Sensors, Signals and Noise

COURSE OUTLINE

- Introduction
- Signals and Noise
- Filtering
- Sensors: SE1 Temperature Sensors

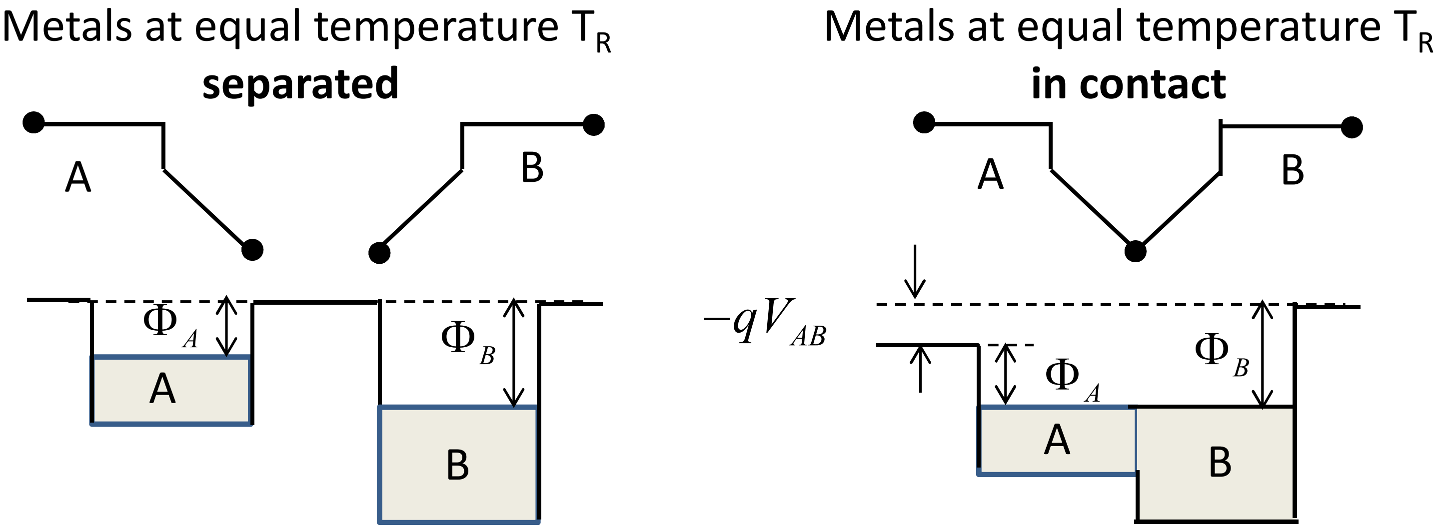
Temperature Sensors

- Thermocouples (TC)
- Resistive Temperature Detectors (RTD)
- Thermistors
- Silicon Diode Sensors

Thermocouples (TC)

- Potential difference of metals in contact
- Seebeck effect
- TC principle and TC standard types
- TC configurations; reference temperature; calibration curve; temperature reading
- TC Remote from the electronics: connection wires and electronic circuits, characteristics and requirements of amplifiers for TCs
- Noise
- Modulation of the TC signal for escaping 1/f noise
- TC dynamic response

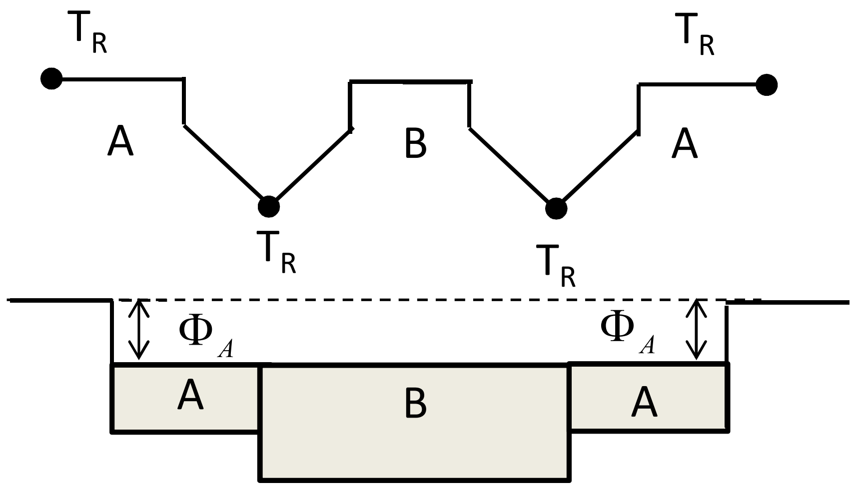
Potential Difference of Metals in Contact



- Φ_A and Φ_B work functions of metals A and B;
- Metals in contact: electrons in higher energy states in metal A move to free states at lower energy in metal B; B gets negative charge, A positive charge
- Voltage builds up between A and B and the difference of Fermi levels is reduced; equilibrium is reached with Fermi energy at equal level in A and B.
  The A-B contact potential difference V_{AB} thus established is

$$V_{AB} = \frac{\Phi_A - \Phi_B}{-q}$$

Potential Difference of Metals in Contact



In a **isothermal** chain of metals wires in contact, **all at temperature T_R :**

- equilibrium is reached with Fermi energy at equal level in all metals
- the potential difference between the ends depends only on the end metals, the intermediate metals do not matter
- if the end metals are equal, the end potential difference is zero

However, in case that one junction is brought to a different temperature $\mathbf{T_S}$ while the rest of the circuit remains at the reference temperature $\mathbf{T_R}$, the situaton changes: a potential difference is established between the ends

Seebeck Effect

- In metal wires with non-uniform temperature (different temperatures in different positions) **thermoelectric effects** generate potential differences.
- Thermocouple sensors operate with negligible electric current in the wires, hence they do not have significant Thomson and Peltier thermoelectric effects. These sensors exploit the **Seeback** thermoelectric effect, which is present also with zero current flowing, i.e. in open circuit.
- Let's consider a wire with higher temperature $\mathbf{T_S}$ at one end, lower $\mathbf{T_R}$ at the other. At higher temperature the random thermal motion of free electrons is enhanced and electrons diffuse away from the hot region. Since the lattice ion space charge is permanent, a net space charge builds up (positive in the hot region, negative in the cold region), which generates a potential variation along the wires. A potential difference between the metal ends is established, which depends on the temperature difference $\Delta T = T_S - T_R$ and on the properties concerning the free carrier motion, which are different in different metals
- In summary: the Seebeck effect is the fact that a temperature gradient along a conductor generates there a voltage gradient.

Seebeck Effect

- The effect is characterized by the **Seebeck coefficient *S*** of the material, defined as the potential difference along the metal wire per unit temperature difference

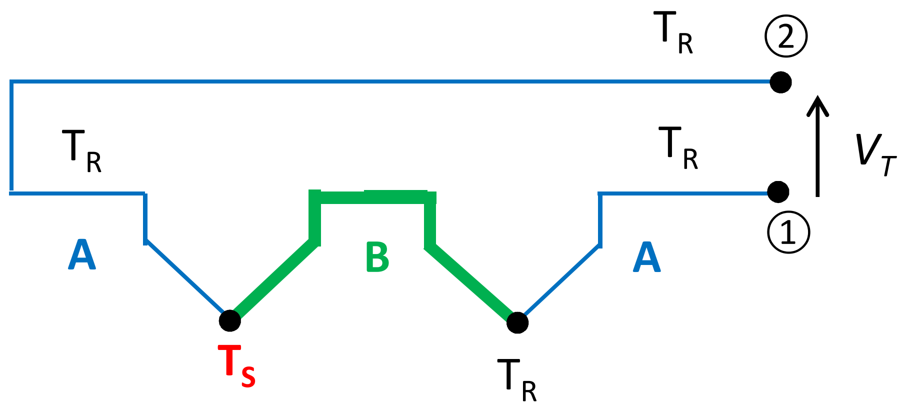
$$S = \frac{dV}{dT}$$

- The value (and the sign) of S depends on the metal properties concerning the free carrier dynamics; it is in a range from about 1 μV/K to a few 10 μV/K
- The voltage difference between two points at different temperatures $\mathbf{T_R}$ and $\mathbf{T_S}$ in a metal is given by

$$\Delta V_{SR} = V_S - V_R = \int_{T_R}^{T_S} S dT$$

- For the metals currently employed in TCs the coefficient *S* is different for different metals and has just small variations with the temperature. Therefore, the Seebeck voltage $\boldsymbol{\Delta V_{SR}}$ is approximately proportional to $\boldsymbol{\Delta T = T_S - T_R}$, with different coefficient *S* for different metals

Thermocouple principle



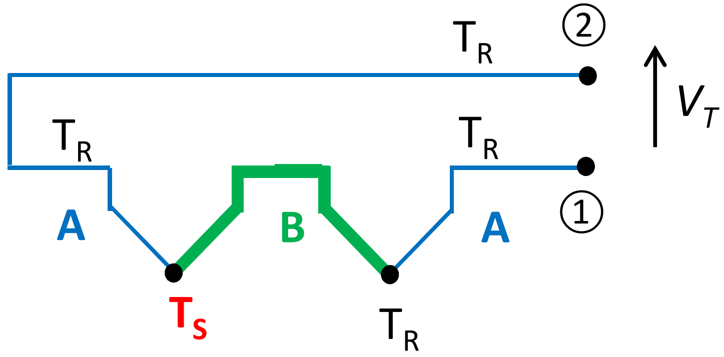
- In the chain of metals previously considered isothermal at temperature $\boldsymbol{T_R}$, let us now bring one of the A-B junctions at a different temperature $\boldsymbol{T_S}$, leaving the rest at $\boldsymbol{T_R}$, and analyse the end potential difference $\Delta V_{21} = V_2 - V_1$ developed
- $\Delta V_{21} = V_2 - V_1 = V_T$ is the sum of the potential differences due to the Seebeck effect in the two wire regions with temperature gradient, i.e. the A and B wires contiguous to the junction at temperature $\boldsymbol{T_S}$

$$V_T = V_2 - V_1 = \left(V_S - V_R\right)_B + \left(V_R - V_S\right)_A = \int_{T_R}^{T_S} S_B dT + \int_{T_S}^{T_R} S_A dT = \int_{T_R}^{T_S} \left(S_B - S_A\right) dT$$

- If $\boldsymbol{T_S}$ and $\boldsymbol{T_R}$ are in a temperature range where S_A and S_B are approximately constant, the dependence of voltage on temperature is approximately linear

$$V_T = V_2 - V_1 \approx \left(S_B - S_A\right)\left(T_S - T_R\right) = S_{BA}\left(T_S - T_R\right) \qquad \text{(where } S_{BA} = S_B - S_A\text{)}$$

Thermocouple operation in linear range



$$V_T = \int_{T_R}^{T_S} S_B dT + \int_{T_S}^{T_R} S_A dT \approx \left(S_B - S_A\right)\left(T_S - T_R\right) = S_{BA}\left(T_S - T_R\right)$$

If the Seebeck effect in the A-B couple of metals is quantitatively known and the temperature $\boldsymbol{T_R}$ **is constant at a known value**, then:

