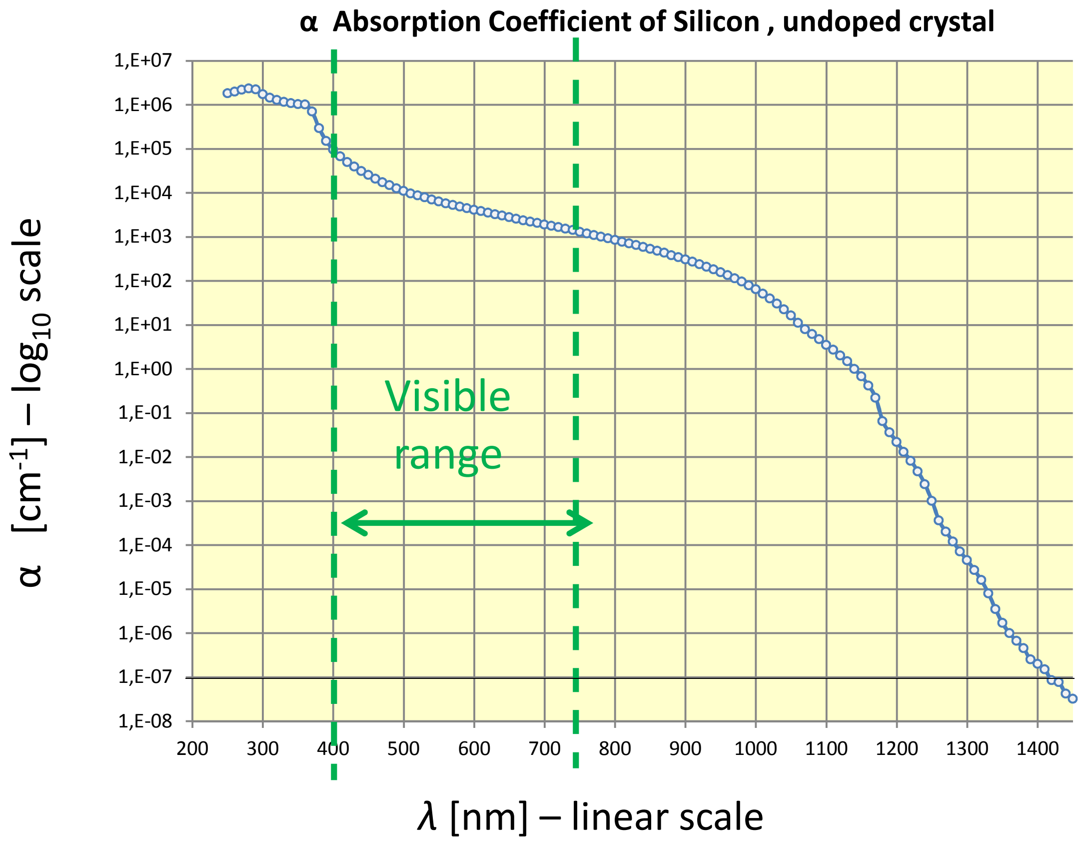
$$P_a = P_{T0} - P_T = P_{T0}\left(1 - e^{-\alpha x}\right) = P_{T0}\left(1 - e^{-\frac{x}{L_a}}\right)$$

which for $x \ll L_a$ leads to $P_a \approx P_{T0} \dfrac{x}{L_a}$

Absorption of Photons

For a given material the optical absorption **STRONGLY** depends on the **WAVELENGTH**.

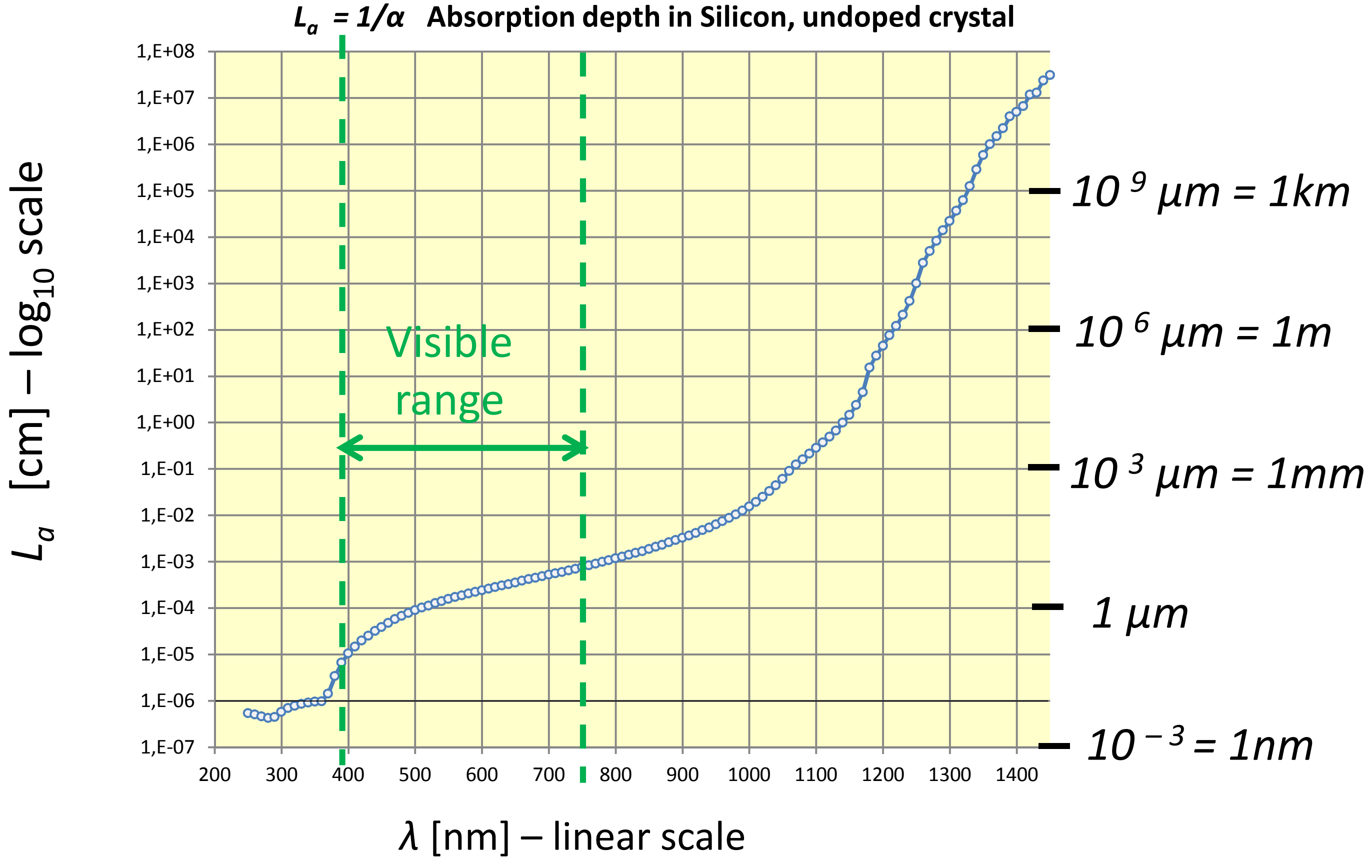
Typical example: Silicon absorption coefficient



