Sensors, Signals and Noise

COURSE OUTLINE

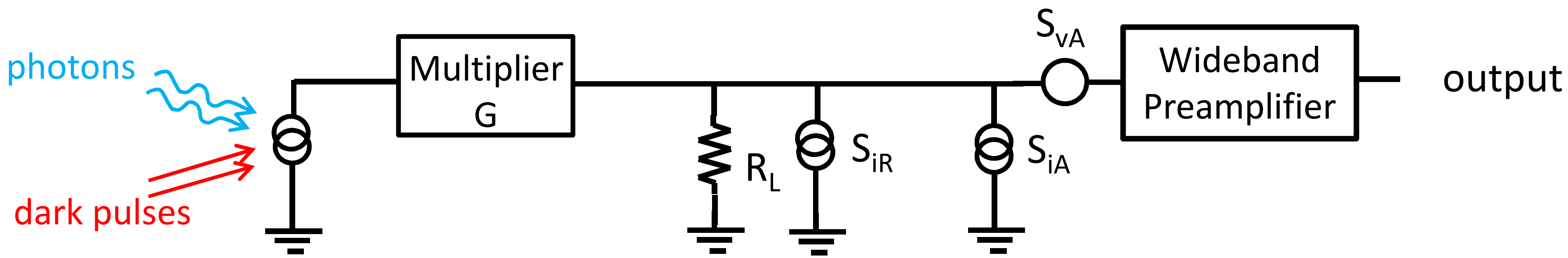
- Introduction
- Signals and Noise
- Filtering
- Sensors: PD 4a - Photon Counting with PMTs

Photon Counting with PMTs

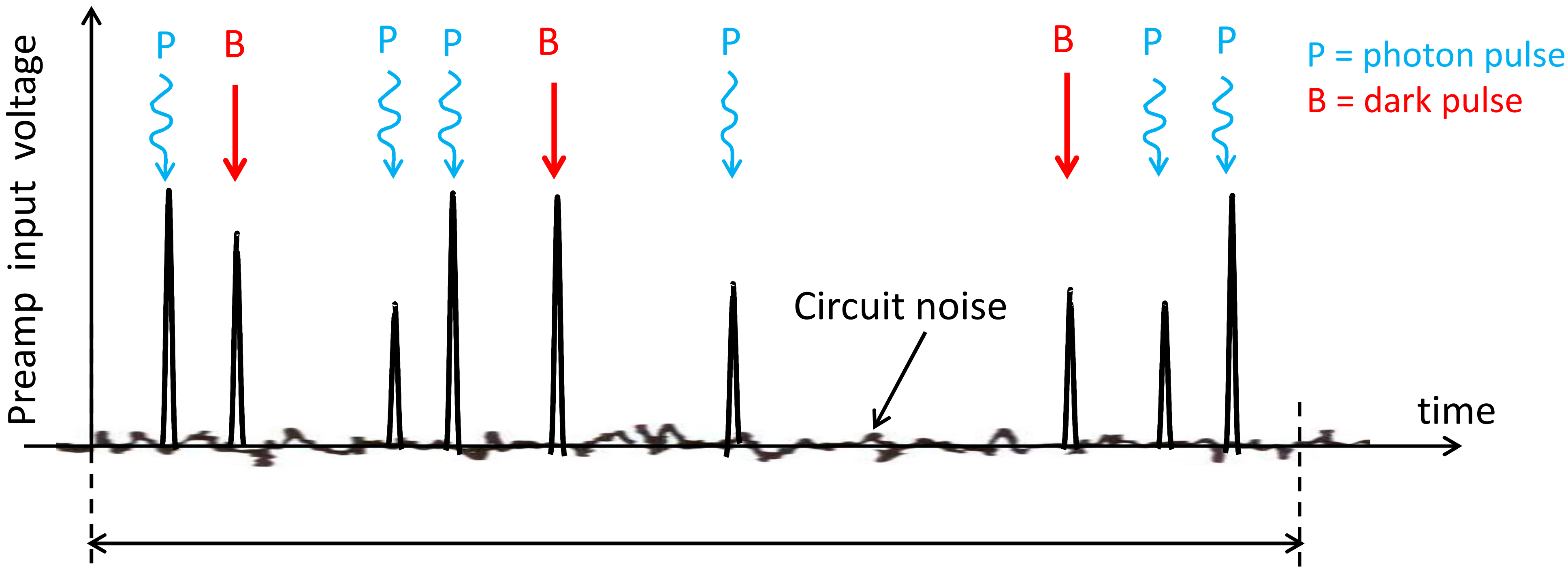
- Limits of PMTs as analog detectors for weak light intensity
- Pulse processing for single-photon counting
- Single Photon Counting with stationary light intensity
- Synchronous Single Photon Counting with modulated light intensity
- Limits of PMTs as analog detectors for measuring fast optical waveforms
- Pulse processing for single-photon timing
- Time-Correlated Single-Photon Counting (TCSPC)

Limits of PMTs as analog detectors for weak light intensity

PMT: excellent analog detector, but ...



By carefully observing signals and noise at the PMT output, we can recognize some drawbacks and understand how they can be avoided



Example: observed interval T = 10 μs, photon count rate $n_p \approx$ 600kHz

PMT: excellent analog detector, but...

PMTs are excellent analog detectors with amplification inside: the high and fast internal gain makes the noise of the following circuits ***almost*** negligible.
However, the performance with weak optical signals is limited by some drawbacks

In **all cases,** with fast and slow signals:

- the random gain fluctuations enhance the noise at cathode by the excess noise factor F, usually in the range F≈1,5-2

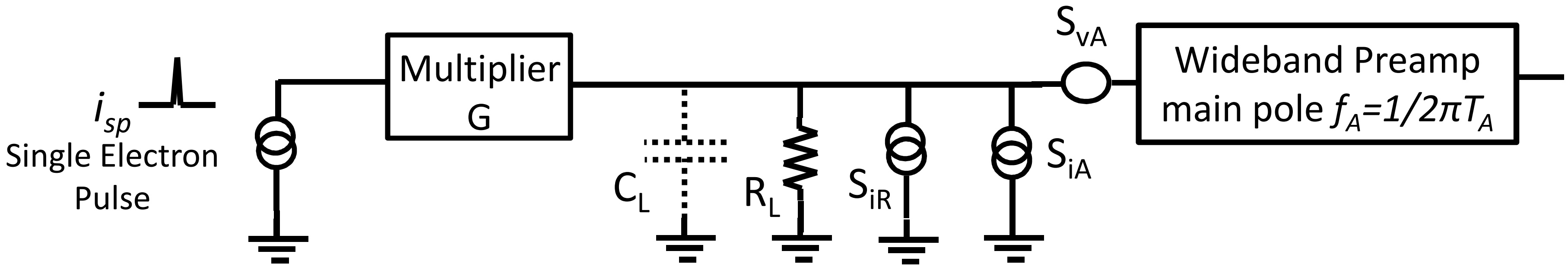
In cases with **stationary** ultraweak light (e.g. bioluminescence studies; fluorescent emission analysis; astrophysics; etc.):