- the temperature difference $\boldsymbol{\Delta T = T_S - T_R}$ is obtained from the measurement of the potential difference V_T . With temperatures in a **range of linear operation**, it is simply

$$\Delta T = T_S - T_R = V_T / S_{BA}$$

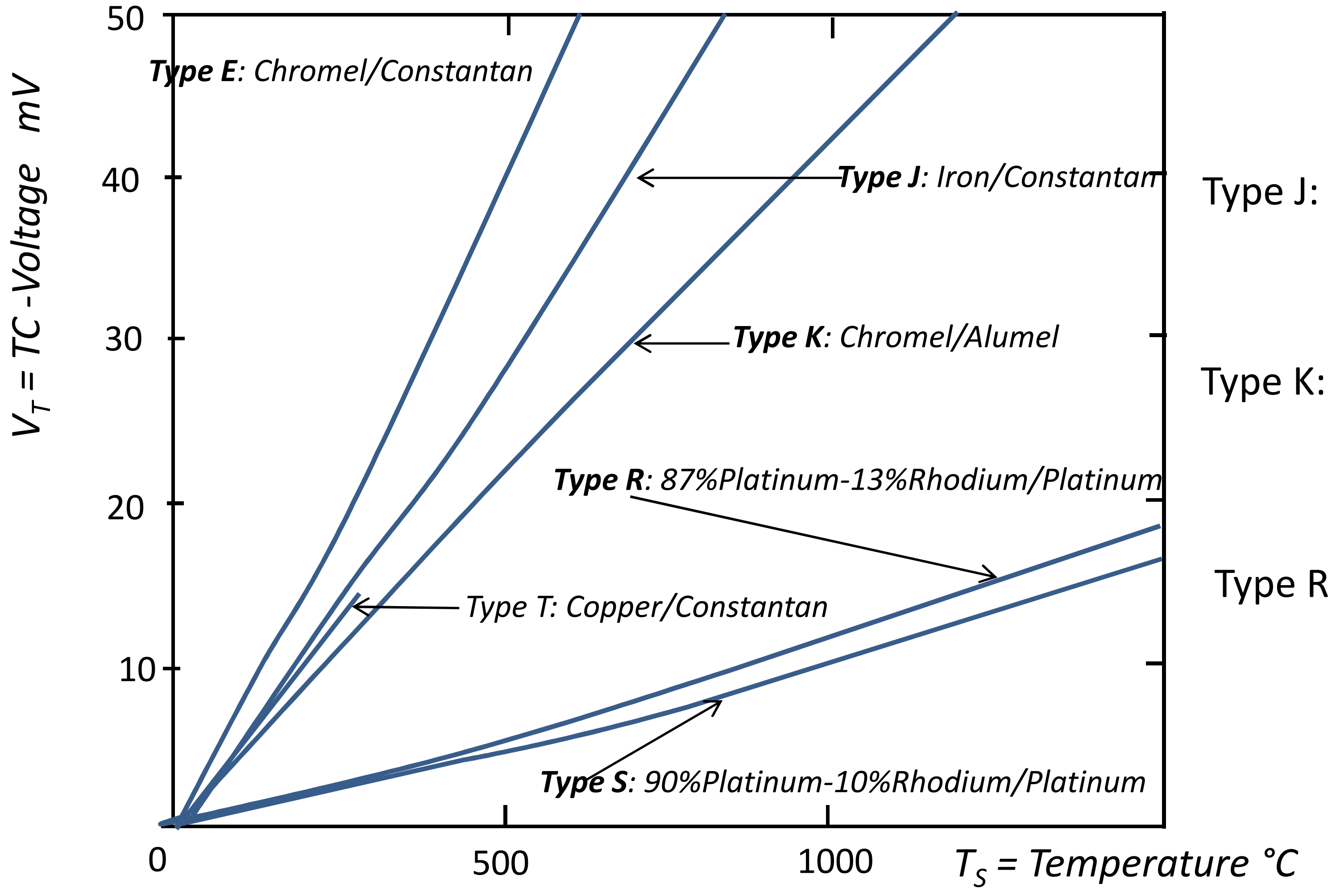
- the temperature T_S at the junction is then obtained by adding the reference temperature

$$T_S = \Delta T + T_R = V_T / S_{BA} + T_R$$

Thermocouple operation over wide T range

- TC can be employed over a very wide temperature range, from 0°C to 1000°C and more, with variation of the TC voltage versus temperature significantly nonlinear
- Next slide shows **calibration plots** for standard TC types, i.e. plots of $V_T(T_S)$ for T_S>0°C measured with a standard reference T_{R0} . They are usually supplied by TC manufacturers as measured with reference $T_{R0} = 0°C$ (at the water triple point, liquid-ice-vapour in thermal equilibrium in a dewar vessel)
- TCs are employed to measure temperatures also below *0°C* . The calibration in this range shows a more evident non-linear behavior, with Seeback coefficient that significantly decreases with the temperature, i.e. with reduced thermometric sensitivity. TC are currently employed down to liquid nitrogen temperature (about 80K), but they are considered unsuitable for cryogenic measurements around liquid helium temperature

Thermocouple Calibration Plots



Type J: $\frac{dV_T}{dT} \approx 60\,\mu V/°C$

Type K: $\frac{dV_T}{dT} \approx 42\,\mu V/°C$

Type R: $\frac{dV_T}{dT} \approx 11\,\mu V/°C$

Reference Temperature T_{R0} = 0°C

TC operation in nonlinear range

OPERATION IN NON-LINEAR RANGE

A TC can be well exploited also in a range of nonlinear voltage-temperature behavior. With calibration table available and TC operating with the standard reference T_{R0} of the calibration:

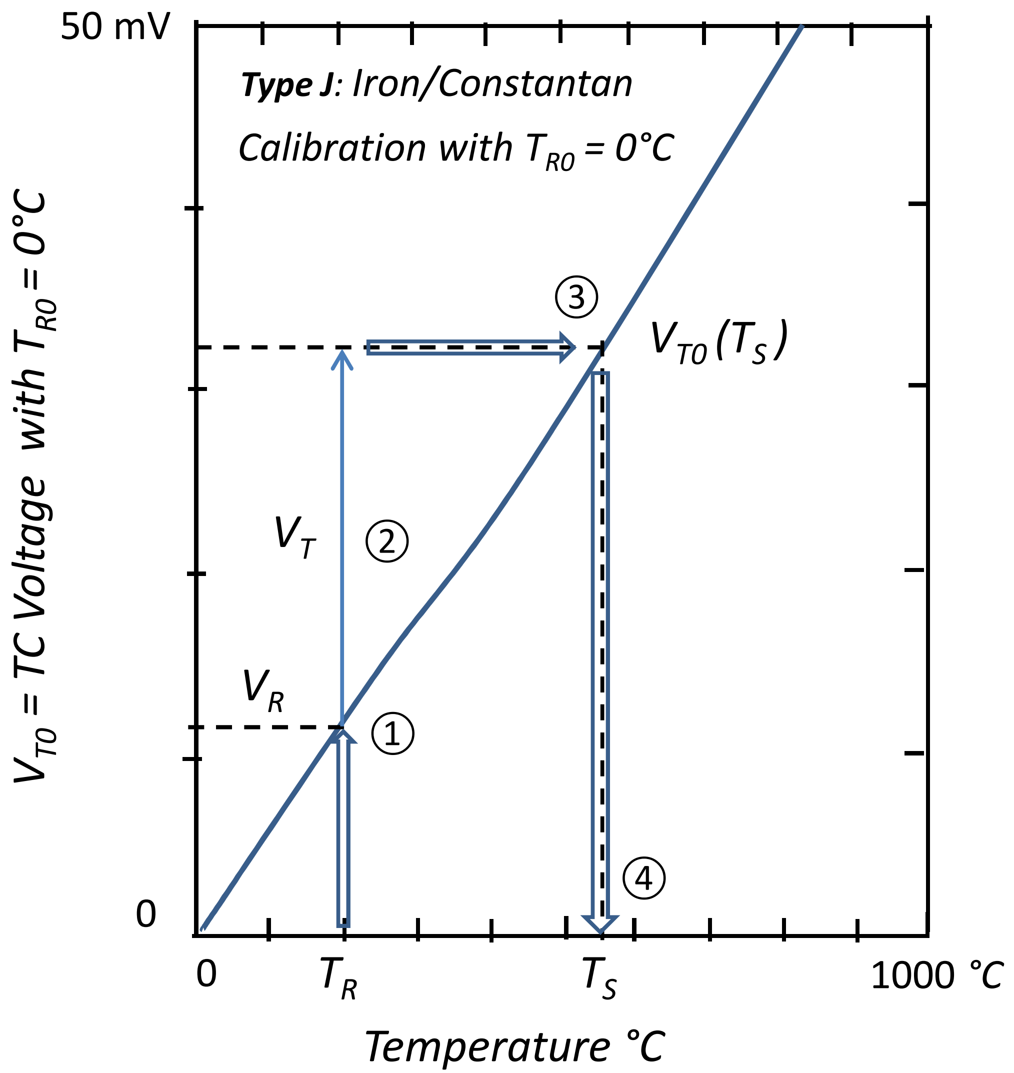
- the temperature difference $\boldsymbol{\Delta T = T_S - T_{R0}}$ is directly obtained from the measurement of the potential difference V_T by **employing the nonlinear calibration table**

REFERENCE TEMPERATURE $\boldsymbol{T_R}$ DIFFERENT FROM THE CALIBRATION $\boldsymbol{T_{R0}}$

If the reference temperature $\boldsymbol{T_R}$ is different from the $\boldsymbol{T_{R0}}$ in the calibration, the measured voltage V_T must be corrected before employing the calibration table because:

- in the calibration, the Seebeck effect is due to the temperature gradient in the wires from the measured temperature T_S to the standard reference $T_{R0} = 0°C$
- in operation, the Seebeck effect is due to the gradient from T_S to the actual reference temperature T_R .
- with $T_R > T_{R0}$ and equal T_S , the difference from V_T (TC voltage in operation) to V_{T0} (TCvoltage in calibration) is the Seebeck effect of the gradient from T_R to T_{R0}.

Measurement with $T_R \neq T_{R0}$



In TC Calibration:

$V_{T0}(T_S)$ measured with ref $T_{R0} = 0°C$

In TC Operation:

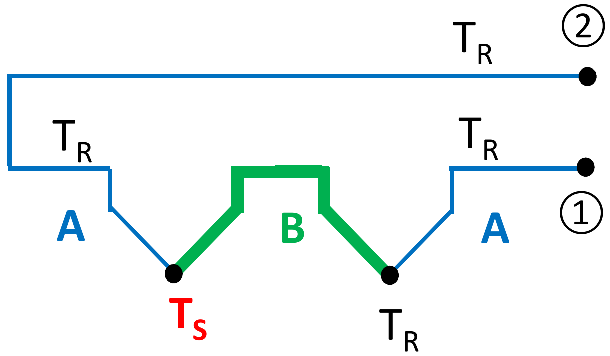
$V_T(T_S)$ measured with ref $T_R \neq T_{R0}$

(in the example T_R =200°C)

Therefore, for obtaining T_S:

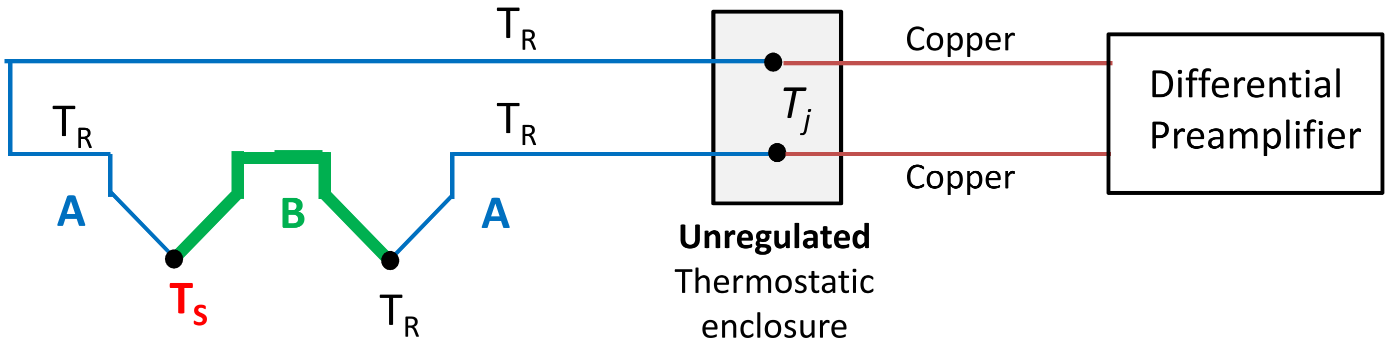
1) T_R known ⟹ $V_{T0}(T_R) = V_R$
2) V_T measured in operation
3) $V_R + V_T = V_{T0}(T_S)$
4) $V_{T0}(T_S)$ ⟹ T_S