Absorption of Photons

For a given material the optical absorption **STRONGLY** depends on the **WAVELENGTH**.

Typical example: Silicon absorption depth



NB: over the visible range L_a varies with λ by two orders of magnitude!!

Thermal Photodetector Principles

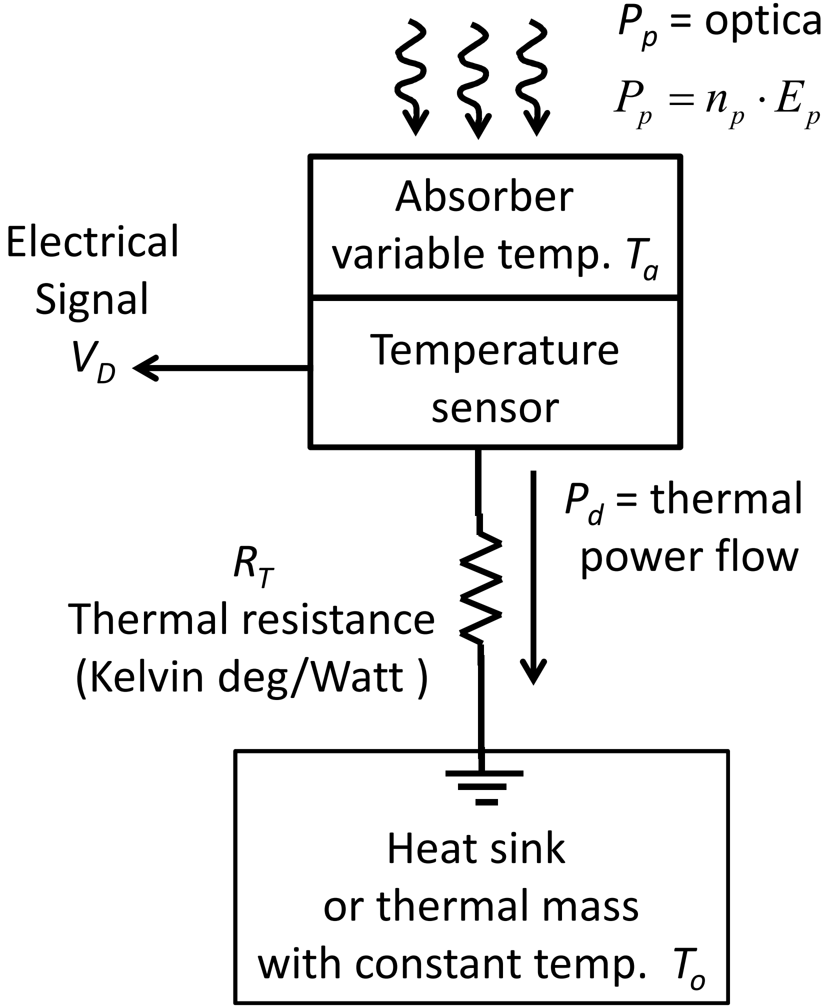
Principle of Thermal Photodetectors

- A principle for detection of light signals is to **employ their energy simply for heating a target and measure its temperature rise *ΔT*** . Detectors relying on this principle are called «Thermal Photodetectors» or «Power Detectors»
- Main advantage: very **wide spectral range**. Since photons just have to be absorbed for contributing to the detection, the range can be extended far into the infrared.
- Main drawback: sensitivity is inherently poor, because a high number of absorbed photons is required for producing even small variations of temperature ΔT in tiny target. For instance: ≈$\mathbf{10^{15}}$ blue photons are required for heating by **ΔT=0,1 K** a water droplet of ≈ **1mm** diameter (*blue photons at λ=475nm have* V_p *= 2,6 eV; water has specific heat capacity* c_T *= 4186 [J/Kg·K]= 2,6* $\cdot 10^{22}$ *[eV/Kg·K] and the mass is 1mg*)
- The dynamic response is inherently slow, because thermal transients are slow. Thermal detectors are mainly suitable for measurement of steady radiation.

Principle of Thermal Photo-Detectors

P_p = optical power; n_p = photon rate

$$P_p = n_p \cdot E_p = n_p \cdot qV_p$$



Absorber:
T_a = temperature, C_a = heat capacitance
$C_a = c_a \cdot m_a$
(m_a = mass; c_a = specific heat capacitance)

$\boxed{T_a - T_o = R_T \cdot P_d}$ analog to Ohm law $V = R{\cdot}I$

Denoting for simplicity $T = T_a - T_o$
the detector energy balance is

$$\boxed{P_p dt = C_a dT + \frac{T}{R_T} dt}$$

Principle of Thermal Photo-Detectors

From the energy balance $$P_p dt = C_a dT + \frac{T}{R_T} dt$$

we get $$\frac{dT}{dt} = \frac{P_p}{C_a} - \frac{T}{R_T C_a}$$ and in Laplace transform $$sT = \frac{P_p}{C_a} - \frac{T}{R_T C_a}$$

The detector transfer function from optical power to measured temperature thus is

$$\boxed{T = P_p R_T \frac{1}{1 + s\, R_T C_a}}$$

- The steady state response (the steady $T = P_p R_T$ obtained with steady P_p) increases as the thermal resistance R_T is increased
- The dynamic response is a single-pole low-pass filter with characteristic time constant $\tau_a = R_T C_a$: as R_T is increased, the bandlimit $f_T = 1/2\pi\, R_T C_a$ is decreased
- For improving the high-frequency response without reducing the steady response it is necessary to **reduce the heat capacitance $C_a = c_a \cdot m_a$** . This implies that
  a) absorber materials with small specific heat capacitance c_a are required
  b) the absorber mass m_a should be minimized.
- Remarkable progress has been indeed achieved in thermal detectors with modern **technologies of miniaturization and integration (of absorber, temperature sensor, etc.)** that make possible to fabricate also multipixel arrays of thermal detectors