- When very long integration time T_F is employed for attaining very high sensitivity (with very narrow-band noise filtering), the 1/f noise of the electronics is no more negligible and it may even set the ultimate sensitivity limit.
- In fact, to make T_F longer reduces the contribution of white noise , but not that of 1/f noise *. For limiting the 1/f noise, the low-pass filtering must be accompanied by a suitable high-pass filtering and making T_F longer reduces by the same factor both band-limits (lowpass and high-pass). The 1/f noise contribution thus is unaltered, since it depends only on the ratio (low-pass bandlimit)/(high-pass bandlimit)

* *Hint: see slide set HPF1 about 1/f noise filtering, in particular about CDF*

Pulse processing
for single-photon counting

Processing SER pulses for photon counting



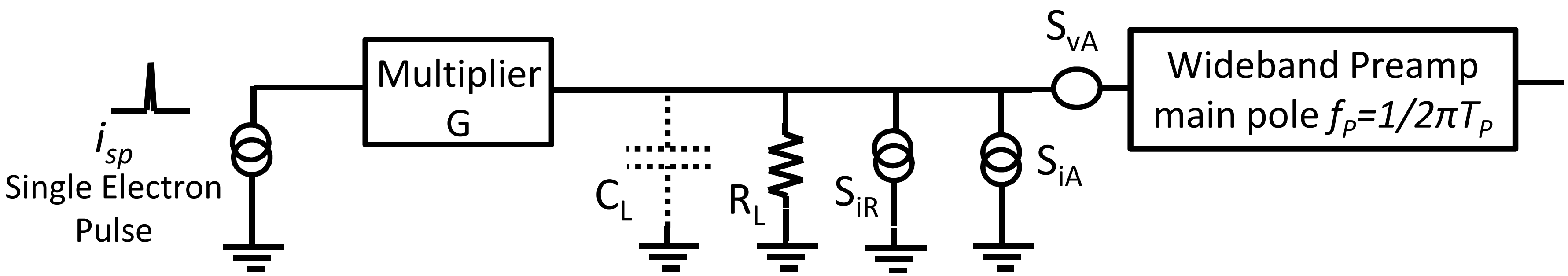
Let's consider a typical PMT example and verify that pulses due to single photons can be efficiently detected and processed with ordinary fast electronics:

- PMT with typical high gain and short SER pulse width: $G=10^6$ and $T_w \approx 4$ns
- load for wide-band operation, i.e. low resistance and capacitance: $R_L=50\ \Omega$, $C_L \approx 2$pF , hence short $T_L=R_LC_L=100$ps and resistor noise $\sqrt{S_{iR}} = \sqrt{4kT/R_L} \approx 18\, pA/\sqrt{Hz}$
- Wide-band preamplifier (T_A=0,5ns main pole time constant) with moderate noise $\sqrt{S_{vA}} \approx 4\, nV/\sqrt{Hz}$ and $\sqrt{S_{iA}} \approx 4\, pA/\sqrt{Hz}$

The SER waveform is practically unmodified by the filtering of load and preamplifier; the mean pulse peak amplitude (referred to the preamp input) is

$$V_p = I_p R_L \approx \frac{Q_p}{T_w} R_L = \frac{G\,q}{T_w} R_L \approx 1{,}6mV$$

Processing SER pulses for photon counting



The circuit noise components in voltage referred to the amplifier input are

$$\sqrt{S_{vA}} \approx 4\,nV/\sqrt{Hz}\ ; \qquad R_L\sqrt{S_{iA}} = 0{,}2\,nV\Big/\sqrt{Hz}\ ; \qquad \sqrt{S_{vR}} = \sqrt{4kT\,R_L} \approx 0{,}9\,nV\Big/\sqrt{Hz}$$

The circuit noise is dominated by S_{vA} and has rms noise

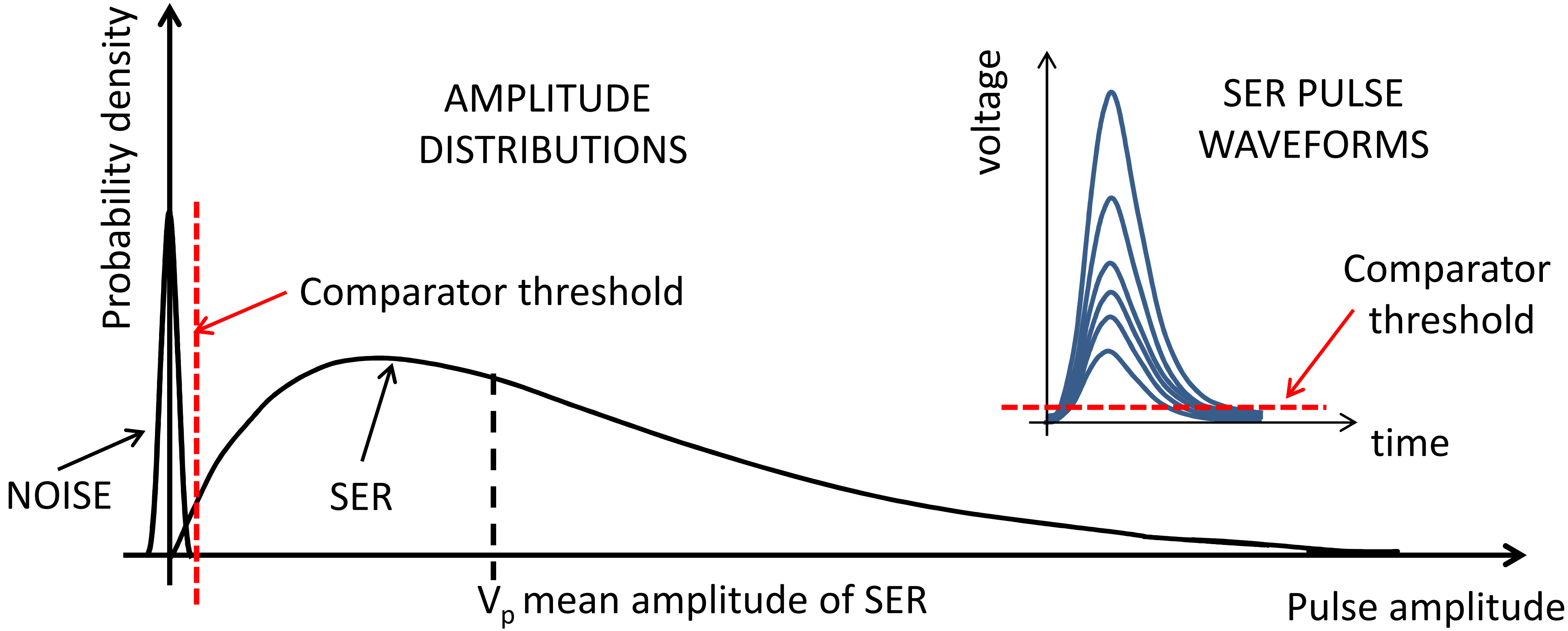
$$\sqrt{\overline{v_{nc}^2}} = \sqrt{\left(S_{vA} + S_{iA}R_L^2 + S_{vR}\right)\Big/4T_A} \approx \sqrt{S_{vA}/4T_A} \approx 90\,\mu V$$

