Sensors, Signals and Noise

COURSE OUTLINE

- Introduction
- Signals and Noise
- Filtering
- Sensors: PD7 - PhotoConductors

PhotoConductors

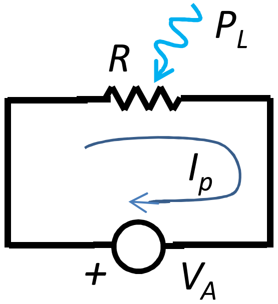
- Typical Characteristics and Gain Principle
- Response to a Single Photon and response to Multi-photon δ-Pulse
- Frequency Response
- Noise in Photoconductors
- APPENDIX 1: Generation-Recombination Noise

Typical Characteristics and Gain Principle

Simple Device and Simple Operation

- Very simple device structure:
  resistor slab in semiconductor material with ohmic contacts at the two ends
- Semiconductor material with conductivity modulated by illumination
  - in dark: negligible conductance *G≈0,*
  - in light: conductance *G* proportional to photon flux P_L
- With constant applied voltage V_A , current proportional to photon flux P_L

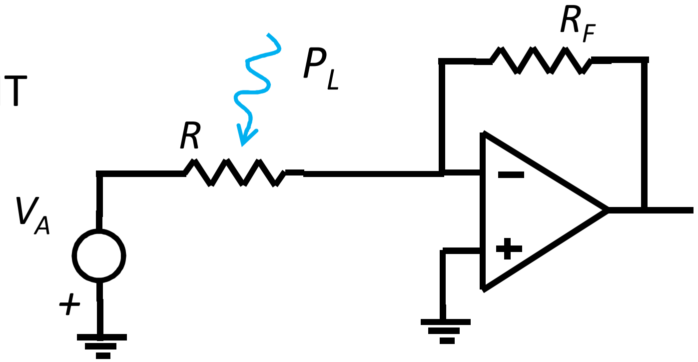
PRINCIPLE



$$G = 1/R \propto P_L$$

$$I_p = G \cdot V_A \propto P_L$$

PRACTICAL CIRCUIT



$$V_O = G \cdot R_F V_A \propto P_L$$

Characteristic Aspects of PhotoConductors

- Simple fabrication technology: just ohmic contacts must be produced, not p-n junctions or other device structures.
- Besides semiconductors currently employed in microelectronics (Si, Ge, GaAs), other materials can be employed: CdS, PbS, CdSe, InSb, HgCdTe et al.
  also organic polymers developed on purpose (polyvinylcarbazole etc.)
- Materials with very low energy gap can be employed for extending the detector operation well into the infrared (IR) spectral region; for instance, HgCdTe detectors work up to $\lambda \approx 10 \mu m$
- Remarkable internal **detector gain** can be obtained in photoconductors, although limited to **low frequencies**. As we will see, it is obtained by exploiting materials where one of the photogenerated free carrier pair (typically the electron) has long lifetime τ_r and the other (the hole) is captured and held by local trap levels.
- The lifetime of free electrons is a technology-dependent property, i.e. it strongly depends also on the fabrication process, not only on the material. In practice, τ_r values of the order of milliseconds and longer are obtained and remarkable gain values can thus be achieved, $M \approx 10^4$ and higher.