Regulated Reference Temperature T_R



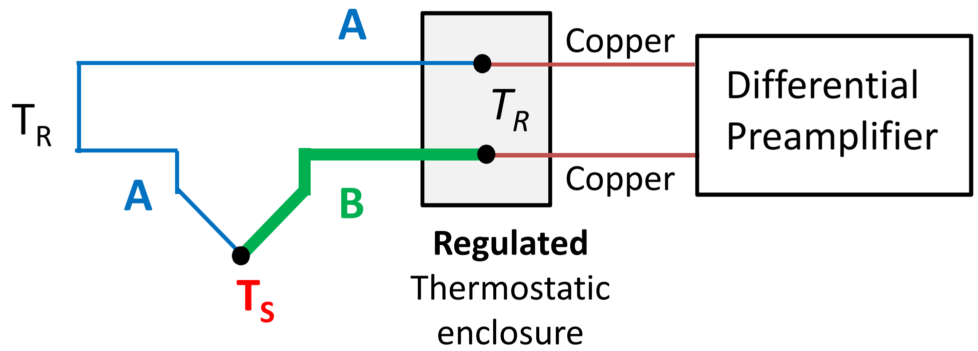
- Accuracy and precision of the temperature measurement are limited by the knowledge of the reference temperature T_R
- Operating with the reference junction at standard temperature T_{R0} is a technically sound approach, but the set-up for establishing and maintaining accurately T_{R0} is fairly complex and is often unsuitable to practical applications
- In alternative, a thermostat at $T_R > T_{R0} = 0°C$ is often used. The value of T_R is measured with a precise auxiliary thermosensor and controlled in a feedback loop with a heater. This control of T_R operates in a narrow range of temperature, therefore it can use a simple RTD, while the TC measures temperature T_S over a much wider range.

Reference Junction and Input Compensation



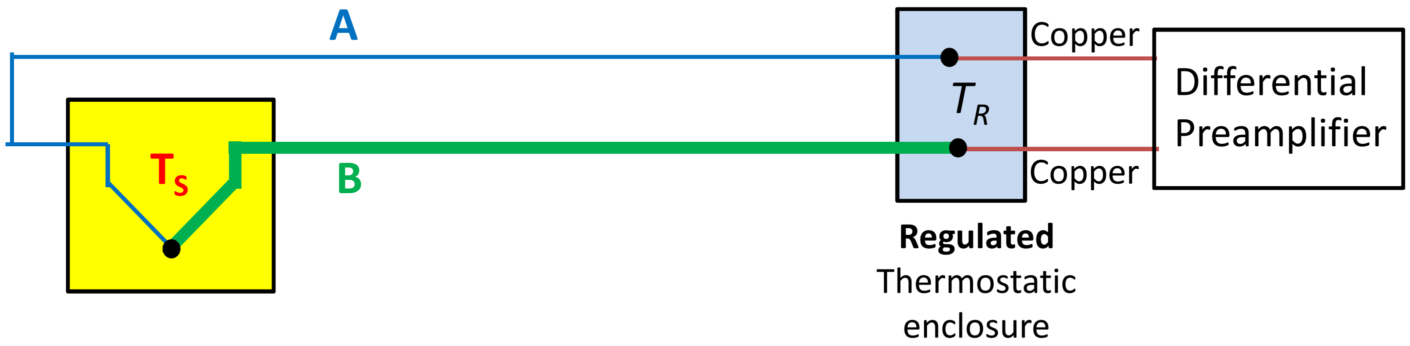
- The TC terminals (metal A) connected to the input wires (Copper) of the electronic circuits introduce a further couple, which brings an unwanted contribution to the measured TC voltage
- This contribution is cancelled simply by holding both junctions A-metal/Copper at the same temperture T_j ,without needing to know the T_j value. With any T_j , the Seebeck voltages of the two couples A-metal/Copper are cancelled (they have same value and opposite sense in the circuit). It is thus sufficient to put the input contacts in a thermostatic enclosure, without regulation.
- The effect on the TC voltage can be completely avoided by holding also the input junctions A/Copper at the reference temperature T_R , but this solution is fairly cumbersome and is not widely adopted. However, it led to devise a simpler and widely adopted configuration, called «Reference Input Configuration».

Reference Input Configuration



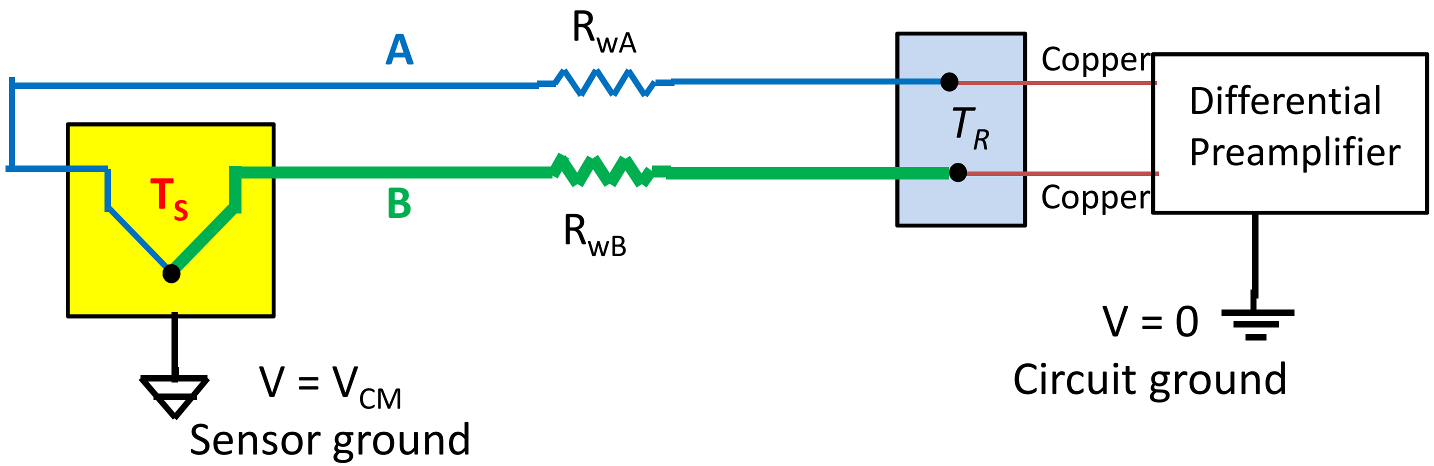
- The input enclosure not only is thermostated to keep the two junctions at the same temperature, but this **temperature is also known**.
- The enclosure temperature can be regulated at a preset reference T_R .
- Alternatively, it can be left free to follow the ambient temperature, but is continuously measured in order to take into account in each measurement the actual T_R value
- With this arrangement the second junction A/B acting as reference is no more necessary: the input junction is now the reference. In fact, if also the thermometric junction A/B is brought to the temperature T_R , it is readily verified that the end potential difference is zero, since we have now an isothermal chain of metals (at temperature T_R) with equal metal ends

Operation with TC Remote from Electronics



- The wires that connect a TC to the electronic circuitry can be **several meter long** in various cases: distributed temperature controls in industrial workshops; measure of high temperature in furnaces; etc.
- We saw that the TCwire/copperwire junctions must be at a **known T_R**, regulated or measured. In order to implement such a control, the TCwire/copperwire junction must be located far from the measured ambient, near to the electronics
- In conclusion, the long connections cannot be made with copper wires: the same A-wire and B-wire of the TC must be used all the way from TC to the reference enclosure
- Special extension wires are also available, made in alloys compatible with TC wires (i.e. with the equal Seebeck coefficient over a limited temperature range of the connection), but with lower cost and better electrical and mechanical properties,

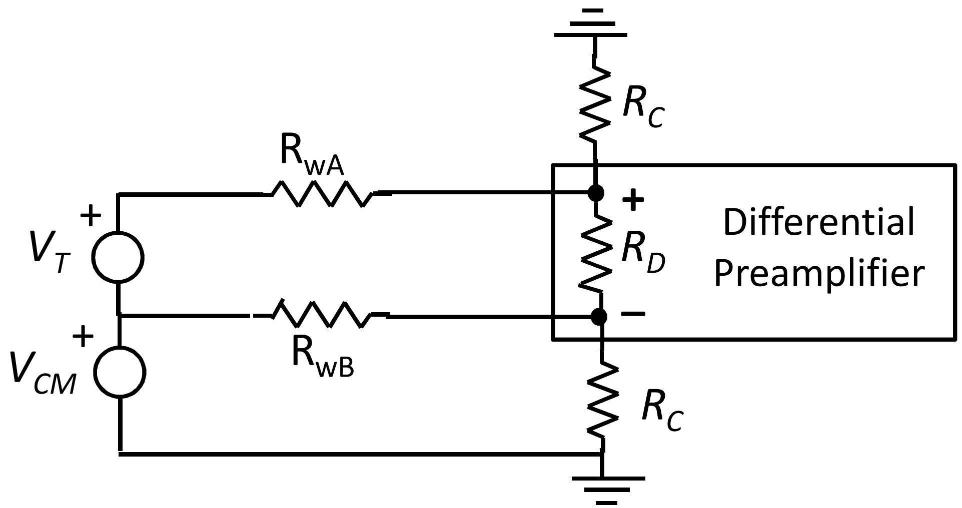
Operation with TC Remote from Electronics



- In general, the two metals of a TC have different and fairly high resistivity and the long connection wires have small section (for low heat transmission, for economy, for passing many wires in small ducts, etc.)
- The connection wires thus have **fairly high resistances** (R_{wA} and R_{wB} ≈ several hundred Ohm) with **remarkably different values** (R_{wA} **-** R_{wB} ≈ various hundred Ohm)
- Specific problems for TC operation ensue from the remarkable difference of the wire resistances. They are added to the general problems caused by ground potential differences, which plague any type of sensor operating remote from the associated circuit

Common Mode Rejection Ratio (CMRR)

Ground potential difference V_{CM} from circuit to remote sensor: it is often remarkable (tens of Volts) particularly in workshops



- The preamplifier must have CMRR sufficient to keep within acceptable limit $\Delta V_{C,max}$ the error caused by V_{CM} in the measurement of V_T

$$CMRR \geq V_{CM} \big/ \Delta V_{C,\max}$$

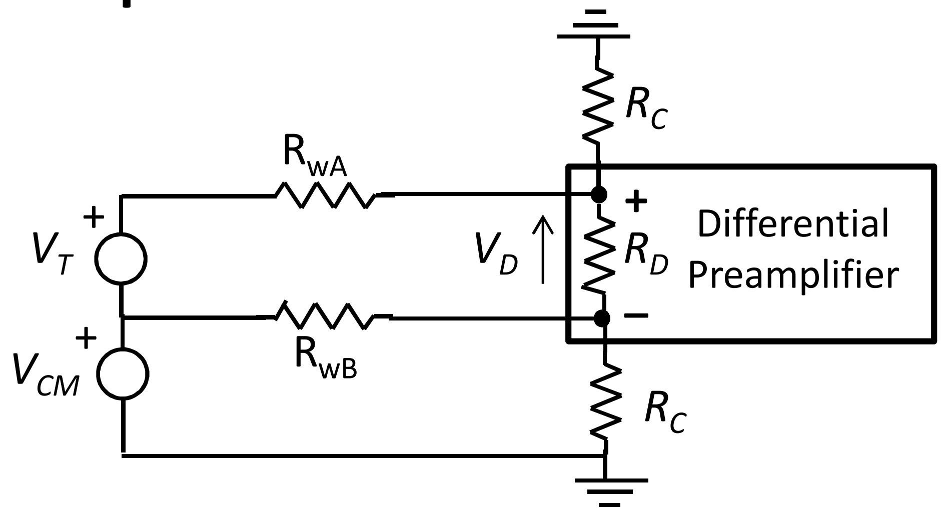
- The $\Delta V_{C,max}$ value is set by the limit ΔT_{max} to the error in temperature and by the Seebeck coefficient S of the TC actually employed

$$\Delta V_{C,\max} = S \cdot \Delta T_{\max}$$

In most cases the **requirements for CMRR are fairly stringent**.
For example: for a TC type R ($S_R = 11\ \mu V/°C$) with $V_{CM} \approx 10V$ and $\Delta T_{max} = 1°C$, we get $\Delta V_{C,max} \approx 11\ \mu V$ and we must require CMRR > 910000 = 120 dB