Since we have a high ratio

$$\frac{\text{mean SER pulse amplitude}}{\text{rms noise of circuitry}}$$

the threshold of a simple fast comparator can be set at a level that efficiently rejects noise pulses and efficiently recognizes SER pulses.

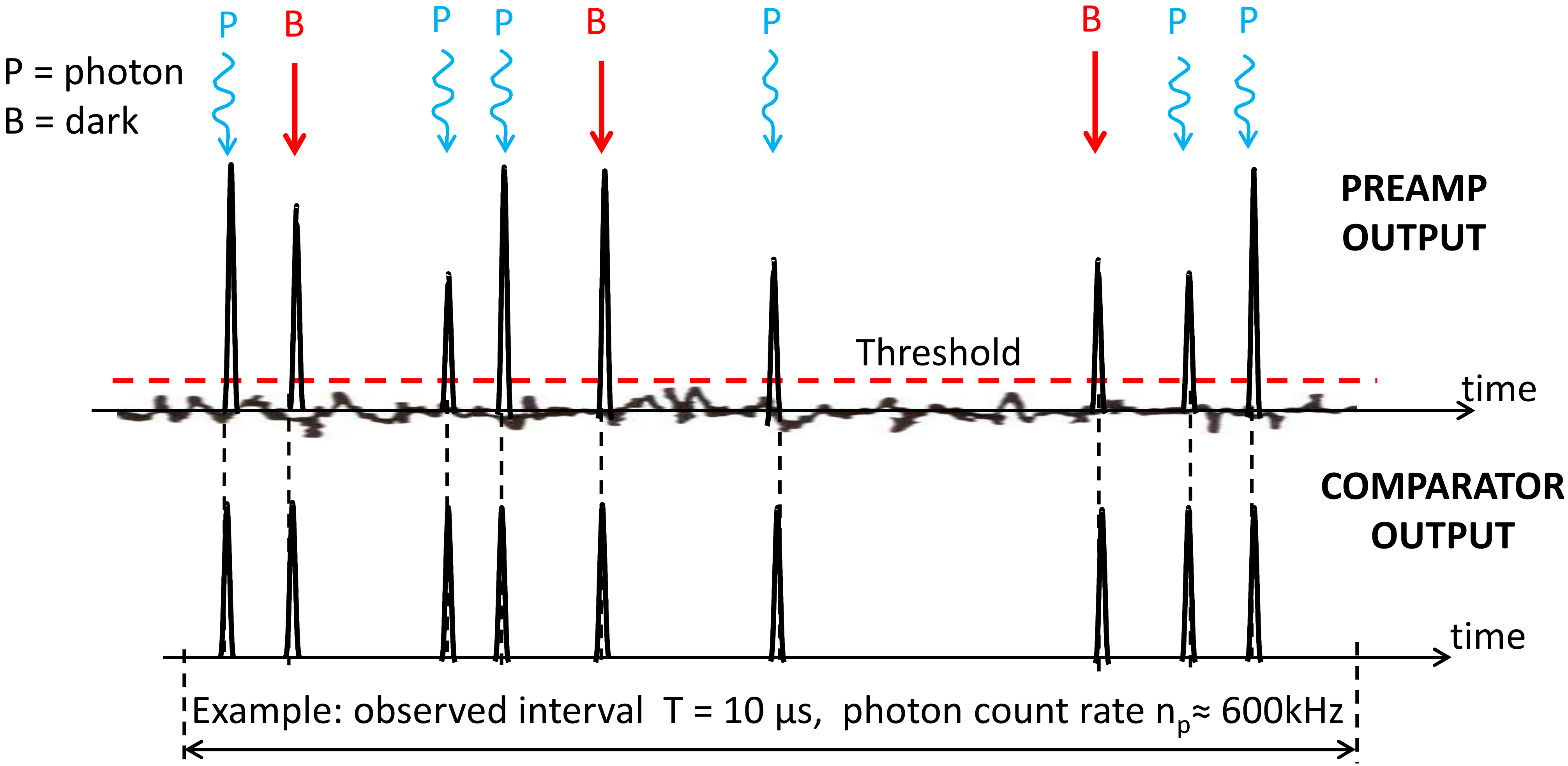
Processing SER pulses for photon counting



- With comparator threshold $V_T \approx 2{,}5\sqrt{\overline{v_{nc}^2}} \approx 220\mu V$ the probability of false triggering by noise is < 1% (gaussian noise distribution)
- Since the SER mean amplitude V_p is much higher than the threshold $V_p \approx 7\ V_T$, only a very small percentage of the SER pulses is discarded and the loss in photon detection efficiency is very small

Single Photon Counting with stationary light intensity

Single Photon Counting (SPC)



- The measurement employs **standard pulses (**comparator output)
- Each pulse carries just the information that a photon (or a dark pulse) is arrived: the PMT acts as a **digital detector**.
- The circuit noise does not produce any output, it is like non-existent.