Characteristic Aspects of PhotoConductors

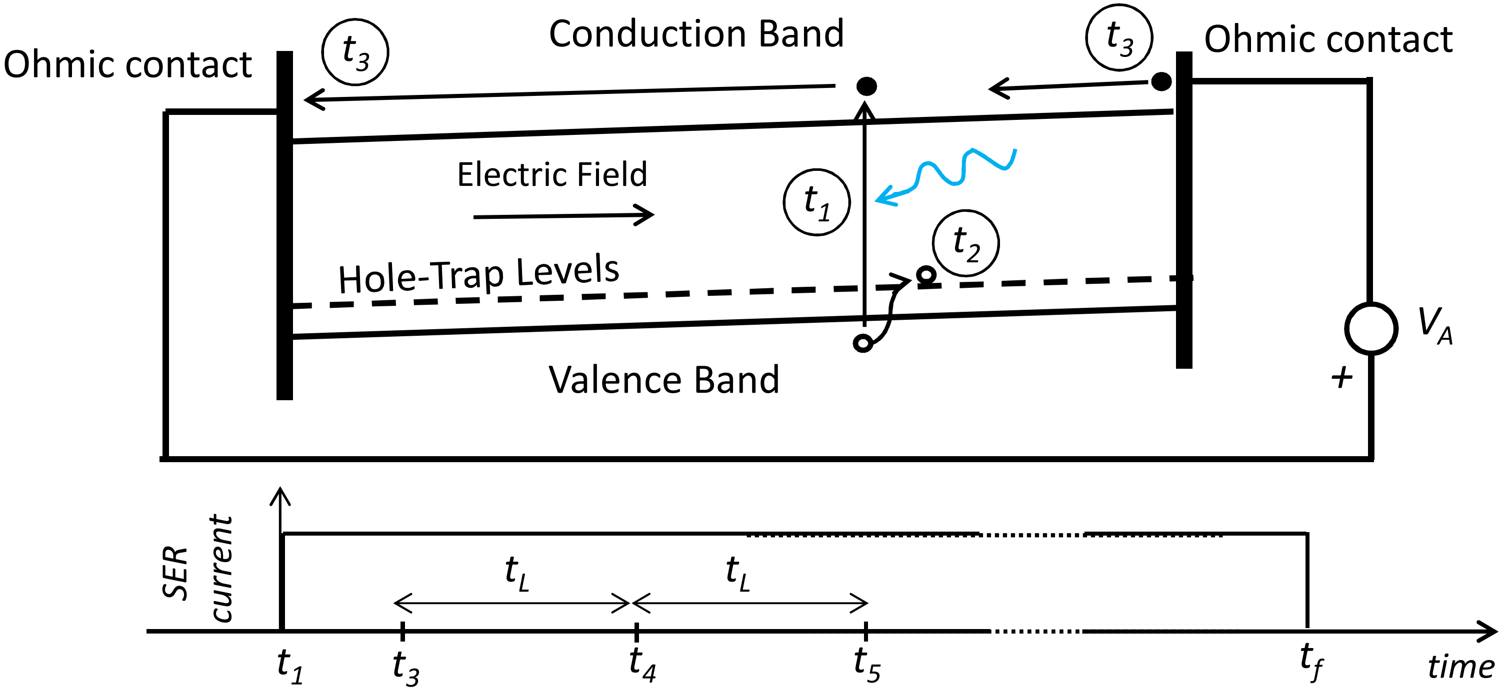
- The photoconductor is a resistor, but with so low conductivity that the current due to a single electron in motion turns out to be similar to that in a reverse-biased pn junction, rather than in a resistance.
- In a conductor, a charge in motion has induction effect on the space charge distribution of free carriers, which act as an electrostatic screen: electrons swiftly change their distribution in order to cancel the field of the moving charge.
- Basic electromagnetic theory (Poisson equation and current continuity equation) shows that the time taken by the space charge to change its distribution in response to a perturbation is ruled by the **dielectric relaxation time constant** $\boldsymbol{\tau_S = \varepsilon/\sigma}$ (ε dielectric constant and σ conductivity of the material). In good conductors $\boldsymbol{\tau_S}$ is very short, picoseconds or less.
- For photoconductors in dark and in weak light the conductivity is very low and the relaxation time constant τ_S is very long, typically seconds, much longer than the transit time of a carrier in the photoconductor. Therefore, the free carriers do not have the time to move for screening a moving charge, which directly induces charge in the metal contacts, just like in a p-i-n photodiode

Principle of Gain in PhotoConductors

- With constant voltage V_A is applied to the photoconductor, if a free electron-hole pair is generated by a photon:
  the hole is swiftly captured by a trap level
  the electron drifts towards the positive contact
- As the electron is collected at the positive contact:
  a further electron is injected from the negative contact to maintain
  zero total charge in the material (contacts «transparent» to electrons)
- As the further injected electron is collected:
  the negative contact re-injects an electron, which drifts towards the positive
  contact; etc...
- The electron **current goes on until the free electron recombines** with a hole. We can consider that the electron circulates many times in the circuit, so that a **total charge of many electrons flows** through the contacts.
- This process is particularly effective when holes are held in trap levels, but it results (with lower efficiency) also if holes drift much slower than electrons: the electron is collected before the hole, a further electron is injected by the other contact, etc.