Differential Input Resistance

- R_D differential input resistance
- R_C common-mode input resistance
- $R_C >> R_D >> (R_{wA} + R_{wB})$



The non-negligible resistance of the connection wires causes a signal loss

$$\Delta V_T = V_T - V_D = V_T \frac{R_{wA} + R_{wB}}{R_D // 2R_C + R_{wA} + R_{wB}} \approx V_T \frac{R_{wA} + R_{wB}}{R_D}$$

With linear TC approximation $V_T \approx S\,(T-T_R)$ we get $\dfrac{\Delta V_T}{V_T} = \dfrac{\Delta T}{T - T_R} = \dfrac{R_{wA} + R_{wB}}{R_D}$

A maximum acceptable error ΔT_{max} requires

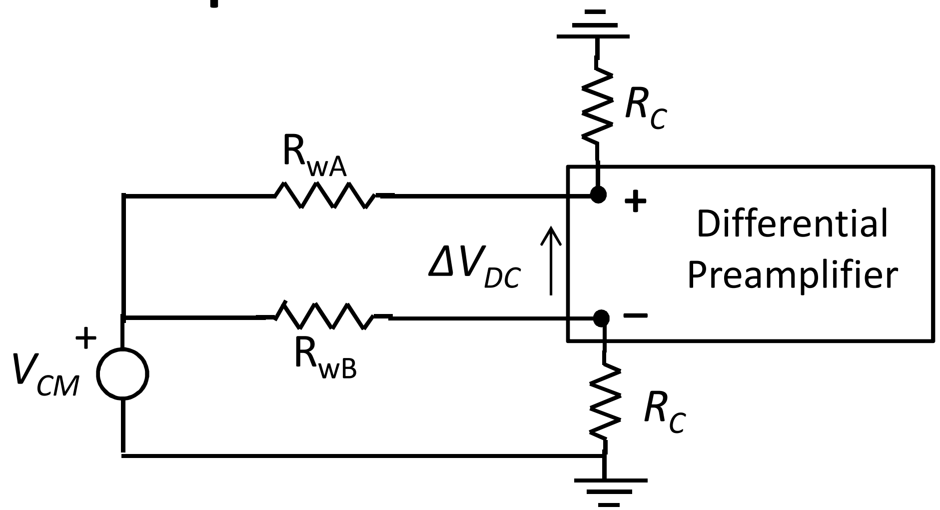
$$\frac{\Delta V_T}{V_T} = \frac{R_{wA} + R_{wB}}{R_D} < \frac{\Delta T_{max}}{T - T_R}$$ i.e. it requires $$\boxed{R_D > \left(R_{wA} + R_{wB}\right) \frac{T - T_R}{\Delta T_{max}}}$$

Quantitative calculations with typical values show that in most cases a **moderate value of R_D (a few MΩ) is sufficient** for justifying the approximation $R_D \rightarrow \infty$

Common-mode Input Resistance

We can consider:

- only V_{CM} active
- $R_D \to \infty$
- $R_C >>> R_{wA}+R_{wB}$



- The currents I_{wA} and I_{wB} in the two wires are very small and practically equal

$$I_{wA} = \frac{V_{CM}}{R_C + R_{wA}} \approx \frac{V_{CM}}{R_C} \qquad I_{wB} = \frac{V_{CM}}{R_C + R_{wB}} \approx \frac{V_{CM}}{R_C} \qquad I_{wA} \approx I_{wB} \approx I_w = \frac{V_{CM}}{R_C}$$

- The different resistances cause different voltage drops on the wires, thus generating a fake differential signal

$$\Delta V_{DC} = I_w R_{wA} - I_w R_{wB} = V_{CM} \frac{R_{wA} - R_{wB}}{R_C}$$

Common-mode Input Resistance

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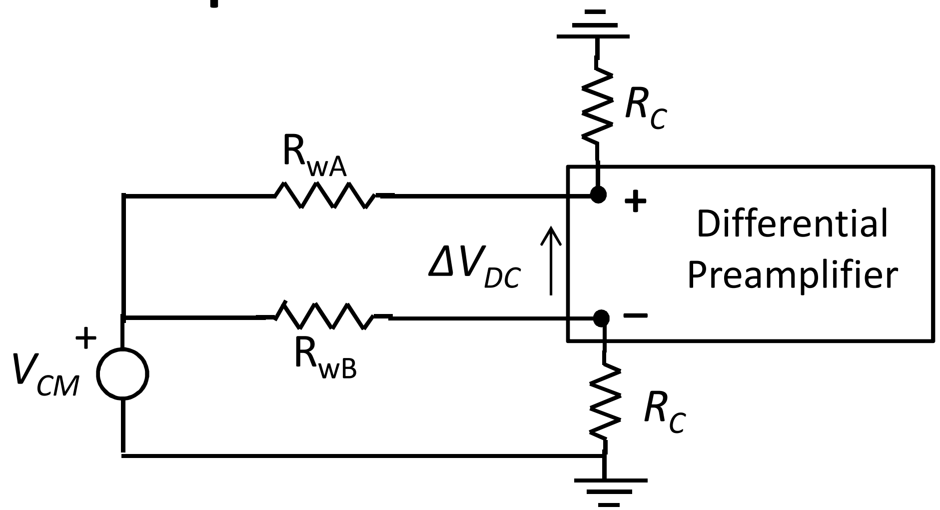
- only V_{CM} active
- $R_D \to \infty$
- $R_C >>> R_{wA}+R_{wB}$



- If $\Delta V_{DC,\,max}$ is the maximum allowed error, for keeping $\Delta V_{DC} < \Delta V_{DC,max}$ we need

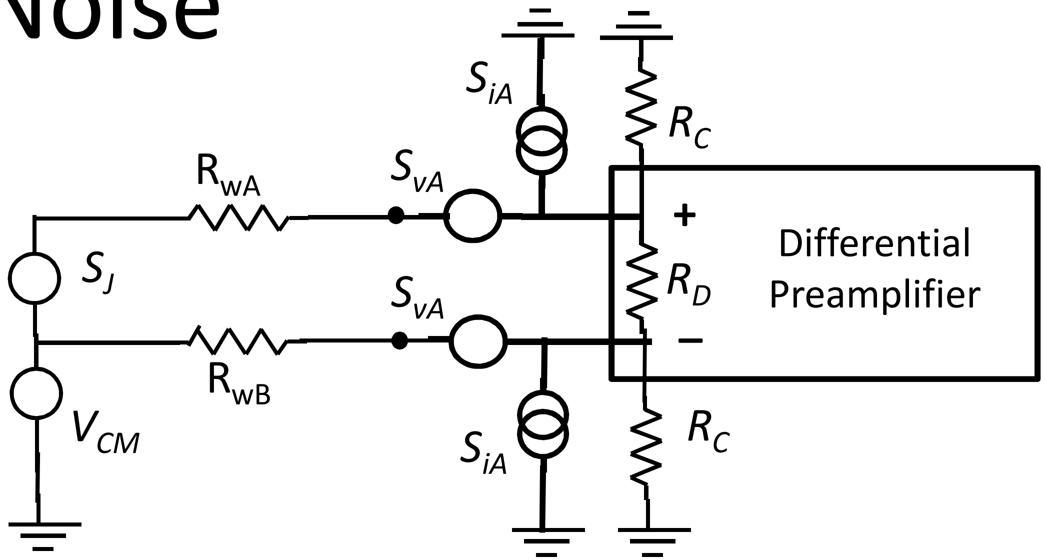
$$\boxed{R_C > \left(R_{wA} - R_{wB}\right)\cdot\frac{V_{CM}}{\Delta V_{DC,\max}}}$$

- In most cases the **requirement for R_C turns out to be severe**: a very high value is required and makes necessary to employ Instrumentation Amplifiers.
- For instance, with $V_{CM} \approx 10V$ and $(R_{wA}-R_{wB}) \approx 500\ \Omega$, employing an R type TC ($S_R=11\mu V/°C$) with allowed error ΔT_{max} = 1°C, which corresponds to $\Delta V_{DC,max} \approx 11\mu V$, the requirement is

$$R_C > 450 M\Omega$$

Noise

- $(R_{wA}+R_{wB})$ from 100 Ω to a few kΩ
- $R_D > 1$ MΩ
- $R_C > 100$ MΩ



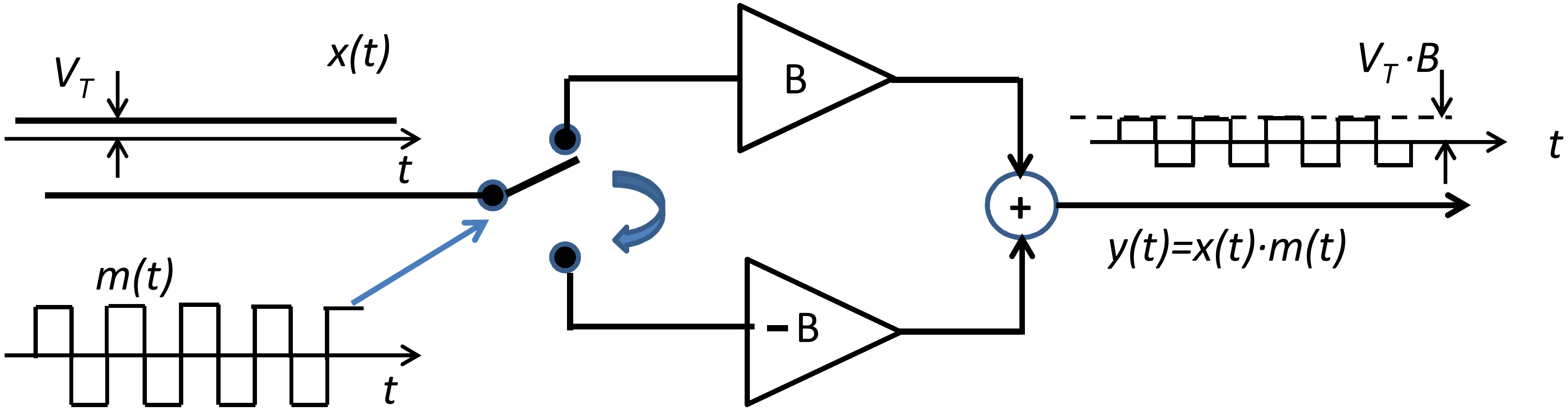
- The sensor is a metal resistor, therefore it has negligible 1/f noise
- The sensor resistance $(R_{wA}+R_{wB})$ ranges from 100 Ω to a few kΩ, hence the effective spectral density of the Johnson noise of the sensor is in the range

$$\sqrt{S_J} \approx 1 \;\; to \;\; 10\, nV\Big/\sqrt{Hz}$$

  i.e. it is not always negligible with respect to the amplifier noise

- Since the source resistance is low or moderate, in most cases the amplifier voltage noise S_{vA} is dominant and the effect of the amplifier current noise S_{iA} is negligible
- The preamplifier noise has also non-negligible 1/f noise components, which can markedly affect the measurements of the slow signals of TCs.