Radiant Sensitivity or Spectral Responsivity

- Thermal detectors transduce the optical power P_P in an electrical output signal V_D of the temperature sensor (voltage signal of thermoresistances in Bolometers and of thermocouples in Thermopiles).
- The basic quantitative characterization of the performance of the detector is given by the **Radiant Sensitivity** (also called Spectral Responsivity) S_D , defined as

$$S_D = \frac{\text{electrical output voltage [in V]}}{\text{optical power on the detector sensitive area [in W]}}$$

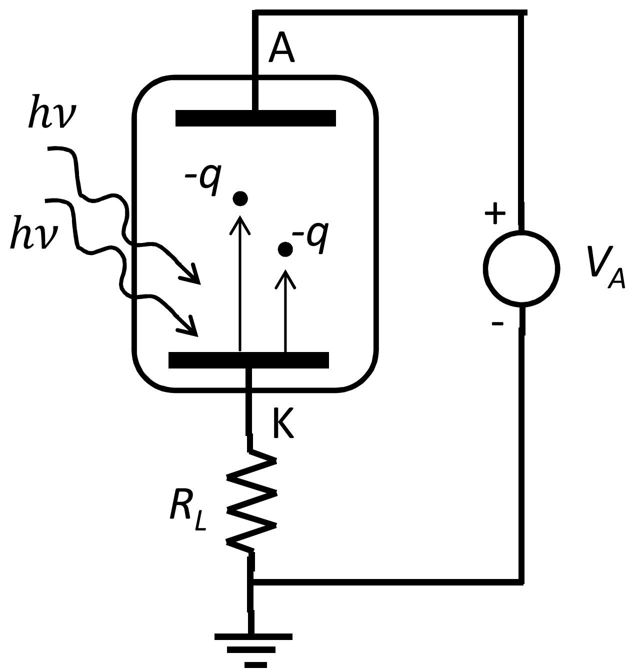
- For a given absorbed power the detector is heated at a given level, independent of the radiation wavelength λ. Therefore, uniform S_D would be obtained at all λ if the reflection and absorption were constant, independent of λ.
- Variations of reflection and absorption vs λ are kept at moderate level with modern absorber technologies. Fairly uniform S_D is achieved over fairly wide wavelength ranges, extended well into the infrared spectral region.

Quantum Photodetector Principles

Principles of Quantum Photodetectors

- A different principle for the detection of light signals is to exploit **photo-electric effects for producing directly an electrical current** in the detector. The energy of the absorbed photons is used for generating free charge carriers, which constitute the elements of the detector current.
- Detectors relying on this principle are called «**Quantum Photodetectors**» or «**Photon Detectors**»
- Photon Detectors can be vacuum-tube or semiconductor devices

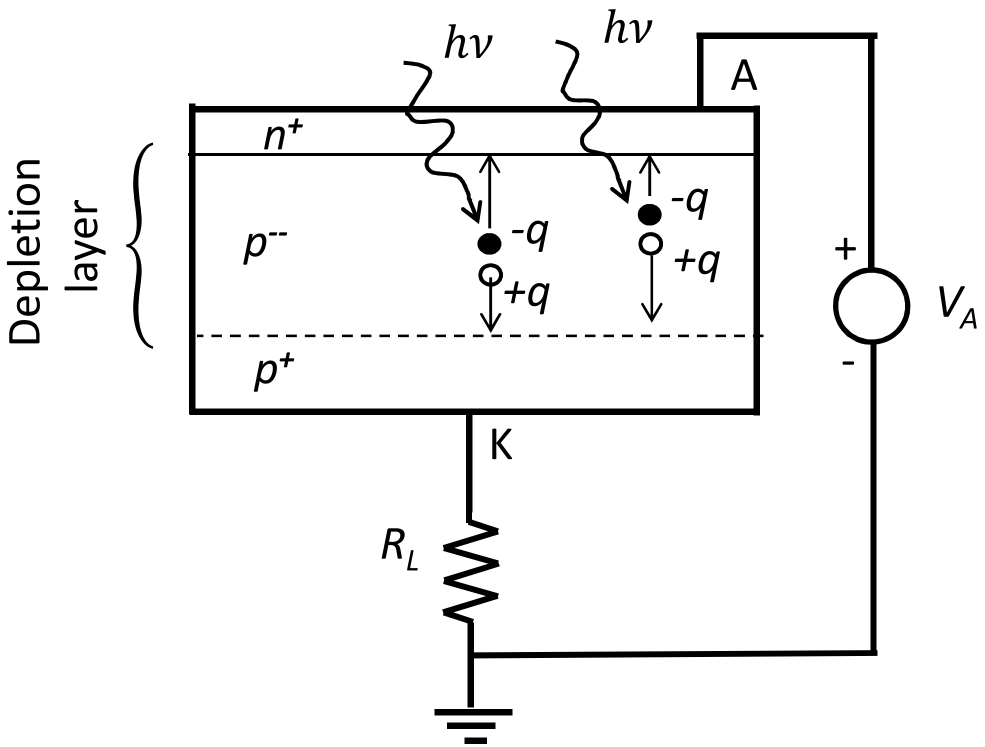
Principles of Quantum Photodetectors



Outline of
Vacuum-Tube detector devices:
Photo-Tubes or Photo-Diodes

- An electrode (cathode K) in a vacuum enclosure receives the photons
- By photo-electric effect the cathode emits electrons in vacuum.
- The electrons are drawn by the electric field to another electrode biased at higher potential (anode A)
- Current flows through the terminals (photocathode and anode).

Principles of Quantum Photodetectors



Outline of Semiconductor detector devices: Photo-Diodes

- Photons impact on a reverse-biased p-n junction diode
- The absorbed photons raise electrons from valence band to conduction band of the semiconductor, thereby generating free electron-hole pairs.
- The free carriers generated in the zone of high electric field **(the depletion layer)** are drawn by the junction electric field (the electrons to the n-terminal and the holes to the p-terminal)
- Current flows through the terminals.