SPC: Minimum Measurable Intensity

- Single Photon Counting (SPC) measures **stationary light intensities** by counting the output pulses of the comparator in known time intervals.
- In the study of analog processing of the PMT output, equations in terms of photon counts N_p and background counts N_B were obtained for Signal-to-Noise Ratio and Minimum Measurable Signal, with the approximation of negligible excess noise (factor F=1) and negligible circuit noise (*hint: see slide set PD4*).
  In SPC the PMT operates as a digital detector, therefore:
  - circuit noise does not matter, it is like non-existent
  - excess noise factor does not matter, standard pulses are processed
  - the equations for S/N and minimum signal in terms of photon counts N_p and Background counts N_B are directly derived in SPC from the Poisson statistics of counts, without any approximation.
- The equations show that with SPC the minimum measurable photon counting rate is extremely low because
  a) only the detector noise is relevant (dark counts and background photons)
  b) The measurement can be extended to **very long** integration time, since counting pulses in a time interval T_F is a **digital** integration over that interval

SPC: Dynamic Range

- SPC measurements have dynamic range extended to much lower light intensity than analog detection, but set lower upper limit to the light intensity. This may cause problems in cases where also high intensity signals have to be measured, for instance in laser ranging with reflections from very different targets
- The upper limit is due to the **count losses**, which increase with the repetition rate of pulses. Two SER pulses are not recognized as separate events if they are less spaced than a **finite time T_D** , which is a characteristic of the electronic processing. E.g. two pulses spaced by less than a pulse-width are recognized as one pulse by a comparator; after a counted pulse, a counter has a finite deadtime where other pulses are not counted; etc.
- Let's consider independent random pulses at true rate n_t , counted over time T by a counter with constant deadtime T_D . The mean number of recorded pulses $N_R=n_R T$ is the true mean number $N_t=n_t T$ in the time T less the true mean number $n_t N_R T_D$ in all the N_R dead time intervals T_D : $N_R = N_t - n_T N_R T_D$
  The recorded count rate will be thus
  $$n_R = \frac{n_t}{1 + n_t T_D}$$
  E.g., with T_D=10ns, for keeping count losses below 1% it is necessary to have $n_t T_D$<0,01, that is, to keep n_t < 1MHz

SPC for ultra-low light intensity

- For measuring the net signal photon rate n_p in presence of high background rate n_B , a **cycle of two counting runs over equal time T_F** is performed, one with signal photons reaching the detector (light on), the other with background only (light off);
  the counts with light off are then **subtracted** from the counts with light on.
- With very long T_F it is expected to achieve very low $n_{p,min}$ (even much lower than the background rate n_B) without being limited by the 1/f noise, which is irrelevant in SPC

$$n_{p,\min} = \sqrt{2n_B / T_F} \qquad \text{that is} \qquad n_{p,\min} \Big/ n_B = \sqrt{2 / n_B T_F}$$

- However, there is still a 1/f noise component which is relevant also in SPC, since it is not due to the analog circuitry, but to physical processes in the detector. Because of temperature fluctuations and slow alteration of the photocathode the dark counting rate of the PMT is affected by drift and slow fluctuations in time. Also the background photon rate may show similar fluctuations in many instances
- The background rate n_B (the pulse probability density in time) thus has
  a) a constant component n_{Bo}
  b) a fluctuating component $n_{Bf}(t)$ with zero mean value and slow variation in time, which has power spectrum with 1/f behavior
- With very long time T_F , the noise contribution of the fluctuating component $n_{Bf}(t)$ pushes the limit $n_{p,min}$ to a level higher than the above quoted value

Synchronous Single Photon Counting with modulated light intensity

SPC for ultra-low light intensity

In intuitive terms, whereas with **constant** background rate

- the mean values of background counts in the two runs T_F are equal and
- the difference of recorded counts is due only to the Poisson statistics of counts

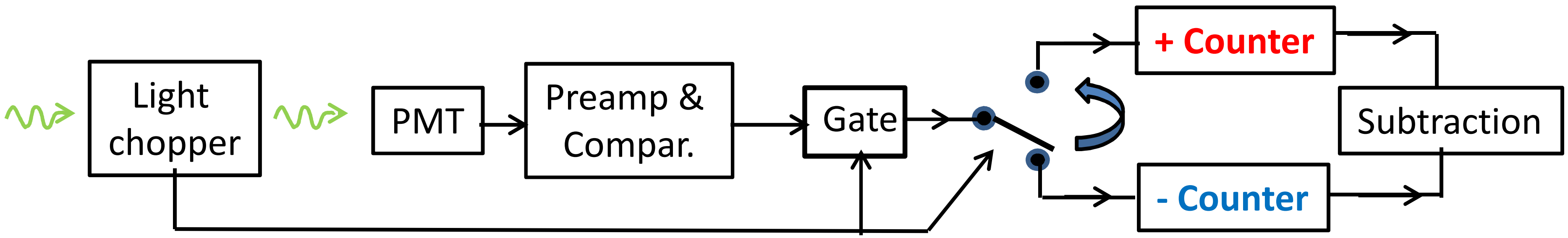
with background rate that **significantly varies over the time T_F**