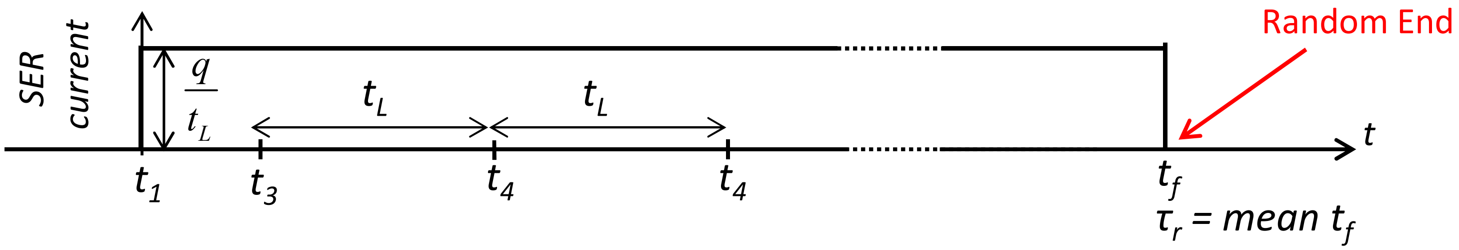
Response to Single Photons and to Multi-photon δ-Pulses

Single Electron Response SER



- t_1 : free electron-hole pair generated by a photon
- t_2 : hole quickly captured while electron drifts and current flows
- t_3 : electron collected at the positive ohmic contact; an electron is injected from the negative contact and drifts; the current goes on
- t_3+t_L : after the **transit time t_L over the conductor length L** the injected electron is collected; an electron is re-injected and the current goes on; etc....
- at random t_f: the drifting electron recombines with a hole, current terminates

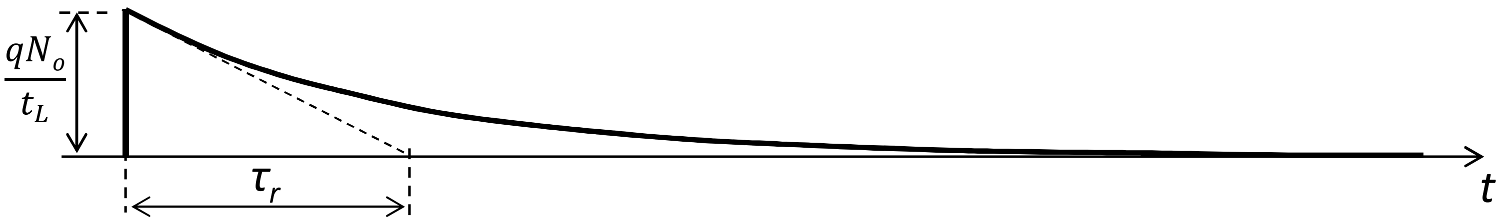
SER as Single-Photon Response



- Electric field *E* is constant over the conductor length *L* , with *E*<2kV/cm in ohmic regime
- Drift velocity is low and **transit time τ_L** over L is fairly long:

  mobility is around $\mu_n \approx 1000$ cm^2/ Vs , hence drift velocity $v_n = \mu_n E < 2 \cdot 10^6$ cm/s
  and unit transit time is $1/v_n >$ *0,5 μs/cm=50ps/μm*
- In each interval t_L a charge *q* is collected, hence the current is $i_q = q/t_L$ (S-R theorem)
- **SER duration and charge are random** , ruled by the electron recombination probability
- With duration much longer than transit time $t_f >> t_L$, the SER pulse charge is much higher than *q*. The **mean value τ_r of the SER duration is the mean lifetime of a free electron;** the **mean SER pulse charge is $Q_q = q \cdot \tau_r / t_L$**
- Note that the photonductor does not amplify the single electron current i_q , but amplifies the charge that the electron carries over a long time τ_r . It is thus able to **amplify slow optical signals** that vary over time intervals **longer than τ_r**

Response to a Multi-Photon δ-Pulse



- For a multi-photon δ-like pulse that generates N_o free electrons (at t=0), the response is superposition of N_o SER pulses. Note that they terminate at random times, when the electron recombines.
- Recombination probability in every dt is constant $p_r dt = dt/\tau_r$. Recombinations are independent; free electron population has Poisson statistics.
- The mean number N of free electrons surviving at time t thus is