Escaping 1/f Noise by Signal Modulation

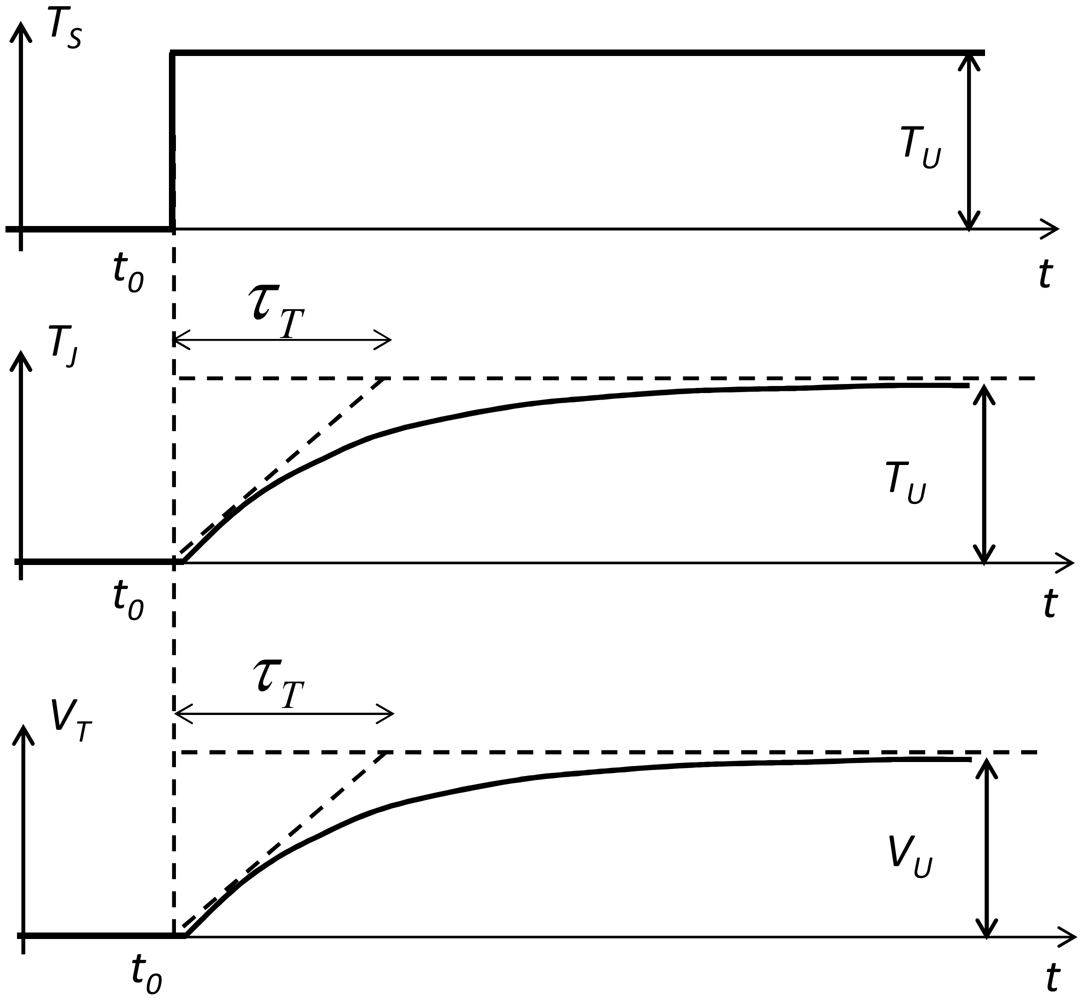


- The 1/f noise of the preamplifier can markedly affect the measurement of slow TC signals. This can be avoided by shifting to higher frequency the TC signal by modulation, provided that the **modulation be made before adding significant 1/f noise**.
- Modulator circuits are unsuitable, since TC signals may be not higher than the modulator 1/f noise. Squarewave modulators implemented with simple switches are suitable, since they have noise referred to the input much lower than analog multiplier electronic circuits.
- Electromechanical metal-contact switches have the lowest noise, but can operate at low switching frequency, typically up to a few 100Hz
- Electronic switches (MOSFET, diodes, etc.) operate up to high frequencies, but have higher noise than metal switches; anyway, MOSFETs in switching operation have lower noise than in linear operation and in many cases they are suitable for TC signal modulation.

TC Dynamic Response

- The response of a TC to a step in the temperature to be measured is dictated by the **thermal transient of the TC junction**. The temperature-to-voltage transfer function of the TC is a low-pass filter, which must be taken into account in all applications and is specially important in feedback control loops
- A fair approximation is to consider the TC in linear operation as a **single-pole lowpass filter**. The thermal transient is characterized by two parameters: the **thermal resistance** R_T and **heat capacitance** C_T (case similar to that of thermal photodetectors, *see slide package PD1)*.
- R_T is between the object measured and the TC junction; C_T is that of the TC junction. Their values strongly vary from case to case, depending on the different fabrication technologies of TCs. For instance, TC often have junction wires encased in a protective metal sheath, filled with Magnesion Oxide powder (MgO) to ensure good thermal contact and good electrical isolation; in other cases, they have junction wires directly exposed to the ambient.
- The **time constant** $\tau_T = R_T C_T$ of the single pole response of the TC has anyway a value remarkably longer than usual in electronic filters and widely variable in the various practical cases, from millisecond range to seconds.

TC Dynamic Response



Object Temperature T_S

$$T_S = 1(t - t_0)\,T_U$$

TC junction Temperature T_J

$$T_J = T_U \left[1 - \exp\left(-\frac{t - t_0}{\tau_T} \right) \right]$$

TC voltage V_T

$$V_T = S \cdot T_U \left[1 - \exp\left(-\frac{t - t_0}{\tau_T} \right) \right]$$

Resistive Temperature Detectors (RTD) and Thermistors

- Metallic RTDs: principle and fabrication
- RTD Electrical Signal
- Circuits for measurements
- Thermistors

Metal RTD principle

Principle:

- Resistance R_S of metal conductors increases monotonically with temperature T
- calibration of resistance versus temperature $R_S(T)$ is accurate and stable
- By measuring resistance variation ΔR_S we get the temperature variation ΔT

Linear behavior of R_S(T) is a good approximation on wide T range for various metals

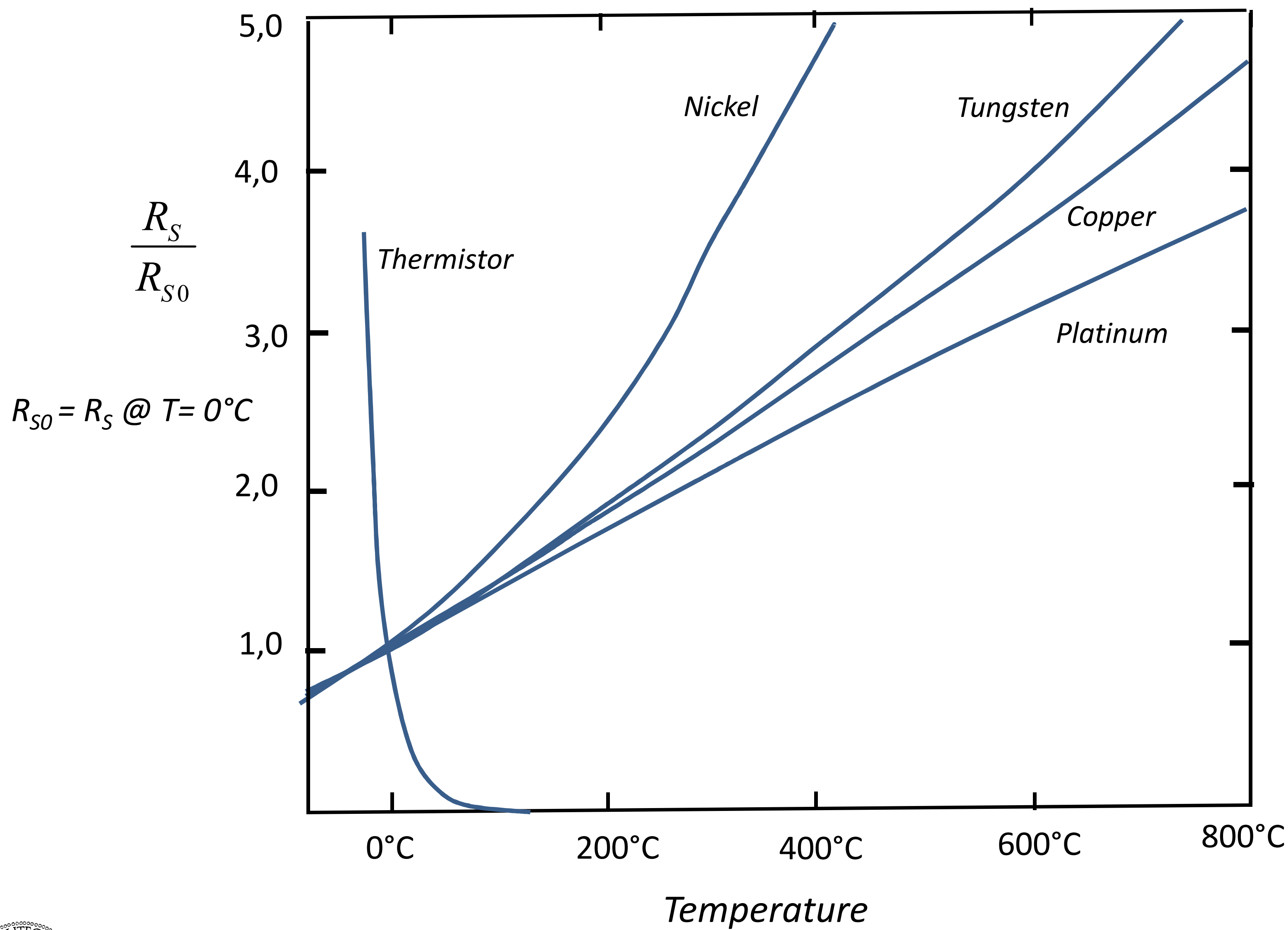
$$R_S = R_0 \left(1 + \alpha \, \Delta T\right) \qquad T_0 = \text{reference temperature};\ R_0 = R_S(T_0);$$

$$\Delta R_S = \alpha \, \Delta T \, R_0 \qquad \Delta T = T - T_0 \ ; \ \ \Delta R_S = R_S - R_0$$

α is called **temperature coefficient of resistance**.
α is around ≈ $4 \cdot 10^{-3}$ for metals currently employed in RTDs

Metal	α
Platinum Pt	$3{,}9 \cdot 10^{-3}$
Copper Cu	$4{,}3 \cdot 10^{-3}$
Tungsten W	$4{,}6 \cdot 10^{-3}$
Nickel Ni	$6{,}8 \cdot 10^{-3}$

Metal RTD principle



Metal RTD technology

Platinum has useful qualities:

- Chemically inert and resistant to contamination, hence stable properties
- R_S(T) linear with very good approximation from -200°C to about 500°C and with small deviation from linearity up to 800°C
- small quantity of Pt necessary in a RTD, cost is not high

Pt is the material of choice in many cases and is used in official metrology to define the International Practical Temperature Scale (from 13,81 K to 903,89 K).