Quantum Detection Efficiency

- Quantum photodetectors transduce optical signals in electrical current signals by collecting the free electrons generated by the photons of the optical radiation.
- The basic quantitative characterization of the performance of the detector is given by the **Quantum Detection Efficiency** (or Photon Detection Efficiency) η_D defined as

$$\eta_D = \frac{\text{number of photogenerated electrons (or electron-hole pairs)}}{\text{number of photons reaching the detector}} = \frac{N_e}{N_p}$$

- However, since in many engineering tasks the focus is on the transduction from optical power to electrical current, the **Radiant Sensitivity** S_D is often employed also for quantum photodetectors, defined as

$$S_D = \frac{\text{electrical output current [in A]}}{\text{optical power on the detector sensitive area [in W]}} = \frac{I_D}{P_L}\,[A/W]$$

Quantum Efficiency and Radiant Sensitivity

Photons of wavelength λ arriving with steady rate n_p on a quantum detector convey an optical power P_L

$$P_L = n_p\, h\nu$$

the electrons (or e-h pairs) photogenerated in the detector with steady rate n_e produce a current

$$I_D = n_e\, q$$

The Radiant Sensitivity is

$$S_D = \frac{I_D}{P_L} = \frac{n_e}{n_p} \cdot \frac{q}{h\nu} = \frac{n_e}{n_p} \cdot \frac{\lambda}{hc/q}$$

and since $\eta_D = n_e / n_p$

$$\boxed{S_D = \eta_D \cdot \frac{\lambda}{hc/q} = \eta_D \cdot \frac{\lambda[\mu m]}{1,24}}$$

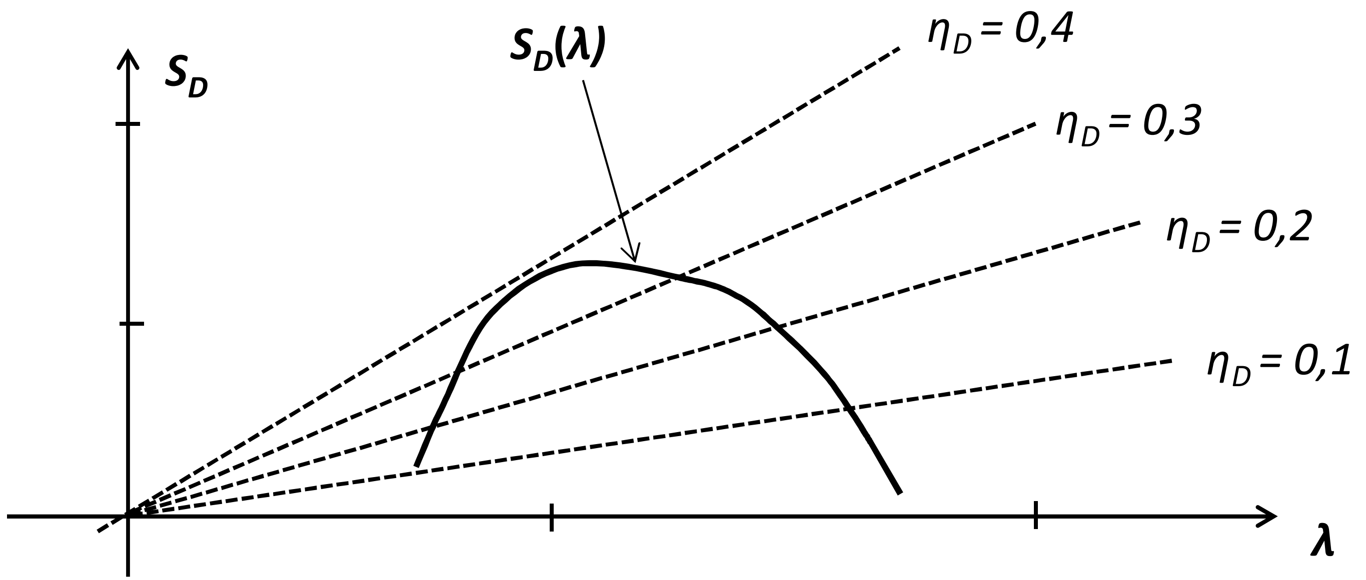
We see that the Radiant Sensitivity of the quantum detectors intrinsically depends on the wavelength λ, that is, even with constant quantum efficiency η_D. This occurs because a given optical power P_L corresponds to different photon rates n_p at different wavelengths λ

Quantum Efficiency and Radiant Sensitivity

The data sheets of the detector manufacturers usually report plots of the Radiant Sensitivity versus wavelength for quantum detectors.
However, the data of quantum detection efficiency can be easily read in such plots: the equation

$$\boxed{S_D = \eta_D \cdot \frac{\lambda[\mu m]}{1,24}}$$

shows how lines corresponding to given values of quantum efficiency can be easily drawn in the plot. In a plot with linear scales for both S_D and λ they are straight lines



Photon Statistics and Noise

Photon Noise

- The optical radiation is composed of photons arriving randomly in time; the photon number N_p in a given time interval *T* is a statistical variable with mean $\overline{N_p}$ and variance $\sigma_p^2 = \overline{N_p^2} - \left(\overline{N_p}\right)^2$
- The random fluctuations of the photons are the noise already present at optical level. This optical noise can be due to a background photon flux and to the actual desired optical signal.
- In most cases the photon statistics is well approximated (see later) by the Poisson statistics, so that it is

$$\boxed{\sigma_p^2 = \overline{N_p}}$$

- The optical power arriving to the detector is composed of quanta with energy $h\nu$ arriving randomly at rate n_p. It is the analog at optical level of a shot electrical current: the mean optical power is $P_p = n_p h\nu$ (analog to $I_e = n_e q$) ; the shot optical noise has unilateral spectral density S_p (analog to $S_i = 2qI_e$)

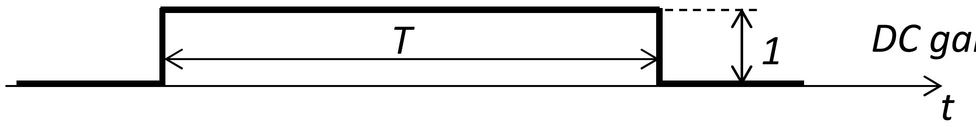
$$\boxed{S_p = 2h\nu P_p = 2\frac{hc}{\lambda} P_p}$$