$$N(t) = N_o \exp(-t/\tau_r)$$

- The current has statistical fluctuations and a **mean waveform**

$$i_\delta(t) = i_q \cdot N(t) = \frac{q}{t_L} N_o \exp(-t/\tau_r)$$

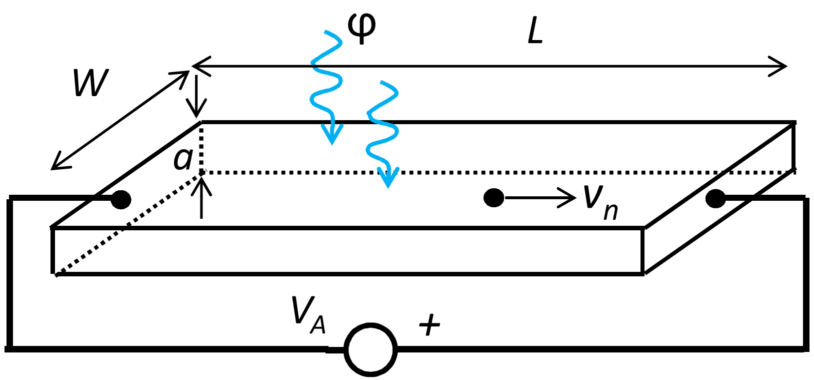
- The mean total pulse charge is

$$Q_\delta = \int_0^\infty i_\delta(t)\,dt = qN_o \frac{\tau_r}{t_L}$$

it is the photogenerated charge **amplified by** $\boldsymbol{\tau_r / t_L}$

Frequency Response

Frequency Response



μ_n electron mobility

$v_n = \mu_n E$ electron drift velocity

$j = nqv_n$ current density

$j = \sigma_n E$ local Ohm law

$\sigma_n = qn\mu_n$ conductivity

Holes are held in trap levels and do not contribute to the current

With uniform material and uniform illumination, the conductance G is proportional to the total number of free electrons $N = n \cdot WaL$

$$G = \sigma \frac{S}{L} = \sigma \frac{Wa}{L} = q\mu_n n \frac{WaL}{L^2} = \frac{q\mu_n}{L^2} N \propto N$$

For a given V_A , the current I thus is proportional to N: $I = GV_A = N \frac{q\mu_n}{L^2} V_A \propto N$

By noting that the transit time is $t_L = L/v_n = L/\mu_n E = L^2/\mu_n V_A$, we see that the current is given simply by the **total mobile charge** divided by the **transit time t_L**

$$\boxed{I = \frac{Nq}{t_L}}$$

Frequency Response

The photocurrent is proportional to the mobile electron population *N(t)*: the Dynamic Response thus is ruled by the response of mobile electron population *N(t)* to the time-varying optical flux φ(t) (photons/cm^2). Denoting

- *η* detection efficiency, i.e. probability for a photon to generate a free hole-electron pair
- $p_r\, dt = dt/\tau_r$ recombination probabilty in *dt* for a mobile electron, with **τ_r lifetime (mean life) of the free electrons**

the *N* balance equation is

$$\frac{dN}{dt} = \varphi\, W\, L\, \eta - N \cdot p_r = \varphi A \eta - N \cdot \frac{1}{\tau_r}$$

In Laplace transform we get $sN = \Phi A \eta - N / \tau_r$

$$N(s) = \Phi \cdot A \eta \tau_r \frac{1}{1 + s\tau_r}$$

and in Fourier transform

$$\boxed{N(\omega) = \Phi \cdot A \eta \tau_r \frac{1}{1 + j\omega\tau_r}}$$

Dynamic Response to Optical Signals

The photoconductor current thus is

$$I = \frac{Nq}{t_L} = \Phi A\eta \cdot q \cdot \frac{\tau_r}{t_L} \cdot \frac{1}{1 + j\omega\tau_r}$$

By noting that

$\Phi a\eta = g_p$ is the photogeneration rate (carriers generated/second)

$g_p q = i_p$ is the primary photocurrent

$\tau_r / t_L = M$ is the mean Charge Gain (amplification of the photoconductor)

we get

$$\boxed{I = i_p \frac{M}{1 + j\omega\tau_r}}$$

The photoconductor output current is the primary photogenerated current i_p amplified by M and low-pass filtered by a single pole filter with time constant τ_r . In fact, the photoconductor transfer function from primary generated current to output current is

$$\boxed{H(\omega) = \frac{I}{i_p} = \frac{M}{1 + j\omega\tau_r}}$$

Dynamic Response to Optical Signals

$$\boxed{H(\omega) = \frac{I}{i_p} = \frac{M}{1 + j\omega\tau_r}}$$

- The 3dB **bandwidth *f*$_p$** depends only on the mobile electron **lifetime τ_r** , i.e. on material properties and technology, but **NOT on device size and bias V_A**

$$f_p = \frac{1}{2\pi} \cdot \frac{1}{\tau_r}$$

- The **gain-bandwidth** product depends on the **transit time *t*$_L$, not on lifetime τ_r;** it thus depends on device design and size and on the bias V_A

$$M \cdot f_p = \frac{1}{2\pi} \cdot \frac{1}{t_L} = \frac{1}{2\pi} \cdot \frac{\mu V_A}{L^2}$$