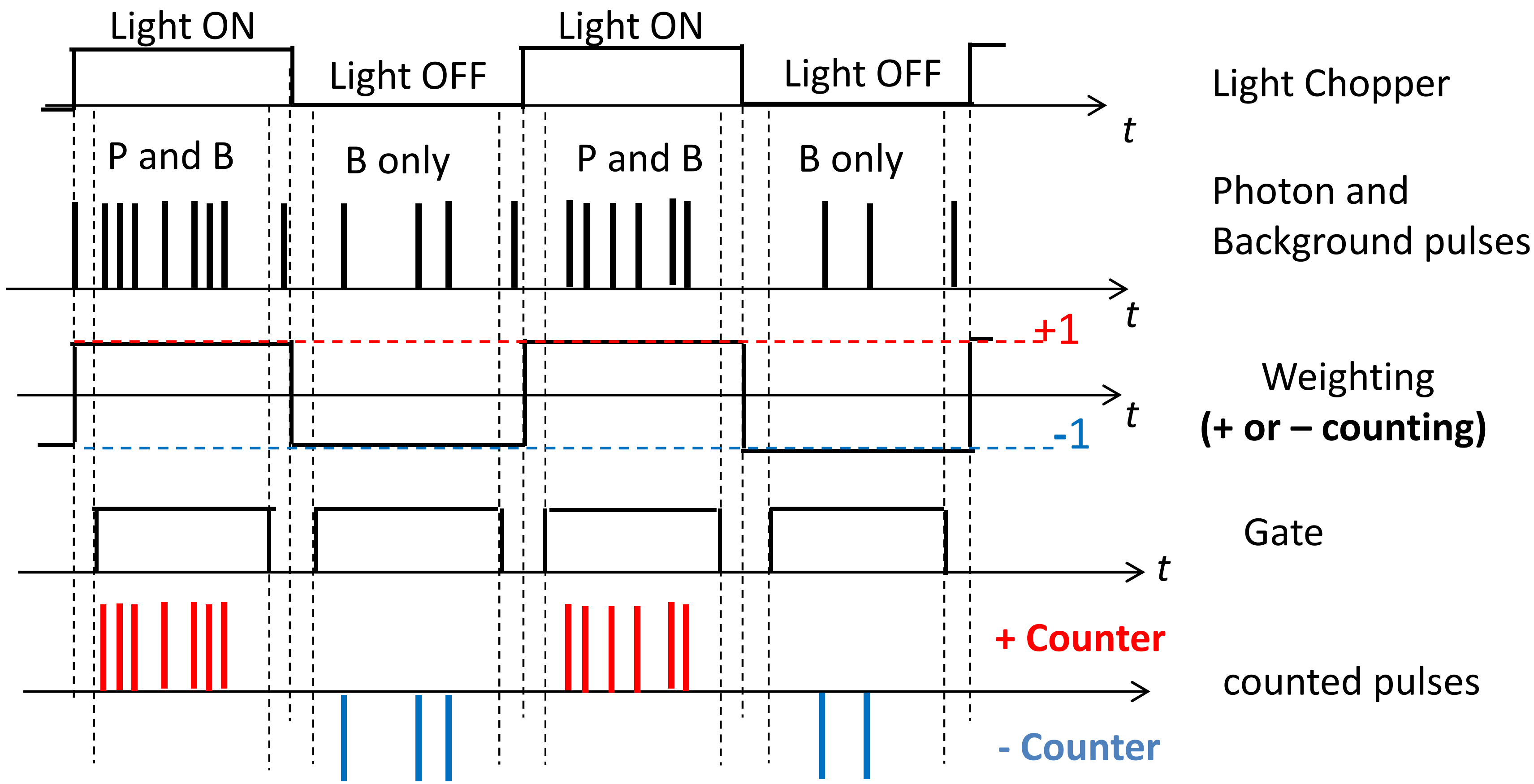
- the mean values of background counts in the two runs T_F are different and
- the difference of recorded counts is greater than the Poisson statistic dispersion, because of the contribution of the slow fluctuation of the background rate, which is at all effects a low-frequency component of the background noise

For reducing the effect of slow fluctuations, fast subtraction is necessary, i.e. short separation between the subtracted counts. However, this impairs the filtering of wideband noise (Poisson dispersion) since it forces also to short integration time T_F

For filtering efficiently both noise components, a modified SPC approach called **Synchronous Single-Photon Counting (SSPC)** can be employed. **Repeated cycles** of light-on counting and light-off counting with short T_F are performed and the recorded counts are **coherently summed** over many cycles, thus combining fast subtraction with long total integration time.

SSPC Synchronous Single Photon Counting





SSPC: a Digital Lock-in Amplifier for SPC

- SSPC is equivalent to a Lock-in Amplifier (LIA) for the signals and noise in digital form provided by the SP detector. The SSPC block diagram is clearly similar to that of an analog LIA that employs switches for the demodulation. Concepts seen and conclusions drawn in the treatment of LIA can be transposed to SSPC and lead to gain a better insight.
- The SSPC weighting function w(t) is a squarewave oscillation between level +1 (pulses counted in addition) and level -1 (pulses counted in subtraction). A symmetrical w(t) is necessary for rejecting the 1/f noise. In analog LIAs the level symmetry of squarewave is limited, but in SSPC the levels are perfectly equal
- The durations of the positive and negative weighting are synchronized with the light chopper, hence the two intervals are equal with moderate precision, with mechanical choppers usually a few percent. The consequent asymmetry of w(t) impaires the 1/f noise filtering by producing a spurious admission band around f=0
- In SSPC the counting intervals can be very precisely defined. A gate can be employed, open within each positive and negative weighting interval with gate time electronically controlled with high precision (0,001% and better). A very good symmetry is achieved for w(t) and consequently a very efficient filtering of the 1/f noise

Limits of PMTs as analog detectors for measuring fast optical waveforms

PMTs for fast optical waveform measurement

- The waveform of PMT current signals in response to fast optical signals is limited by the bandwidth of the PMT as analog detector. The observed waveform is the convolution of the true optical waveform (distribution in time of the photons) with the multiphoton δ-response (in practice the SER waveform). The optical waveform is thus smoothed by the SER width T_w , typically a few nanoseconds.
- The information about the arrival time of a single photon is available from the PMT with precision much better than T_w , since the SER pulse centroid has a statistical dispersion with width T_j (time jitter) 5 to 10 times smaller than T_w
- If the photon distribution in time were obtained by collecting the arrival time of every photon seen by the PMT, a better result would be obtained. The observed photon distribution in time would be the convolution of the true distribution of photons with the statistical distribution of the SER centroid. The waveform would thus be smoothed by the SER time jitter, typically a few 100 ps.
- For achieving in reality such a result, two basic issues must be faced :