- Note that for a given optical power P_p the shot noise density decreases as the wavelength λ is increased

Photon Noise

The photon shot noise is consistent with the Poisson statistics of photons.
This can be directly verified by considering for a given optical power $P_p = n_p h\nu$ to filter the optical signal and noise with a gated integration

GI weighting function



Noise bandpass $\Delta f = \frac{1}{2T}$

DC gain = T

GI output signal $\quad E_p = P_p T = h\nu\, n_p\, T = h\nu \overline{N_p}$

GI output noise $\quad \sigma_E^2 = S_p\, T^2 \Delta f = 2h\nu P_p\, T^2 \Delta f = h\nu P_p T = h\nu E_p = (h\nu)^2\, \overline{N_p}$

It is thus confirmed that $\quad \sigma_p^2 = \dfrac{\sigma_E^2}{(h\nu)^2} = \overline{N_p}$

In fact, photons obey the Bose-Einstein statistics. However, in most cases of interest for us the Poisson statistics is a satisfactory approximation of the B-E statistics. For instance, this is valid for lasers operating above threshold and for light emitted from hot matter in thermal equilibrium (black body radiation).

Photon Noise and B-E statistics

For the **thermal radiation (black body radiation)**, the treatment based on the B-E statistics leads to correct the equations obtained with the Poisson statistics as follows

$$\sigma_p^2 = \overline{N_p}\left(1+n_m\right) \qquad\qquad S_p = 2h\nu P_p\left(1+n_m\right)$$

The correcting term n_m is the mean number of photons at the energy level considered, computed with the B-E statistics as

$$n_m = \frac{1}{\exp\left(\dfrac{h\nu}{kT}\right)-1}$$

The quantitative evaluation of this correction shows that deviations from the Poisson approximation are appreciable only at long λ in the IR spectral range, beyond a limit that depends on the temperature.

For instance, the correction is > 1% with $n_m > 0{,}01$, that is for $h\nu < 4{,}6\ kT$, which corresponds

at T= 300K to $h\nu < 0{,}115\ eV$, i.e. to $\lambda > 11\mu m$

at T= 1300K to $h\nu < 0{,}498\ eV$, i.e. to $\lambda > 2{,}5\mu m$

Current Signals of Quantum Photodetectors

Detector Current Pulse Signal

- In the transduction of optical signals to current signals by Quantum Photodetectors the dynamic response has a cut-off at high frequency. Ultrafast optical pulses are transduced to current pulses that are still fast, but have longer duration.
- The response to a multi-photon optical signal is the linear superposition of the elementary responses to individual photons. The response to a single photon is also called **Single-Electron-Response SER** because a photon generates just one free electron (or one electron-hole pair).
- It is simply **wrong** to consider the SER a **δ-like current pulse** occurring at the time where the photogenerated charge carrier impacts on the collector electrode.
  The carrier **induces** a charge in the collector electrode **before** reaching it; the induced charge varies with the carrier position, so that current flows during all the carrier travel in the electric field.
- The waveform of the current signal is obtained by taking the derivative of the charge induced on the collector electrode as a function of time. To compute this charge is an electrostatic problem not easy to solve in general. However, the mathematical treatment can be remarkably simplified by preliminarily computing the motion of the charge carriers and exploiting then the **Shockley-Ramo theorem**.

Shockley-Ramo theorem

The output current due to an electron traveling towards the collector electrode can be obtained by applying the Shockley-Ramo theorem in three steps

1. The motion of the electron must be computed; i.e. the trajectory and the velocity v_c at every point of it must be known
2. A reference electric field E_v must be computed, which is the field that would exist in the device (in particular along the electron trajectory) under the following circumstances:

   electron removed

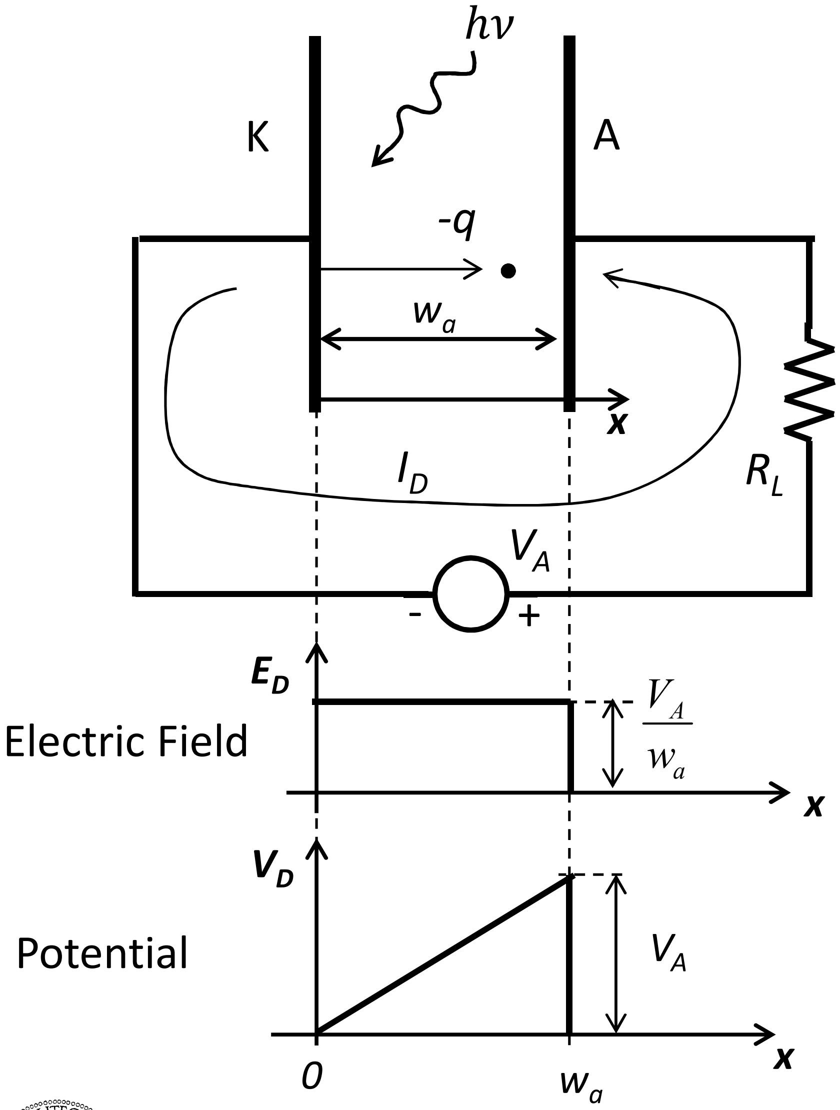
   output electrode raised at unit potential

   all other conductors at ground potential
3. The **Shockley-Ramo theorem** states that the current i_c that flows at the output electrode due to the electron motion can be simply computed as

$$\boxed{i_c = q\overrightarrow{E_v} \bullet \overrightarrow{v_c} = qE_{vc}v_c}$$

where • denotes scalar product and E_{vc} is the component of the field $\overrightarrow{E_v}$ in the direction of the velocity $\overrightarrow{v_c}$