- For **slow** signals, detectors with long lifetime τ_r are suitable. High gain can be obtained also with fairly long t_L , i.e. with **wide sensitive area** (long *L*) and low bias voltage V_A
- For **fast** signals, short τ_r are necessary; for obtaining gain it is necessary to have even shorter t_L , i.e. **small sensitive area** (short *L*) and high bias voltage V_A

Noise in Photoconductors

Noise in Photoconductors

Johnson Noise

- As in any resistor, it consists in fluctuations of current (and voltage) due to the random thermal motion of the free carrier population. The spectral density is white and independent from the applied voltage

$$S_J = 4kTG \text{ (unilateral)}$$

- In most cases S_J is a minor component (G is very low) with respect to the preamplifier circuit noise and other internal noise components of the detector

1/f Noise

- As all resistors in semiconductor materials, photoconductors have 1/f noise components that strongly depend on the material and on fabrication technology

Generation-Recombination Noise (G-R noise)

- Fluctuations in the current of the free carriers that drift in the electric field. Similar to shot noise, but with a basic difference: not only the starting times of pulses due to single generated electrons are random, but also the pulse duration is random , ruled by the electron recombination.

G-R Noise

- The G-R noise has two components:
  - a) from photogeneration with primary current i_p and output current $I_p = Mi_p$
    This component is analogous to the **photocurrent noise** in PMTs
  - b) from thermal generation with primary current i_d and output current $I_d = Mi_d$
    This component is analogous to the **dark current noise** in PMTs
- For drift currents and the associated G-R noise, the situation is similar to a reverse-biased pn junction rather than to a high resistance. The spectral densities (unilateral) of photocurrent and dark-current GR-noise are (see Appendix 1)

$$S_{GR,p}(\omega) = 4QI_p \frac{1}{1+\omega^2\tau_r^2} \quad \text{and} \quad S_{GR,d}(\omega) = 4QI_d \frac{1}{1+\omega^2\tau_r^2}$$

- Setting in evidence the mean gain M and the primary currents i_p and i_d we get

$$S_{GR,p} = 2\cdot 2qi_p \frac{M^2}{1+\omega^2\tau_r^2} \quad \text{and} \quad S_{GR,d} = 2\cdot 2qi_d \frac{M^2}{1+\omega^2\tau_r^2}$$

which point out that G-R noise is given by:

a) the shot noise of the primary current, but

b) amplified and filtered by the photoconductor transfer function and

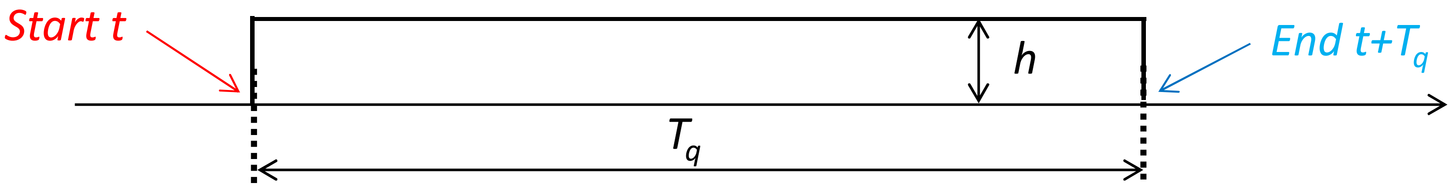
c) increased by the excess noise factor F=2 due to the random gain

Photoconductor Noise and Minimum Signal

- For PMTs we have seen that the detector internal gain reduces the weight of the circuit noise with respect to signal and to dark-current noise, thus making possible to attain much higher S/N and much lower minimum signal
- For photoconductors the internal gain brings less significant advantages, because of the remarkable quantitative differences from PMTs in gain bandwidth and value
- First, the gain is high only in the low frequency range, below the detector pole frequency. In most cases this means below 1kHz, a range plagued by 1/f noise (from circuits and from the photoconductor) that should be avoided if possible.
- Second, although the M values obtained are in various cases remarkable ($M \approx 10^4$ and higher), they are anyway much lower than a typical PMT gain. Furthermore, the gain-bandwidth product is limited to the inverse of the transit time in the device structure, hence getting higher gain implies accepting lower bandwidth
- In conclusion, the internal gain of photoconductors is useful in the detection of slow optical signals (with spectrum limited to low frequencies), where it remarkably reduces the weight of the circuit noise. However, it does not make the circuit noise really negligible as PMTs do: with photoconductors the contribution of the circuit noise must always be evaluated and compared to the detector noise.