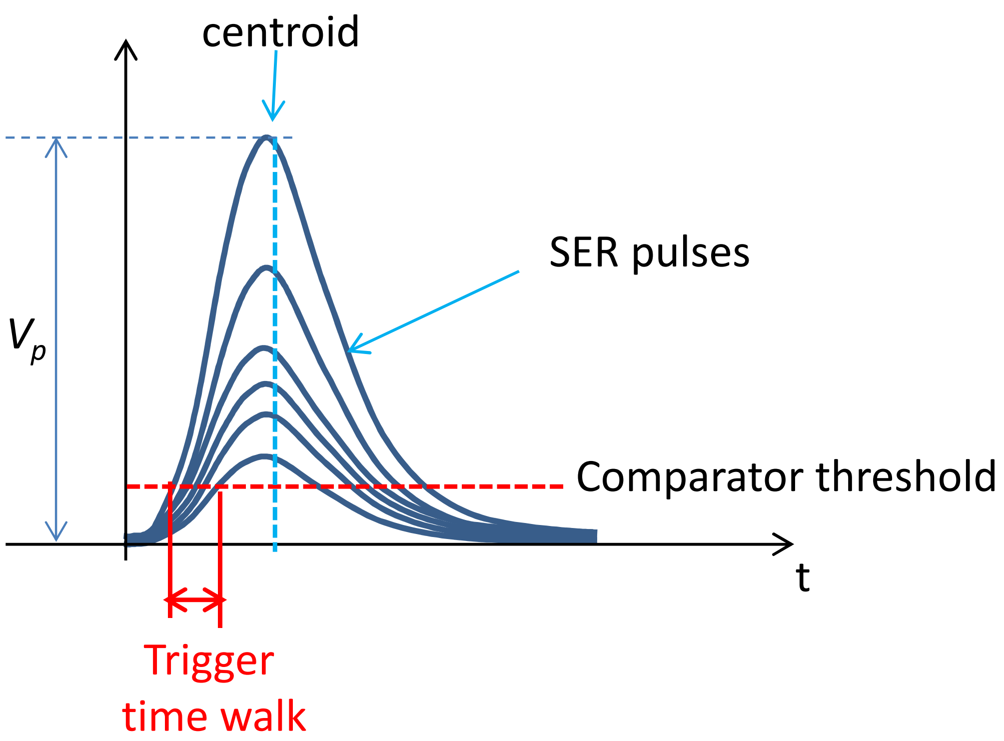
  a) how to extract correctly the information about the arrival time of SER pulses ?

  b) how to collect in reality the distribution of arrival times of photons of a fast optical signal?

Pulse processing for single-photon timing

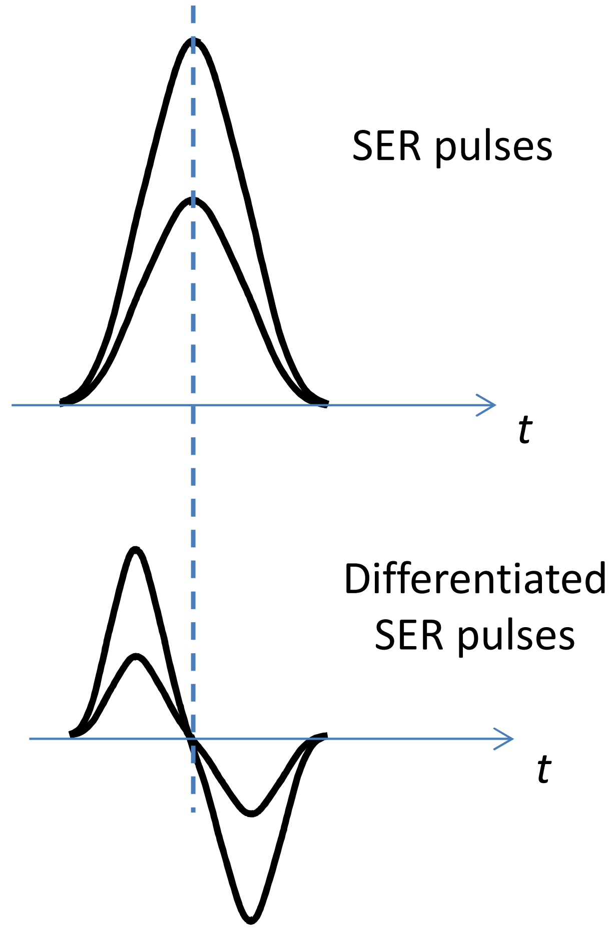
Processing SER pulses for photon timing

- A comparator cannot give a correct information of the time position of the SER pulses (i.e. of single photon arrival time) because the pulse amplitude V_p is random.
- The crossing of the threshold is delayed with respect to the onset of the pulse and the delay systematically depends on the pulse amplitude V_p . As V_p decreases the delay increases: the triggering suffers a «time-walk»
- Since V_p is widely fluctuating, the time walk causes strong fluctuations of the observed pulse arrival time, much greater than the jitter Tj of the pulse centroid

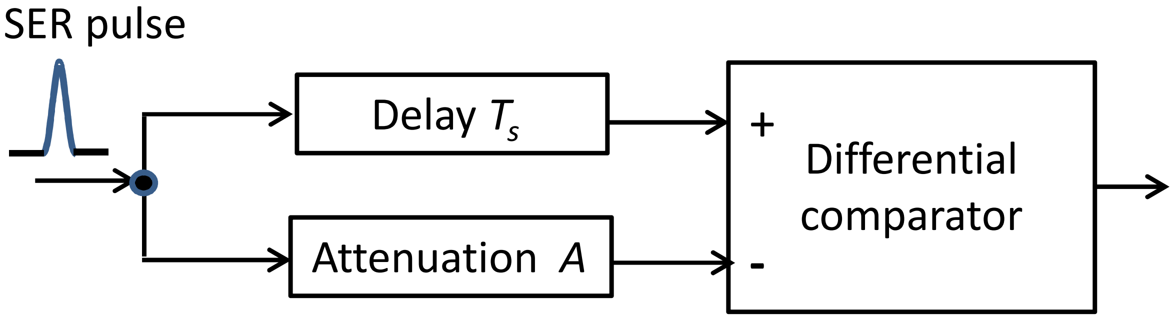


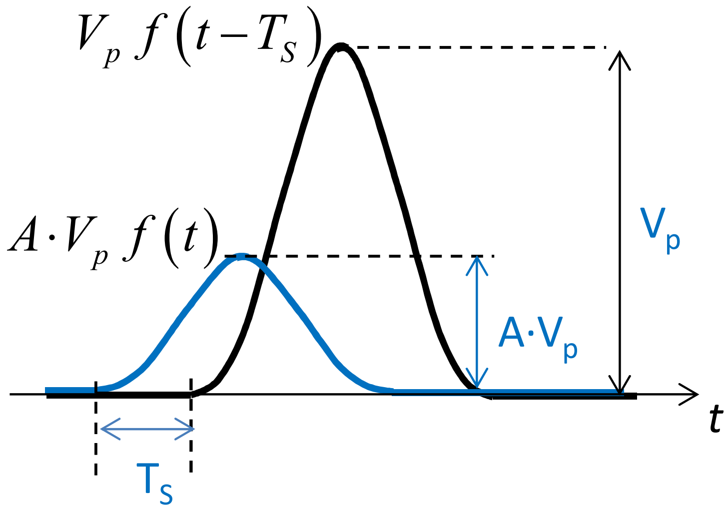
Processing SER pulses for photon timing

- For extracting correctly the photon arrival time, it's necessary a pulse-timing circuit arrangement with triggering time independent from pulse amplitude
- A known solution employs a **differentiator linear filter** followed by a **zero-crossing-trigger** circuit.
- The constant-parameter linear filter reshapes the SER pulse to a zero-crossing waveform, with zero-crossing time independent from pulse amplitude (it approximates the SER pulse derivative, the zero-crossing is at the SER peaking time)
- The trigger circuit receives the differentiated SER pulse and is designed to trigger at the zero-crossing of the waveform