Carrier motion in a phototube (PT)



VACUUM PHOTOTUBE WITH PLANAR GEOMETRY

w_a = cathode to anode distance

V_A = bias voltage

$E_D = \frac{V_A}{w_a}$ true electric field (in the - x direction)

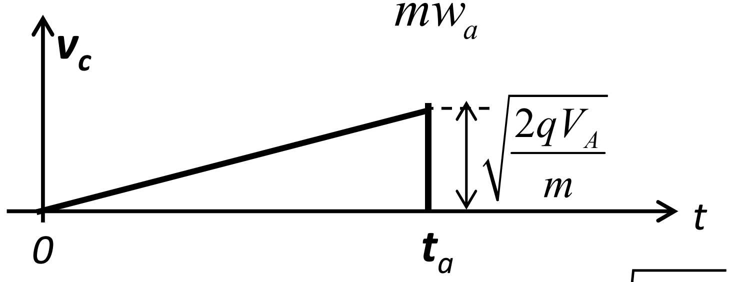
$V_D = V_A \frac{x}{w_a}$ potential distribution

ELECTRON MOTION **IN VACUUM**

(*-q* charge; *m* mass)

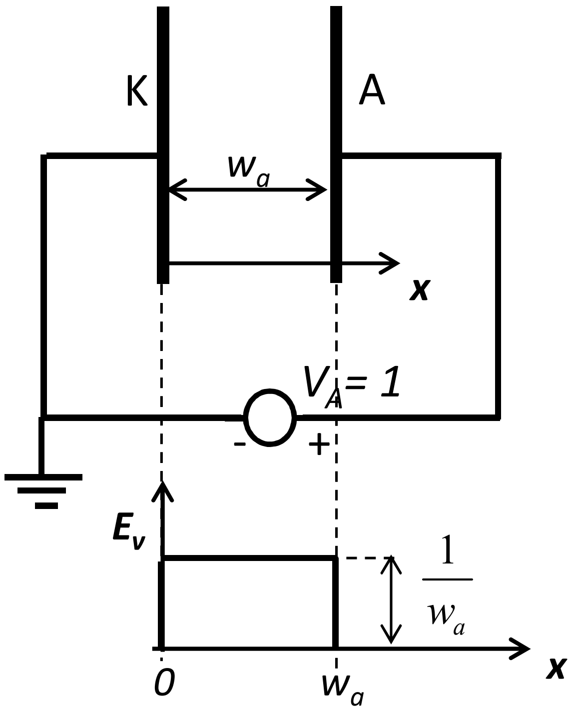
acceleration $a_c = \frac{qE_D}{m} = \frac{qV_A}{mw_a}$

Velocity $v_c = a_c t = \frac{qV_A}{mw_a} t$



Transit time $t_a = w_a \sqrt{\frac{2m}{qV_A}}$

SER current in a phototube (PT)

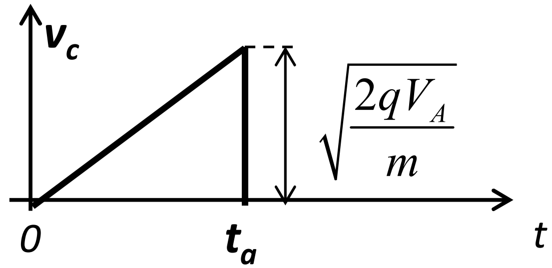


Reference electric E_v field computed with electron removed; $V_A = 1$; $V_K = 0$

$$E_v = \frac{1}{w_a}$$ parallel to the x-axis

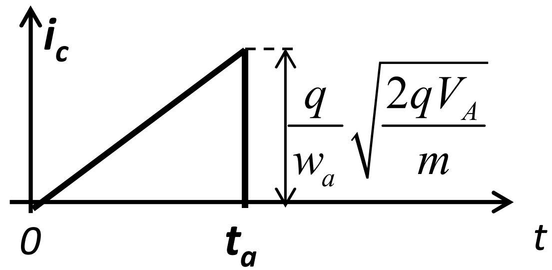
True electron velocity

$$v_c = \frac{qV_A}{mw_a}t$$ parallel to the x-axis



SR theorem: the output current due to a single electron is

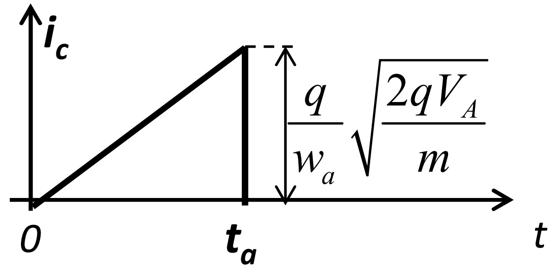
$$i_c = qE_v v_c = \frac{q^2 V_A}{m w_a^2} t$$



SER current in a phototube (PT)

In a phototube with planar geometry the single electron response (SER) is a pulse with triangular waveform

$$i_c = qE_v v_c = \frac{q^2 V_A}{m w_a^2} t \qquad \left(0 \le t \le t_a\right)$$