APPENDIX 1:
Generation-Recombination Noise

Generation-Recombination Noise Pulses



- The current *I* is a sequence of shot pulses of constant amplitude $h = i_q = q/\tau_r$ with random start time t_i **and random duration, i.e. random charge**
- λdt probability of pulse starting (carrier generation) in a dt.
  Carrier generations are independent events with Poisson statistics
- $p_r dt = dt/\tau_r$ probability of pulse termination (carrier recombination) in a dt.
  Carrier recombinations are independent events with Poisson statistics
- The probability of duration ≥ T_q is the probability P_0 of no recombination in the interval T_q . It starts from $P_0(0) = 1$ for $T_q=0$ and decreases as T_q increases; in every dT_q the decrease is given by the probability of recombination

$$dP_0 = -P_0\, dT_q / \tau_r$$

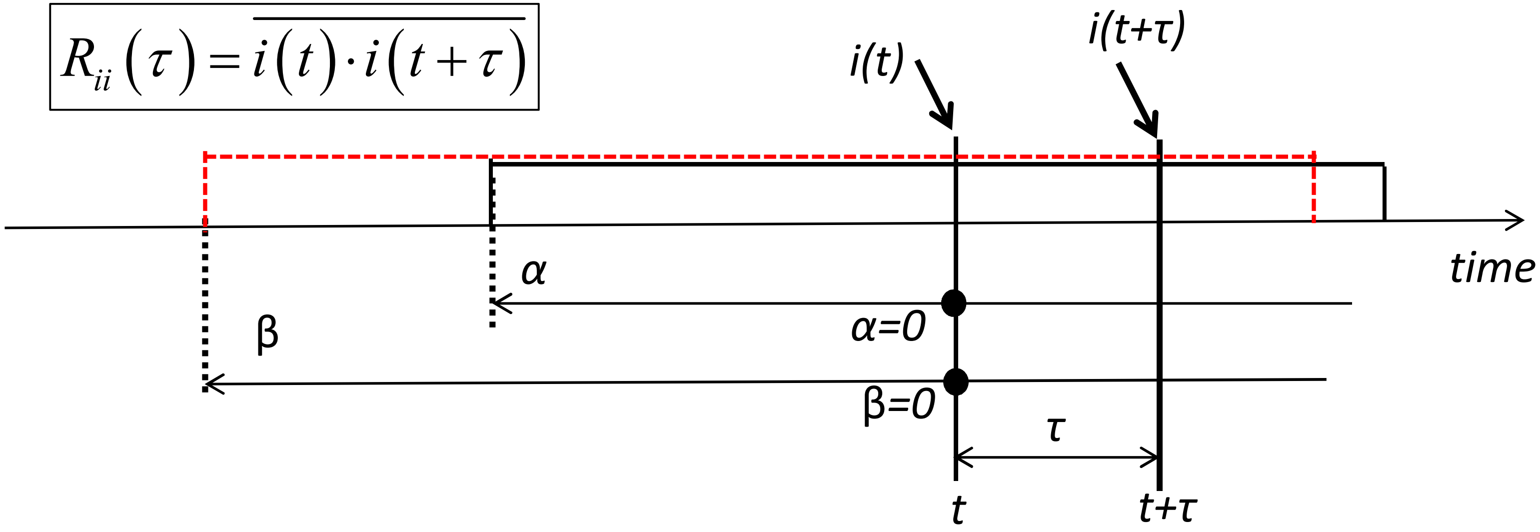
This leads to

$$P_0\left(T_q\right) = \exp\left(-T_q/\tau_r\right)$$

which confirms that $\boldsymbol{\tau_r}$ **is the mean pulse duration**