Processing SER pulses for photon timing





- An alternative solution (similar to zero-crossing but better implementable with current fast circuits) is more frequently employed: the «Constant Fraction Trigger».
- The SER pulses is routed in two circuit paths leading to the two inputs + and - of a differential comparator. The + path gives a short delay T_s , comparable to the pulse width T_w ; the - path attenuates the pulse by a factor A (usually from 1/3 to ½).
- The triggering time occurs when the delayed pulse overrides the attenuated pulse. It's does not depend on V_p , as evident and confirmed by the equation that defines it, which does NOT contain V_p . In fact, denoting by *f(t)* the SER waveform normalized to unit peak amplitude, the triggering time is occurs when

$$f(t-T_S) = A\, f(t)$$

Time-Correlated Single-Photon Counting (TCSPC)

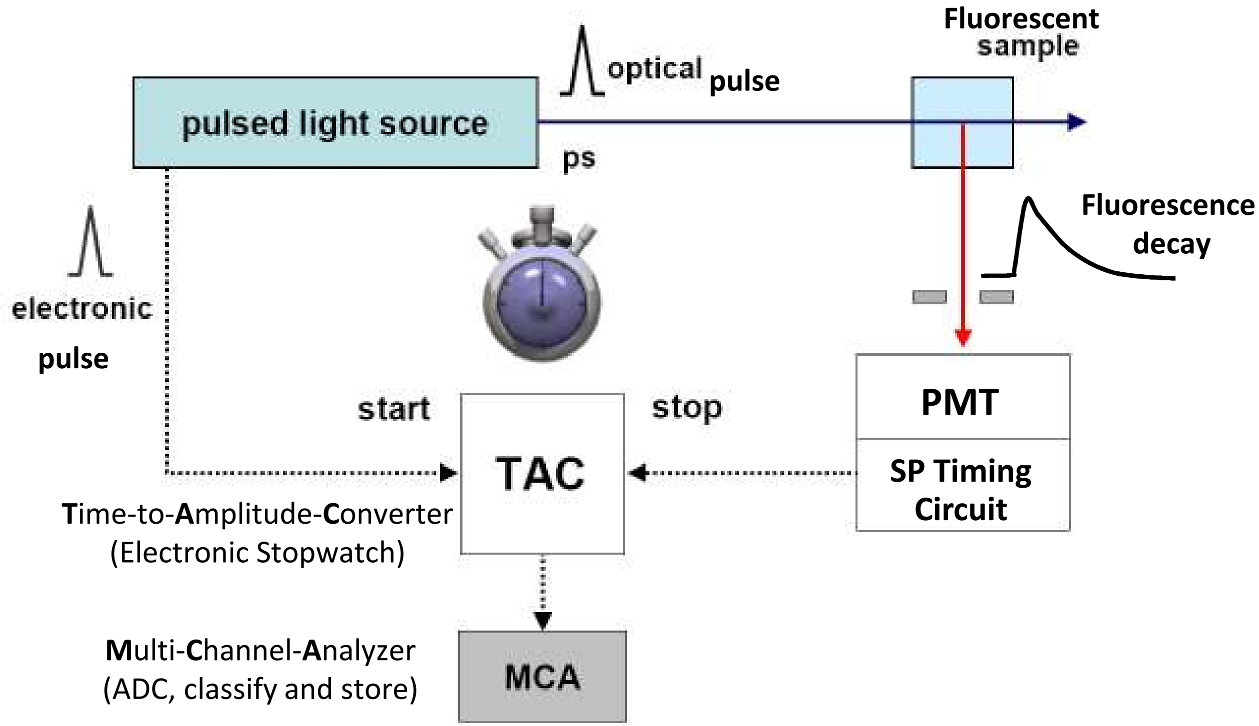
Single Photon Timing Applications

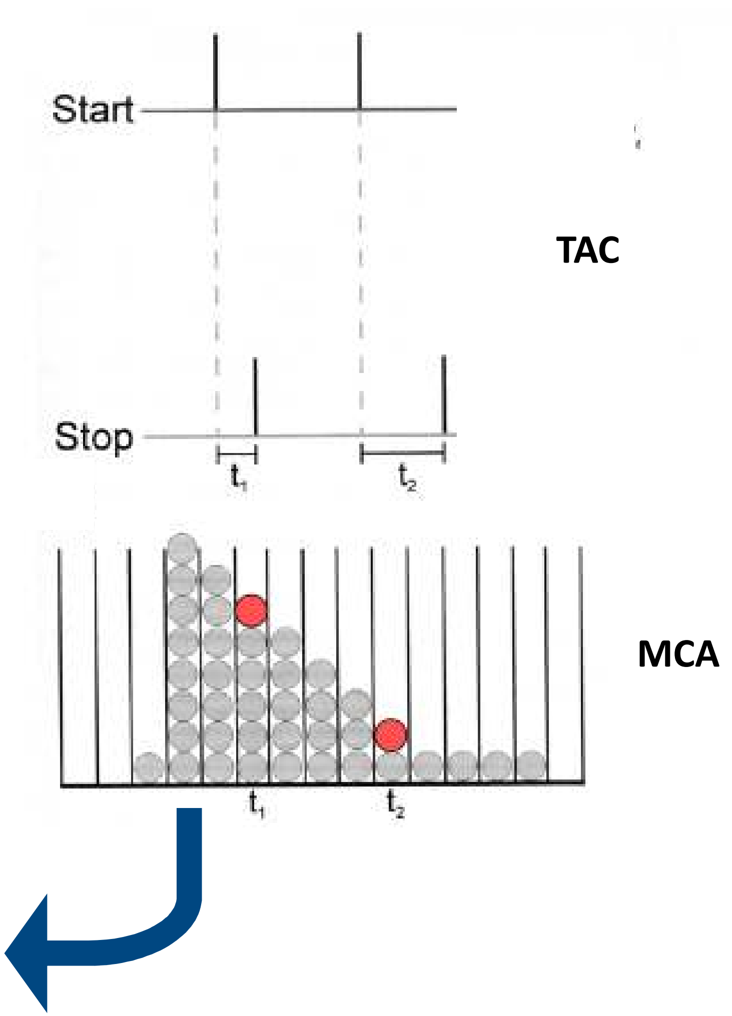
- SP timing is exploited in **Laser Ranging (LR)**, an optical technique similar to RADAR: a short laser pulse is directed onto a target and the measured delay of the reflected pulse gives a measurement of the target distance.
- SP timing brings various advantages to LR: extension to longer distances and to targets with poor reflection; high precision in time measurement, enhanced by averaging over many repetitions; capability of working at lower laser power, safe for operation in free space; etc.
- **Satellite Laser Ranging (SLR)** with SP timing is currently exploited at high performance level: satellite position is measured with precision better than 1cm
- SP timing is exploited also in measurements of optical waveforms and fluorescence lifetimes in various fields, from chemical analysis to biomedical diagnostics (Fluorescence Lifetime Imaging FLIM, et al.) to studies on single molecules of biomedical interest.
- The SP technique for measuring fast optical waveform is called **Time Correlated Single Photon Counting (TCSPC)**. It can be better clarified by considering first studies on single molecules for measuring their fluorescence lifetimes and then measurements of optical waveforms in general .