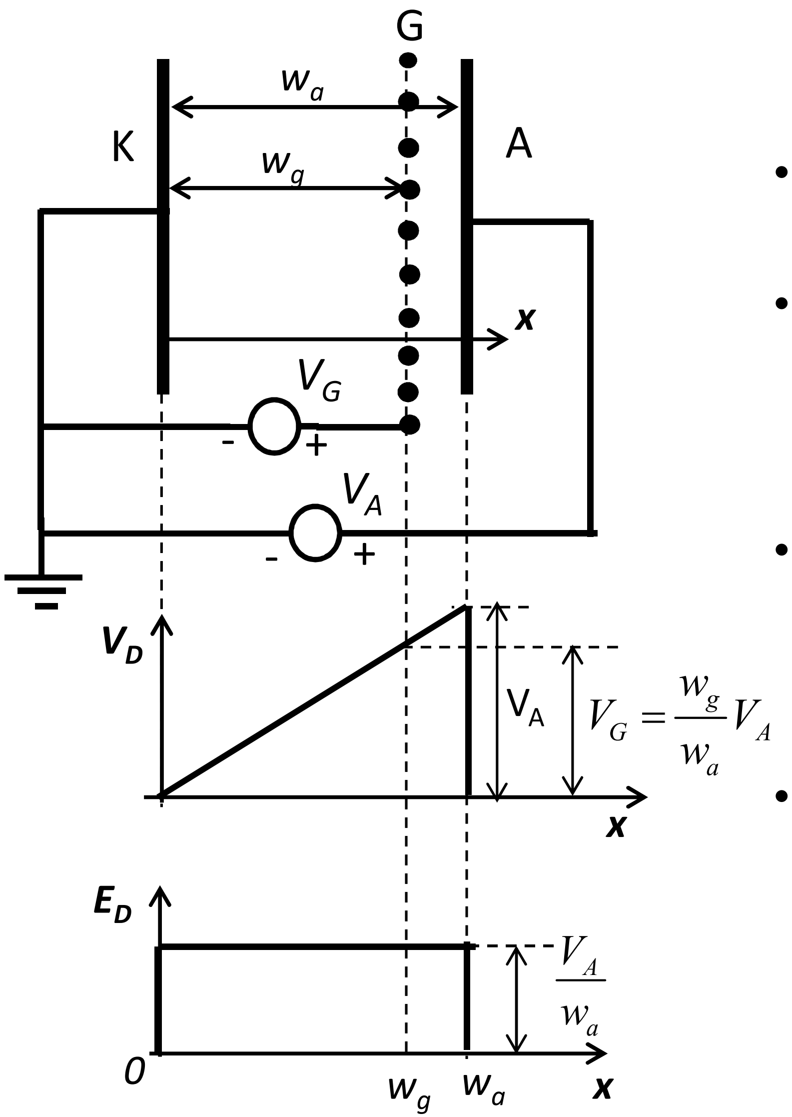


The frequency response is the Fourier transform of the SER pulse, which has a high frequency cutoff inversely proportional to the pulse width.
The pulse width is set by the transit time t_a of the electron from cathode to anode

$$t_a = \sqrt{2\frac{m}{q}} \cdot \frac{w_a}{\sqrt{V_A}} = 3{,}37 \cdot 10^{-6} \frac{w_a}{\sqrt{V_A}}$$

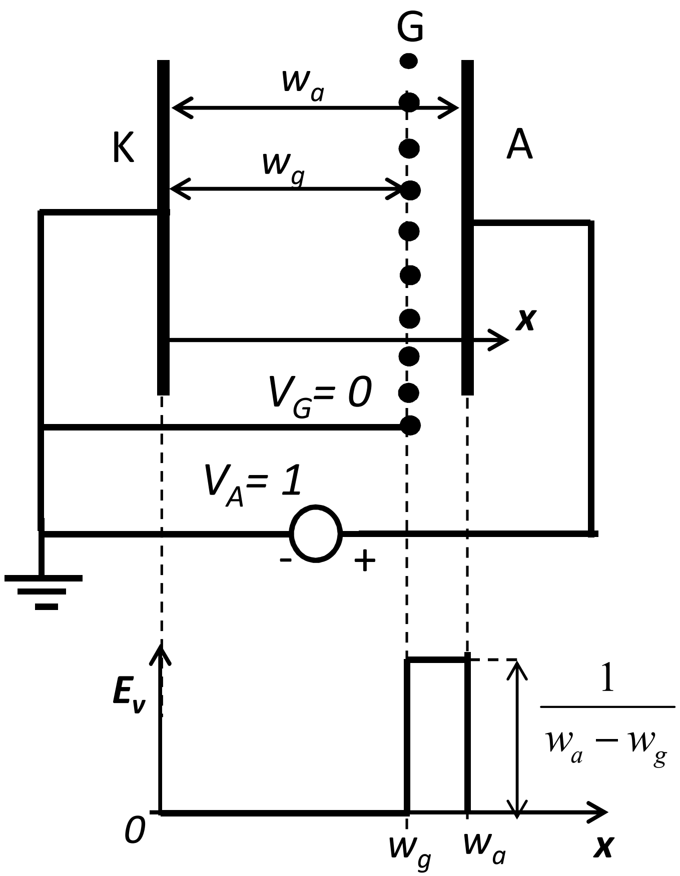
Typical values for phototubes are around *w = 1cm = 0,01m* and V_A= 100V, which correspond to transit time around $t_a \approx 3{,}3$ ***ns***

Screened-Anode PT: carrier motion



- A shorter SER pulse can be obtained by inserting a metal wire grid in front of the anode
- The basic idea is that the grid acts as electrostatic screen that does not allow an electron traveling from $x=0$ (cathode) to $x=w_g$ (grid) to induce charge on the anode.
- The grid bias voltage is selected to minimize the perturbation to the electron motion; i.e. it is set to the potential V_G corresponding to $x=w_g$ in absence of the grid (or slightly below it).
- In these conditions, the electric field is practically the same as in the phototube structure without grid and the motion of an electron in vacuum is also the same.