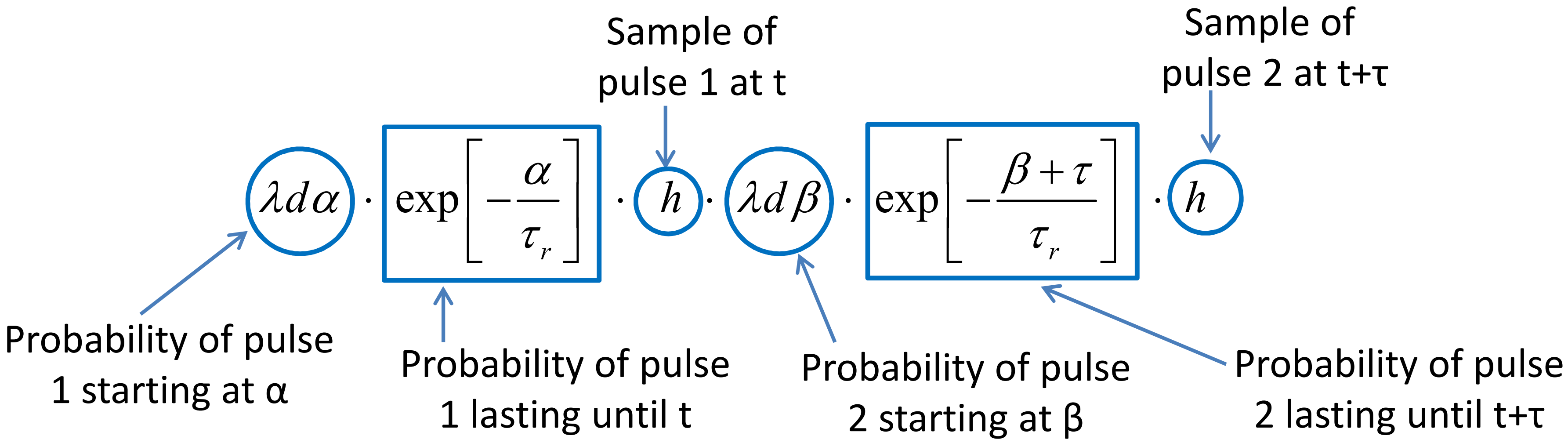
Autocorrelation Function of G-R Noise

$$\boxed{R_{ii}(\tau) = \overline{i(t) \cdot i(t+\tau)}}$$



- The samples of the total current *i(t)* at times t and t+τ are sum of contributions of all the SER pulses that start at some previous instant and are still alive at the sampling time
- The product *i(t)·i(t+τ)* is a sum of «square» and «rectangular» product terms
- «**Square**» terms are the products of samples at times t and t+τ **of the same pulse**, which starts at some previous time *α* before *t* and is still alive at t+τ
- «Rectangular» terms are products of samples taken at times *t* and *t+τ* respectively on **two different pulses**, which start at different previous times *α* and β before *t* and and are still alive at time *t* and *t+τ* respectively

«Rectangular» terms of the Autocorrelation



$$\lambda d\alpha \cdot \exp\left[-\frac{\alpha}{\tau_r}\right] \cdot h \cdot \lambda d\beta \cdot \exp\left[-\frac{\beta+\tau}{\tau_r}\right] \cdot h$$