Because of requirements for correct operation, the **RTD fabrication technology is not so simple** :

- The package must be compact and ensure good **thermal contact** of the resistor to the object measured and good **electrical isolation** from it
- Small size is required with R_0 > some 10 Ω , typically R_0 =100 Ω, in order to have to measure not very small ΔR_S. Thin wire wrapped in spiral on a support is used
- The mechanical structure must **avoid strain** of the metal wire due to thermal expansion or contraction: the **piezoresistive effect** would cause unwanted resistance variations and consequent errors in ΔT

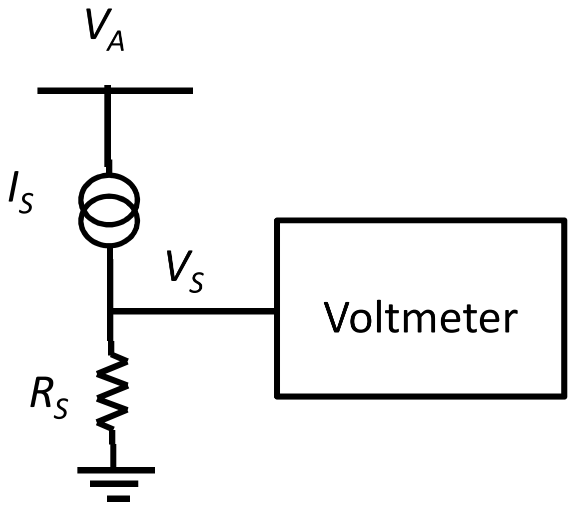
Generation of RTD Electrical Signal

- RTD do not generate an electrical signal, a **power supply is necessary** to get current and voltage in the RTD
- Joule **self-heating** makes the RTD temperature T_S higher than the temperature T_a of the object measured; the difference $\Delta T_S = T_S - T_a$ increases with power dissipation P_S and sensor-to-object thermal resistance R_{th}.
- The maximum tolerable ΔT_S in a given RTD configuration sets a limit P_{Smax} to the power dissipated in the RTD, hence to the **maximum voltage V_S** on the RTD

$$P_S = V_S^2 / R_S \qquad P_S \leq P_{S,\max} \qquad \boxed{V_S \leq \sqrt{R_S \cdot P_S}}$$

- The allowed voltage V_S on the RTD is fairly small: e.g. with $R_S \approx 100\ \Omega$ and limit $P_{Smax} = 100\mu W$, the voltage is limited to $V_S < 100mV$.
- The **voltage variations** to be measured for small variations of temperature are a small fraction of V_S , i.e. they are **definitely small**.

RTD Operation at Constant Current



$$\Delta R_S = f\left(\Delta T\right) \approx \alpha R_0 \cdot \Delta T$$

$$V_{S0} = I_S R_0$$

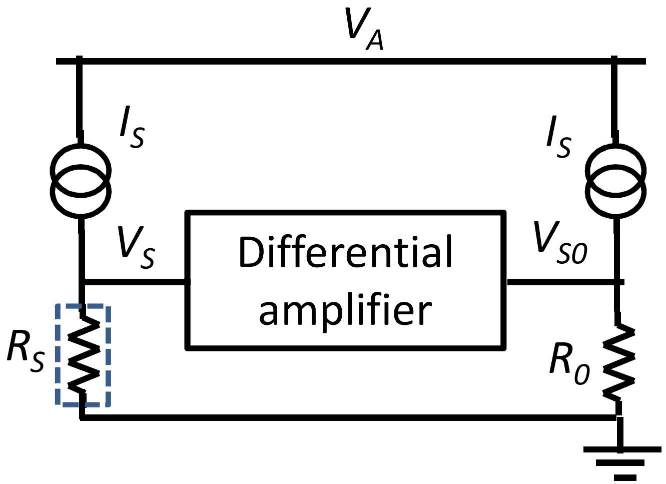
$$\Delta V_S = V_S - V_{S0} = I_S \cdot \Delta R_S =$$

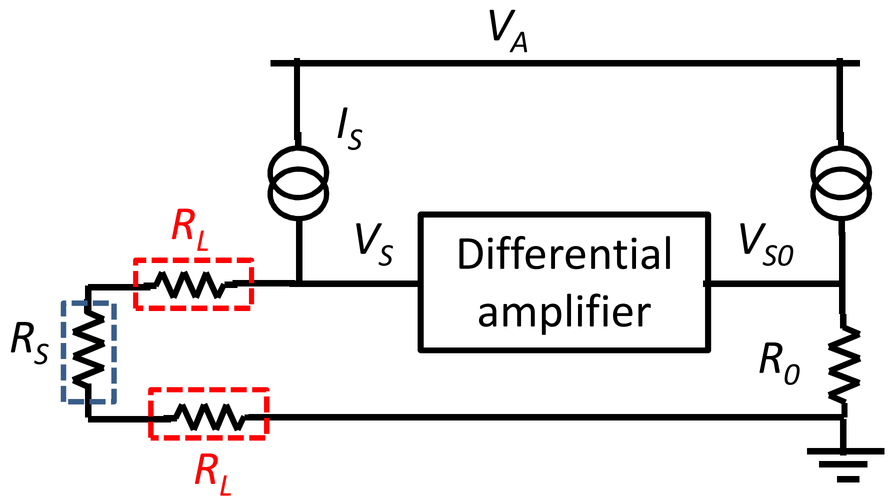
$$= V_{S0} \frac{\Delta R_S}{R_0} \approx V_{S0}\, \alpha \Delta T$$

In modern electronics a simple approach is possible and practical thanks to the routine availability of current generators :

- R_S is biased with a **constant current** generator I_S ,
- voltage V_S on R_S is measured
- at any T, ***V_S* is exactly proportional to *R_S*** : the difference ΔV_S from measured V_S to reference voltage V_{S0} gives an accurate measure of ΔR_S
- ΔR_S is an accurately known function of $\Delta T = T - T_0$, in many cases approximately linear

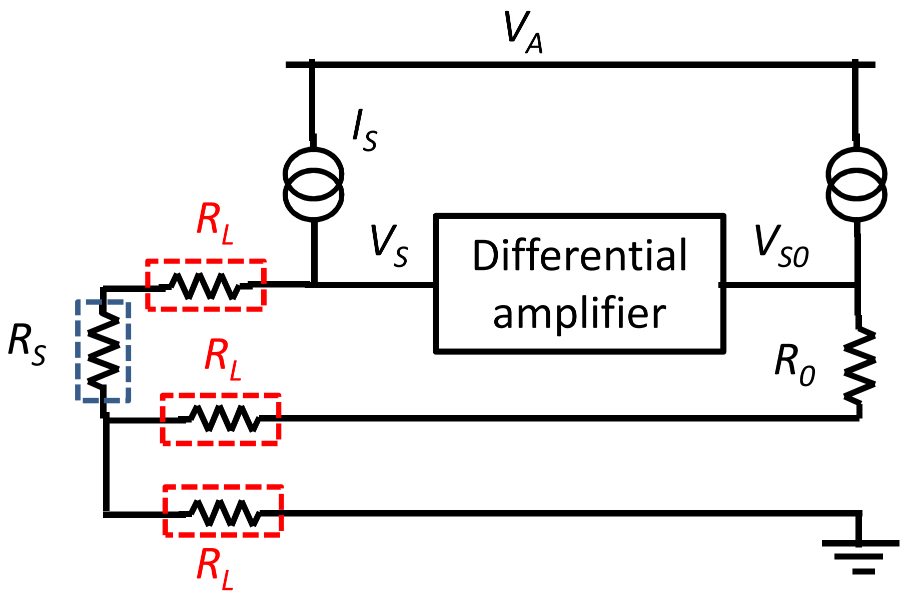
Differential Signal at Constant Current





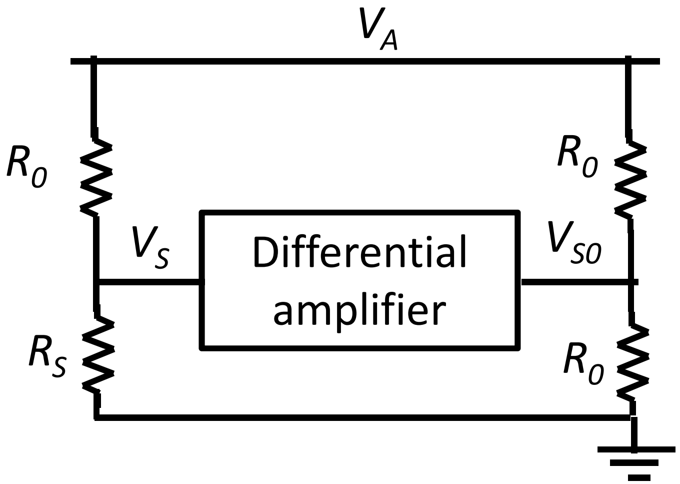
- Since ΔV_S is much smaller than V_S, it is advisable to include in the circuit a reference V_{S0} and take **directly differential measurements of ΔV_S**, instead of measuring V_S and then subtracting V_{S0}
- However, in various cases the RTD is placed on a measured object not near to the circuit, the **long connecting wires** have resistance R_L not negligible with respect to R_S and their **effect is significant** and must be taken into account
- In the simplest configuration, called «Two-wire-connection», the two wire resistances are in series with R_S and their voltage drop $2I_S R_L$ is added to V_S, thus causing a significant error in the measured ΔV_S

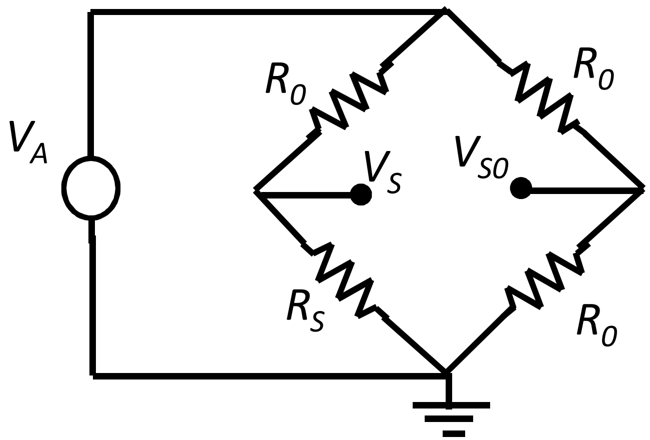
Remote RTD Operation



- Errors in ΔV_S due to wire resistances R_L are avoided by a **«Three-wire-connection».** Both the reference arm and the RTD arm include in series a wire resistance R_L ; the third wire resistance R_L is inserted in the common return to the circuit ground

RTD Operation in Wheatstone Bridge





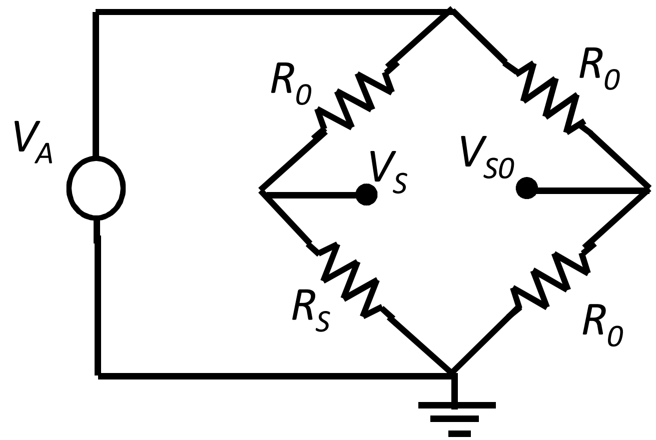
- An alternative configuration, devised when current generators were not available, requires only resistors and due to its simplicity is still widely exploited
- A **voltage divider** is implemented by the ***R_S* of the RTD** in series with a **reference resistor *R_0*** and the variations of the divider output voltage corresponding to the variations of R_S are measured
- This is the principle of the **Wheatstone bridge**, invented in 1833 by Samuel Hunter Christie and popularized by Charles Wheatstone and usually drawn as sketched above at right

RTD Linear Operation in Wheatstone Bridge

$$R_S = R_0 + \Delta R_S$$

$$V_{S0} = V_A \frac{R_0}{R_0 + R_0} = \frac{V_A}{2}$$

$$V_S = V_A \frac{R_S}{R_0 + R_S}$$



For **small resistance variation** $\Delta R_S < 0{,}05\ R_0$ the voltage variation ΔV_S is **approximately linear** with ΔR_S and can be computed by first-order development