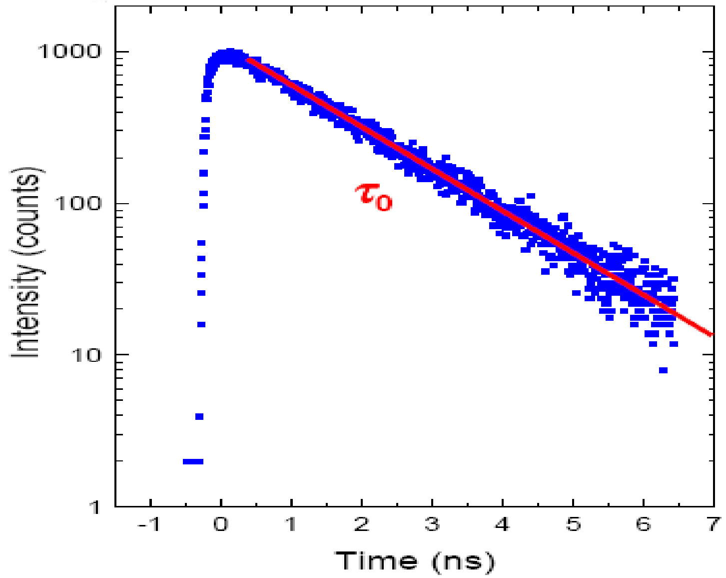
Single Molecule Fluorescent Decay Studies

- Molecules of biological interest (proteins, etc.) have dynamically varying configuration, hence properties that vary in time and from molecule to molecule. Experiments on single molecules are necessary for studying such properties: even very small samples of the substance contain huge numbers of molecules and give values averaged over the molecule population.
- With suitable biochemical techniques a single fluorescent molecule can be maintained in a stable position on a non-fluorescent substrate. A fast laser pulse can thus be repeatedly pointed to the molecule for exciting the fluorescence and a suitable optics can collect the emission from the molecule onto a PMT
- The excited molecule gets back to quiescent state by emitting a fluorescence photon; the emission is random, with emission probability density in time ***p(t)*** that decays exponentially with the characteristic lifetime of the level.
- The electronic instrumentation has to measure and classify the time interval from level excitation (laser pulse) to level decay (fluorescent photon detection). Besides a pulsed laser (with synchronous electric pulse) and a PMT (with SP timing electronics), it includes an electronic stopwatch. There is no ambiguity, inherently only one photon per excitation is emitted.
- We have just to observe a high number of excitation-decay cycles and collect the histogram of number of decay events versus measured delay

Time Correlated Single Photon Counting







Hystogram of many trials

≡

fluorescence decay curve

Time Correlated Single Photon Counting

- Let us now consider the fluorescence of an ordinary small sample, which contains many fluorescent molecules so that the laser pulse excites more than one molecule
- If the laser intensity and/or the efficiency of photon collection at the detector is low, the situation looks similar to the case of a single molecule: single photon pulses are observed and there is a small probability of detecting a photon after an excitation. One might deem that a TCSPC set-up can be employed without any new problem
- In fact the case is different. After excitation, more than one photon is emitted and only the first one arriving to the detector stops the electronic stopwatch, the others do not contribute any measurement. The issue is that the photon selection is not random, but systematic: first arrived is best served, others are lost.
- The TCSPC histogram will thus reflect not the probability density $p(t)$ of detecting **A photon at time *t*** , but the probability density $p_1(t)$ of detecting **at time *t* the FIRST photon** among a few. However, if the probability of detecting a photon in the time range observed is very small, the difference beween $p_1(t)$ and $p(t)$ is practically negligible, as it will be shown quantitatively
- The conclusions that are drawn for the measurement of a fluorescent decay waveform can be readily extended to the measurement of any optical waveform

Time Correlated Single Photon Counting

- The detected photons are independent events ruled by Poisson statistics.
  The probability density of having **at time *t* the FIRST detected photon** is

$$p_1(t) = P_0(t)\, p(t)$$

  with $P_0(t)$ probability of zero detected photons from t=0 to t.

- Denoting by m(t) the mean number of detected photons from t=0 to t per excitation

$$m(t) = \int_0^t p(\alpha)\, d\alpha$$

  and with Poisson statistics $P_0(t) = e^{-m(t)} = e^{-\int_0^t p(\alpha)\, d\alpha}$

- Therefore $p_1(t) = p(t)\, e^{-m(t)} = p(t)\, e^{-\int_0^t p(\alpha)\, d\alpha}$

  and with small m(t) << 1 $p_1(t) \approx p(t)\left[1 - m(t)\right] = p(t)\left[1 - \int_0^t p(\alpha)\, d\alpha\right]$

  By keeping m(t) ≤ 0,01 the difference between $p_1(t)$ and p(t) is kept below 1%, so that the p(t) derived from the histogram of the experiment can be considered correct.

- Furthermore, a correction equation for computing the correct p(t) from the data representing $p_1(t)$ can be obtained on the basis of Poisson statistics. It is thus possible to carry out TCSPC also with somewhat higher m(t), say m(t) from 0,1 to 0,2.