Screened-Anode PT: SR theorem



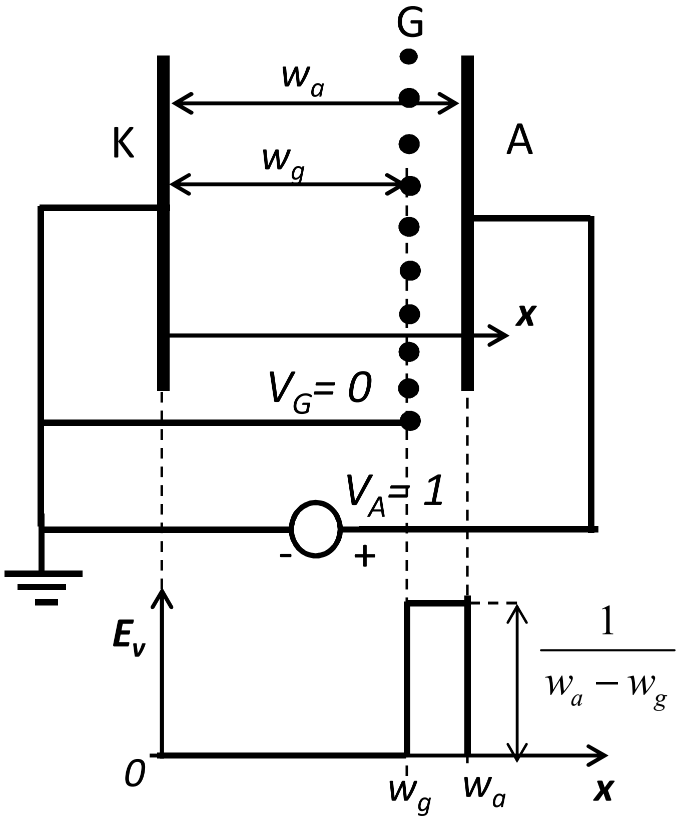
- Same electron motion as in the phototube without grid
- Different evolution in time of the induced charge on the anode.
- In fact, the reference field $\boldsymbol{E_v}$ is now very different and neatly shows that charge is induced on the anode only during the last part of the electron trajectory, i.e. from $x=w_g$ (grid) to $x=w_a$ (anode)

$$\begin{cases} E_v = 0 & for \quad 0 < x < w_g \\ E_v = \dfrac{1}{w_a - w_g} & for \quad w_g < x < w_a \end{cases}$$

- The SR theorem states that the SER current is

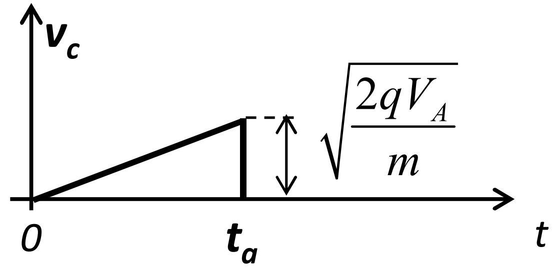
$$i_c = qE_v v_c$$

Screened-Anode PT for faster response



True electron velocity

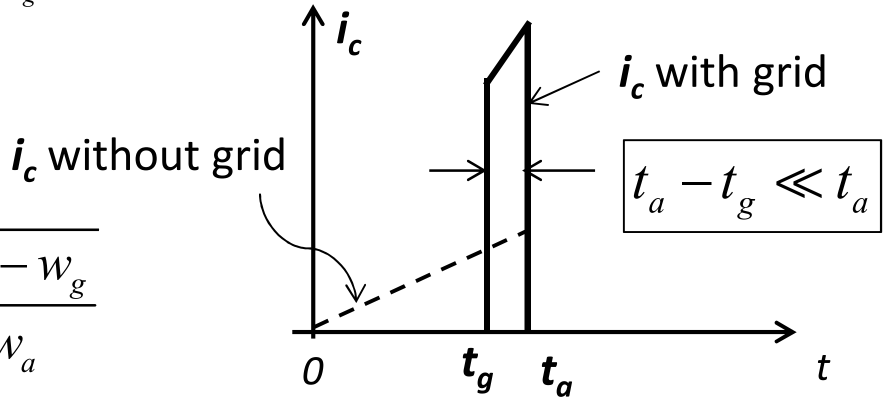
$$v_c = \frac{qV_A}{mw_a} t$$



Reference field of SR theorem

$$\begin{cases} E_v = 0 & for \quad 0 < x < w_g \\ E_v = \dfrac{1}{w_a - w_g} & for \quad w_g < x < w_a \end{cases}$$

SR theorem $\quad i_c = qE_v v_c$



$$t_a - t_g = t_a \sqrt{\frac{w_a - w_g}{w_a}}$$

Appendix: S-R theorem demonstration

- The theorem was independently demonstrated for the motion of carriers in vacuum in 1938 by William Shockley[1] and in 1939 by Simon Ramo[2]
- Extension to the motion of carriers in presence of a space charge (as it is the case in semiconductor devices) was demonstrated in 1971 by Emilio Gatti et al[3]
- These demonstrations were based on high-level concepts in electrostatic theory. Later on, a simple demonstration based on the "principle of virtual work“ (which exploits the energy conservation) was found. This demo is here summarized
- The S-R theorem considers situations where linear superposition of effects is valid (i.e. all media are linear). In such conditions:
  a) the **total** charge induced on a given electrode is the linear superposition of the contributions induced by the other electrodes at different potential and by the point charges present in the surrounding.
  b) the charge induced **specifically by a given carrier** on a given conductor depends **only on the position and charge of that carrier**, it does not depend on the potentials of the electrodes and on other point charges present.

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1. W. Shockley, J.Appl.Physics vol 9, 635-636 (1938)
2. S. Ramo, Proc. IRE, 584-585 (1939)
3. E. Gatti et al, Nucl.Instr.&Meth vol 92, 137-140 (1971)

Appendix: S-R theorem demonstration

- Let us consider a carrier of charge q in a given point of its trajectory (position vector $\vec{\boldsymbol{x}}$) and assume that the collector electrode is kept at constant voltage V=1 by a battery and all other electrodes are grounded. Note that in these conditions the electric field is just the Reference Field E_v of the S-R theorem.
- We do not intend to calculate the charge that the carrier in position $\vec{\boldsymbol{x}}$ induces on the collector electrode. We just want to calculate its **variation** caused by the carrier motion. Let us suppose to displace by $\boldsymbol{d}\vec{\boldsymbol{x}}$ the carrier.
- Because of the displacement the carrier gains an energy

$$dW_c = q\overrightarrow{E_v} \bullet d\vec{x} = q\overrightarrow{E_v} \bullet \overrightarrow{v_c} dt = qE_{vc} v_c dt$$

- The battery keeps the potential of the collector electrode constant by bringing a charge dQ from ground to the electrode and therefore by supplying an energy

$$dW_b = V_b dQ = 1 \cdot dQ$$

- There is no other energy exchange, hence for energy conservation

$$dW_b = dW_c$$ and therefore $$\boxed{i_c = \frac{dQ}{dt} = qE_{vc} v_c}$$