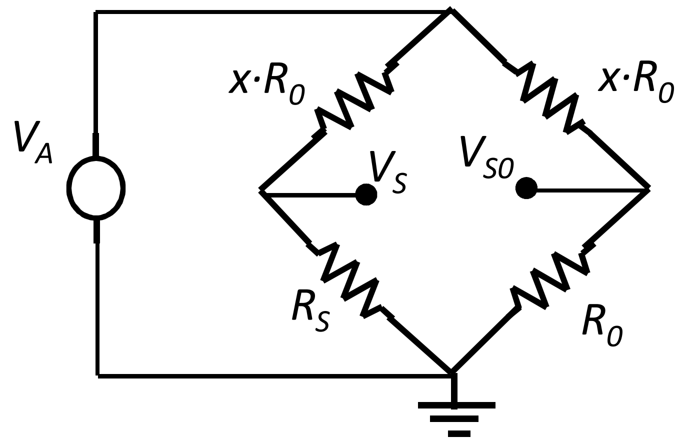
$$\Delta V_S = \Delta R_S \left(\frac{dV_S}{dR_S} \right)_{R_S = R_0} = \frac{V_A}{4} \frac{\Delta R_S}{R_0} = \frac{V_A}{4} \alpha \Delta T$$

RTD Linear Operation in Wheatstone Bridge

$$R_S = R_0 + \Delta R_S$$

$$V_{S0} = V_A \frac{R_0}{R_0 + xR_0} = \frac{V_A}{1+x}$$

$$V_S = V_A \frac{R_S}{xR_0 + R_S}$$



The Wheatstone bridge can be employed with **any ratio x** of the voltage divider, i.e. R_S can be in series with a resistor x·R_0 with any value of the factor x. However, it is intuitive and readily verified that **with x=1 the highest output ΔV_S** is obtained

$$\Delta V_S = \left(\frac{dV_S}{dR_S}\right)_{R_S=R_0} \Rightarrow \Delta R_S = V_A \frac{x}{(1+x)^2} \frac{\Delta R_S}{R_0}$$

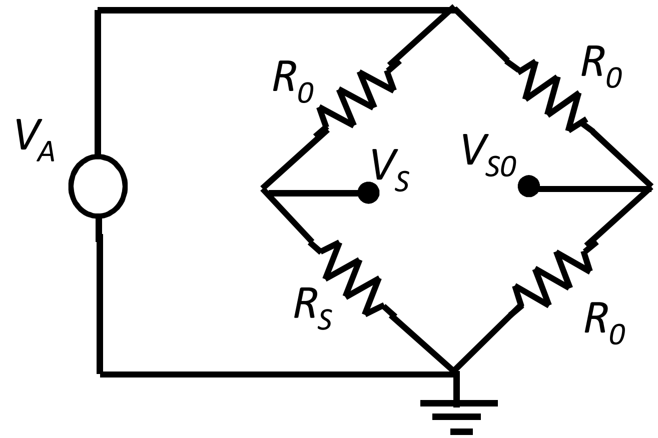
$$\max\left[\frac{x}{(1+x)^2}\right] = \frac{1}{4} \quad for \quad x = 1$$

RTD Nonlinear Operation in Wheatstone Bridge

$$R_S = R_0 + \Delta R_S$$

$$V_{S0} = V_A \frac{R_0}{R_0 + R_0} = \frac{V_A}{2}$$

$$V_S = V_A \frac{R_S}{R_0 + R_S}$$



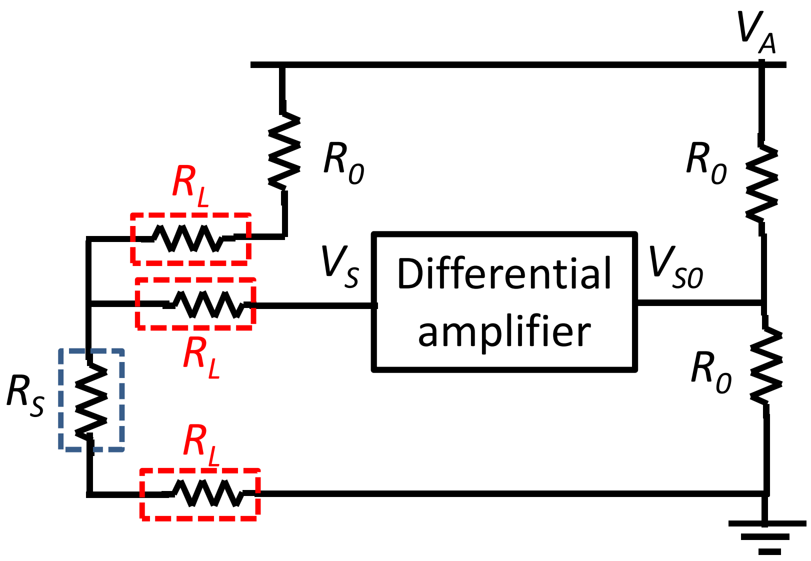
The cheap availability of integrated electronics for digital data processing and storage makes practical to extend the application of the Wheatstone bridge also to cases with **greater variations ΔR_S**, that have a **non-linear but known dependance** of ΔV_S on ΔR_S

$$\Delta V_S = V_S - V_{S0} = V_A \frac{R_0 + \Delta R_S}{2R_0 + \Delta R_S} - \frac{V_A}{2} =$$

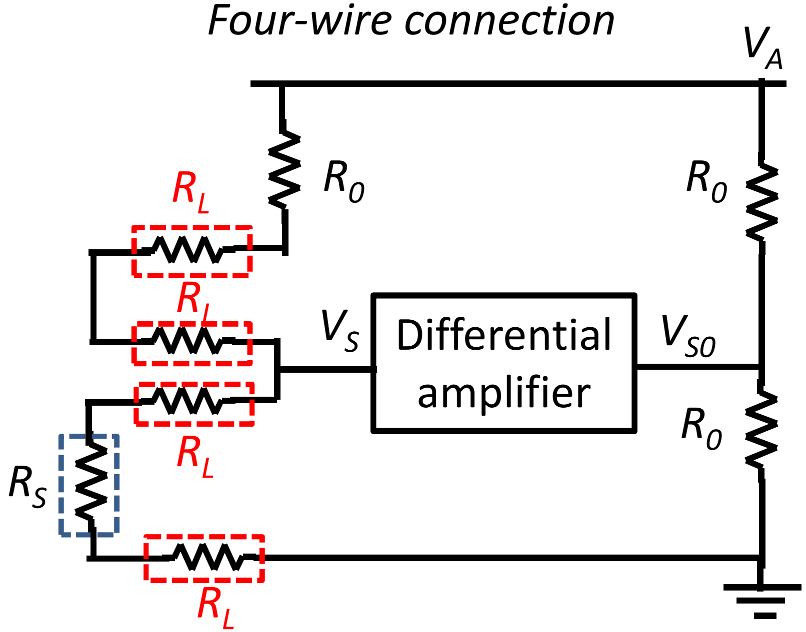
$$= \frac{V_A}{2} \cdot \frac{\dfrac{\Delta R_S}{2R_0}}{1 + \dfrac{\Delta R_S}{2R_0}}$$

Remote RTD Operation

Three-wire connection

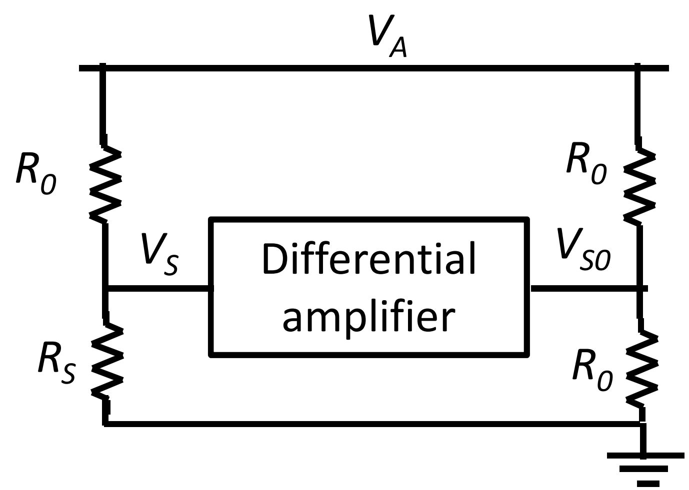


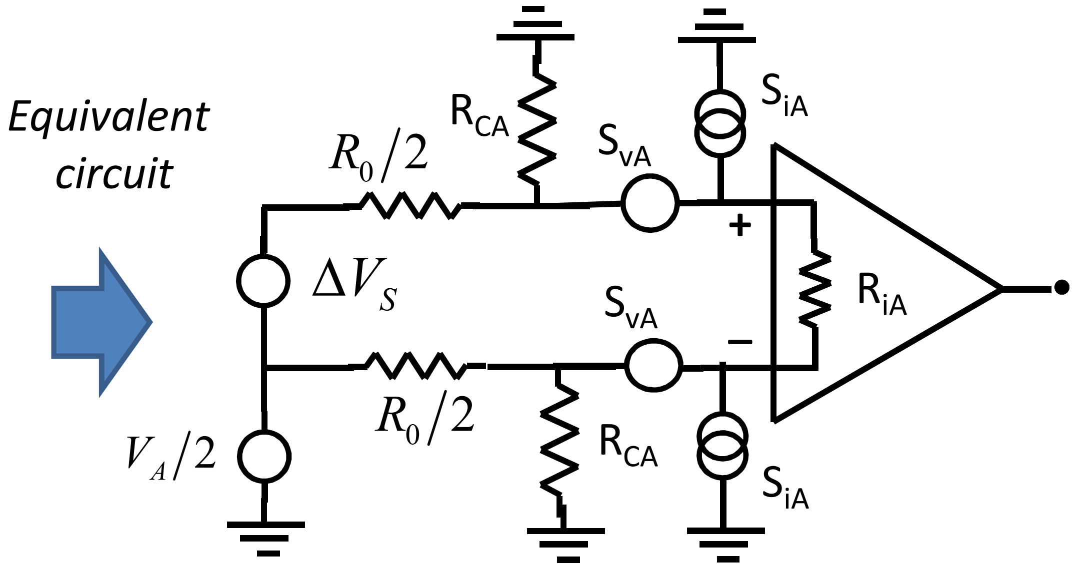
Four-wire connection



- «**Two-wire connection**» causes error also in this case by adding $2R_L$ to R_S
- «**Three-wire-connection**» adds one R_L to the RTD and one to the balancing resistance R_0 . The R_L of the connection to the differential amplifier is not compensated, but its effect is negligible because the current in it is negligible
- «**Four-wire-connection**» achieves complete simmetry between RTD arm and balancing arm, with complete cancellation of the errors due to wire resistances (and also cancellation of other minor thermoelectric effects caused by electrical current flowing in conductors with a temperature gradient)

About RTD Preamplifiers





Since the **source resistance is low**, typically R_0=100 Ω:

- for the input differential resistance $\mathbf{R_{iA}}$ and the input-to-ground resistance $\mathbf{R_{CA}}$ **moderately high** values are sufficient
- the contribution of the input current noise generators is reduced, the input **voltage noise generators are dominant**

Since the differential signal ΔV_S is accompanied by a **high common mode signal $V_A/2$** :

- adequate **CMRR** is required **at the frequency of the supply V_A** , which can be selected at several kHz for reducing the 1/f noise contribution

Thermistors

- Commonly used temperature transducers called Thermistors are made of semiconductor ceramic materials, oxides of Cr, Mn, Fe, Co, Ni
- The dependence of thermistor resistance R on temperature is strikingly different from RTDs (see the plot in slide 29): strongly **nonlinear**, **decreases with increasing temperature** and the R values are **much larger** (some 100 kΩ at room temperature) and have much **greater relative variation**
- The resistance-temperature relationship can be described by the equation

$$R = \exp\left(B/T\right)$$

where T is the absolute temperature in Kelvin degrees, A and B are constants, B is called characteristic temperature of the termistor and usually ranges from 2000 K to 4000 K.
Making reference to the resistance value R_0 at a known reference temperature T_0 we get

$$R = R_0 \exp\left[B\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$

Thermistors

- Thermistors can be made much smaller than RTDs.
- The smaller mass enables them to respond more quickly to temperature variations
- The smaller size, however, makes less efficient the dispersion of the self-heating power, which must be limited to low level
- The basic advantage of thermistors with respect to RTDs is higher sensitivity, i.e. larger relative variation $\Delta R/R$ for a given ΔT, which eases measurements of very small ΔT
- The main disadvantages are lower accuracy and lower reproducibility and strongly nonlinear characteristics, which limit the application of thermistors in automatic control systems