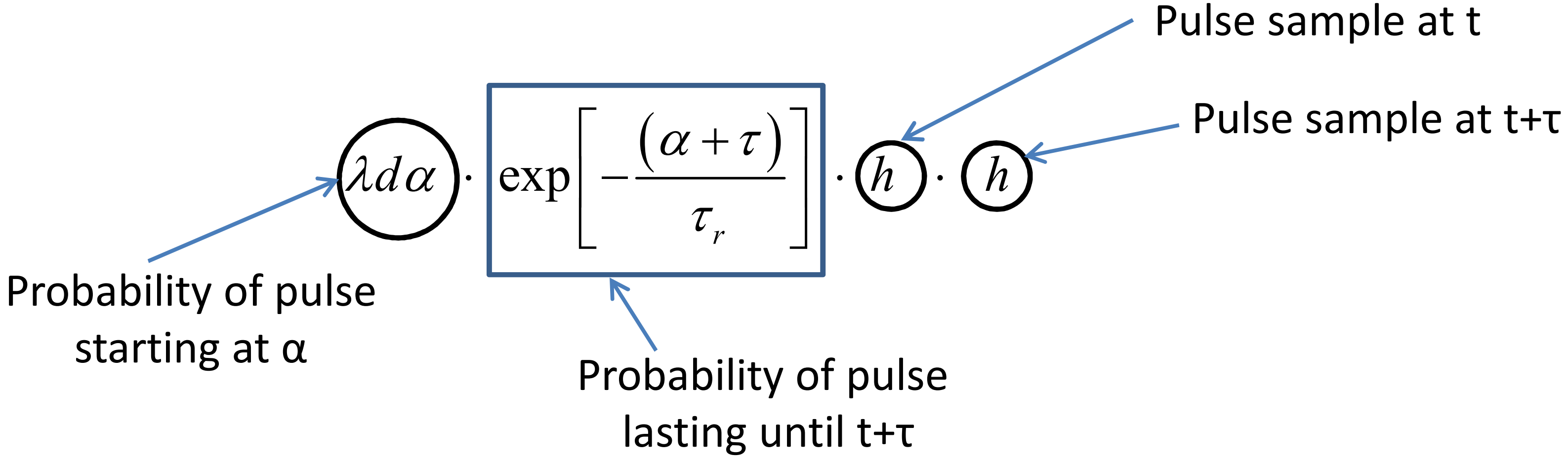
Sum of the «Rectangular» terms

$$\left[R_{ii}\right]_{REC} = \int_0^\infty h\,\lambda\,d\alpha\,\exp\left[-\frac{\alpha}{\tau_r}\right]\quad \int_{-\tau}^\infty h\,\lambda\,d\beta\,\exp\left[-\frac{\beta+\tau}{\tau_r}\right] = \lambda^2 h^2\left[\int_0^\infty \exp\left(-\frac{\alpha}{\tau_r}\right)d\alpha\right]^2$$

Since $Q = h\tau_r$ is the mean pulse charge and $I = \lambda Q$ is the mean current, we get

$$\left[R_{ii}\right]_{REC} = \lambda^2 h^2 \tau_r^2 = \lambda^2 Q^2 = I^2$$

«Square» terms of the Autocorrelation



$$\lambda d\alpha \cdot \boxed{\exp\left[-\frac{(\alpha+\tau)}{\tau_r}\right]} \cdot h \cdot h$$

Pulse sample at t

Pulse sample at t+τ

Probability of pulse starting at α

Probability of pulse lasting until t+τ

Sum of the «Square» terms

$$[R_{ii}]_{SQ} = \lambda\, h^2 \cdot \int_0^\infty \exp\left[-\frac{(\alpha+\tau)}{\tau_r}\right] d\alpha = \lambda\, h^2 \cdot \exp\left(-\frac{\tau}{\tau_r}\right) \int_0^\infty \exp\left(-\frac{\alpha}{\tau_r}\right) d\alpha = \lambda\, h^2 \tau_r \cdot \exp\left(-\frac{\tau}{\tau_r}\right)$$

Since $Q = h\tau$ is the mean pulse charge, we get

$$[R_{ii}]_{SQ} = \lambda \frac{Q^2}{\tau_r} \cdot \exp\left(-\frac{\tau}{\tau_r}\right)$$

Autocorrelation and Spectrum of G-R Noise

Finally, the autocorrelation R_{ii} of the photoconductor current is

$$R_{ii}(\tau) = \overline{i(t) \cdot i(t+\tau)} = \lambda \frac{Q^2}{\tau_r} \cdot \exp\left(-\frac{\tau}{\tau_r} \right) + I^2$$

The noise autocorrelation function R_{nn} is the autocovariance C_{ii} of the current

$$C_{ii}(\tau) = R_{ii}(\tau) - I^2 = \lambda \frac{Q^2}{\tau_r} \cdot \exp\left(-\frac{\tau}{\tau_r} \right) = R_{nn}(\tau)$$

The Fourier transform of R_{nn} gives the bilateral noise power spectrum $S_{gr,b}$

$$S_{gr,b} = 2\lambda Q^2 \frac{1}{1+\omega^2 \tau_r^2} = 2QI \frac{1}{1+\omega^2 \tau_r^2}$$

and the unilateral spectrum $S_{gr,u}$ is

$$\boxed{S_{gr,u} = 4QI \frac{1}{1+\omega^2 \tau_r^2}}$$

Spectrum of G-R Noise

The spectral density equation can be written in terms that better clarify the nature and typical features of G-R noise. By recalling that

$M = \tau_r / t_L$ is the mean Charge Gain and $Q = Mq$

i_p is the primary photocurrent and $I = Mi_p$

we see that the G-R noise spectrum is given by:

a) the shot noise of the primary photocurrent but
b) amplified and filtered by the photoconductor transfer function and
c) increased by the excess noise factor F=2 due to the gain fluctuations

$$S_{gr,u} = 4QI \frac{1}{1+\omega^2\tau_r^2} = \underbrace{2qi_p}_{\text{Primary Shot Noise}} \cdot \underbrace{\frac{M^2}{1+\omega^2\tau_r^2}}_{\text{Detector Amplification and Filtering}} \cdot \underbrace{2}_{\text{Excess Noise Factor F=2}}$$