Silicon Diode Sensors

- Silicon junction diode as temperature sensor: features and limitations
- Improving the Silicon diode sensor by a differential measurement
- Single diode sensor with switching supply
- Matched dual diode sensor with DC supply

Silicon Diode Temperature-Sensor Principle

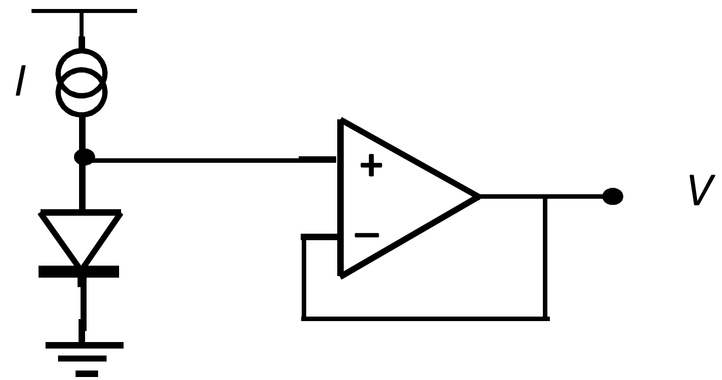
- Silicon is an interesting material also for temperature sensors: besides Thermistors with good performance, also junction diode sensors are available.
- The principle of a pn-junction sensor directly arises from the Shockley equation

$$I = I_S\left[\exp\left(qV/kT\right)-1\right]$$

where I_S is the saturation reverse current is. By inverting it we get

$$V = \frac{kT}{q}\ln\left(\frac{I}{I_S}-1\right)$$

At first sight, it seems that a diode biased at constant forward current I is an excellent sensor that produces a voltage V proportional to the absolute temperature T



Silicon Diode as Temperature Sensor

Let us analyze the situation more in detail

- The Shockley equation is really accurate only for junctions free from generation-recombination centers in the depletion layer (i.e. fabricated with special technologies that minimize local defects in the Silicon lattice). The equation can be extended to all cases with a simple correction

$$I = I_S \left[\exp\left(qV / m \cdot kT \right) - 1 \right]$$

  where *m* is a numerical correction factor, which depends on the fabrication technology and ranges from 1 to 2. With this correction we get

$$V = m \cdot \frac{kT}{q} \ln\left(\frac{I}{I_S} - 1 \right)$$

- The saturation current I_S is very small, hence with good approximation we have

$$\boxed{V = m \cdot \frac{kT}{q} \ln\left(\frac{I}{I_S} \right)}$$

  Since I_S is lower than *1pA* in good quality diodes, the approximation is valid down to very low bias current *I*, e.g. in the microampere range.

Silicon Diode as Temperature Sensor

However, the presence of the saturation current $\boldsymbol{I_S}$ **invalidates the sensor principle above described** because:

a) I_S has strong and nonlinear dependance on T

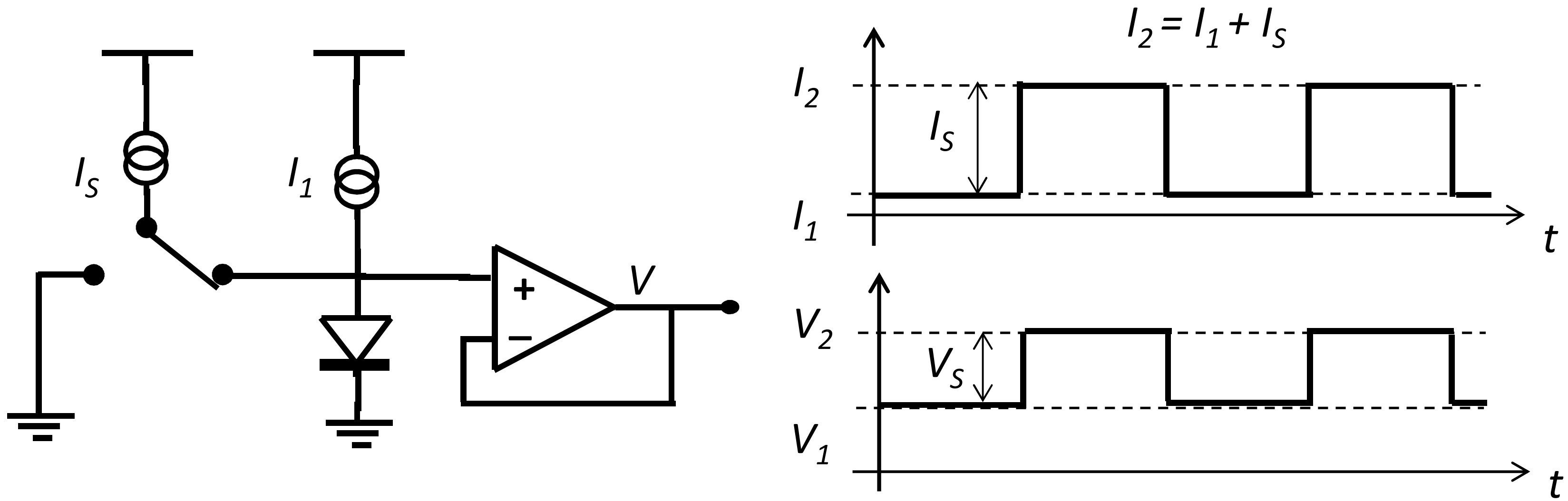
$$I_S \propto \exp\left(-\frac{E_G}{kT}\right)$$

b) the value of I_S strongly depends on the fabrication process; it varies from a fabrication run to another and even within the same run there are strong differences between individual diodes

c) it follows that the dependance of voltage *V* on T is not linear and is different in different diode samples, which therefore should be individually calibrated

Also the correction factor *m* is process-dependent and varies with the temperature; however, this is a moderate variation and is avoided in diodes fabricated with special «clean» Silicon technologies that ensure $m \approx 1$

Differential Operation Validates the Sensor



Since the diode voltage V has **logarithmic dependance** on the bias current I

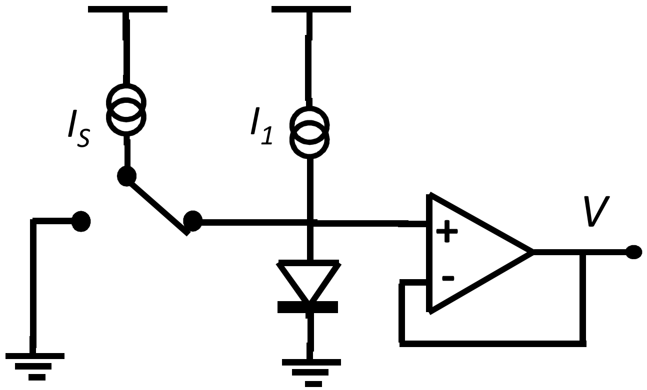
$$V = m \cdot \frac{kT}{q} \ln\left(\frac{I}{I_S}\right)$$

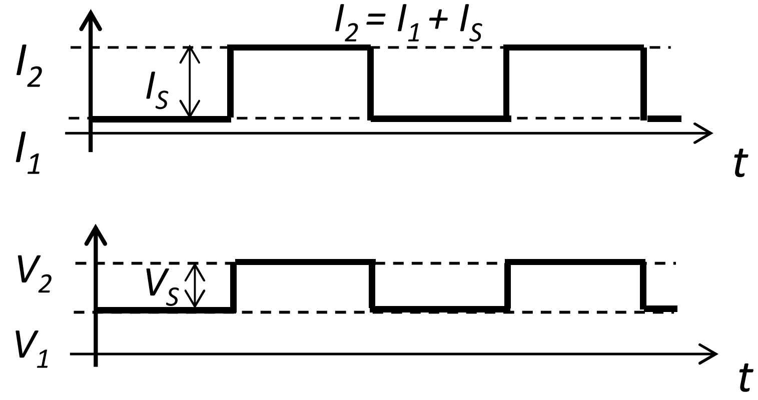
the **dependance on the saturation current *I_S* can be cancelled** by taking a measure of the difference V_S of diode voltage at two different bias levels I_1 and I_2

$$V_S = V_2 - V_1 = m \cdot \frac{kT}{q} \ln\left(\frac{I_2}{I_S}\right) - m \cdot \frac{kT}{q} \ln\left(\frac{I_2}{I_S}\right) = m \cdot \frac{kT}{q} \ln\left(\frac{I_2}{I_1}\right) = m \cdot \frac{kT}{q} \ln \beta$$

With a periodic current switching as shown, a squarewave voltage with amplitude V_S is generated and can be measured with a Lock-in Amplifier (LIA) arrangement

Differential Operation Validates the Sensor





Employing a diode fabricated in «clean» technology we can have m=1.
We can thus get an absolute thermometer with linear scale and calibration dependent only on fundamental constants (k and q) and on the current ratio β, which can be selected and accurately controlled

$$V_S = \frac{kT}{q} \cdot \ln \beta$$

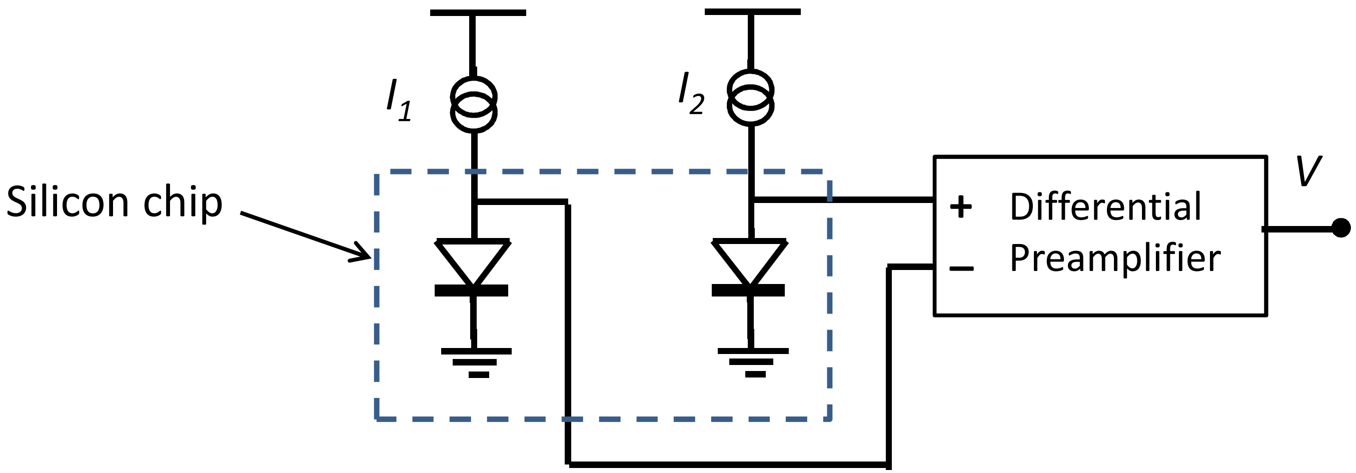
The sensitivity obtainable in the temperature measurement is remarkable

$$\frac{dV_S}{dT} = \frac{k}{q} \cdot \ln \beta$$

for instance, selecting β = 10 we get $\frac{dV_S}{dT} = 198\,\mu V/K$

Integrated Dual-Diode Sensor

- The squarewave signal obtained by a current switching arrangement can be measured with high precision employing a LIA arrangement, but in various cases a simpler set-up working on DC signals is preferable for practical reasons, notwithstanding it leads to lower precision.
- The principle of differential measurement can be employed also in DC operation, thanks to the capability of fabricating «twin diodes»; that is, two diodes with identical structure and properties, integrated at short distance in a silicon chip so that they operate at the same temperature T



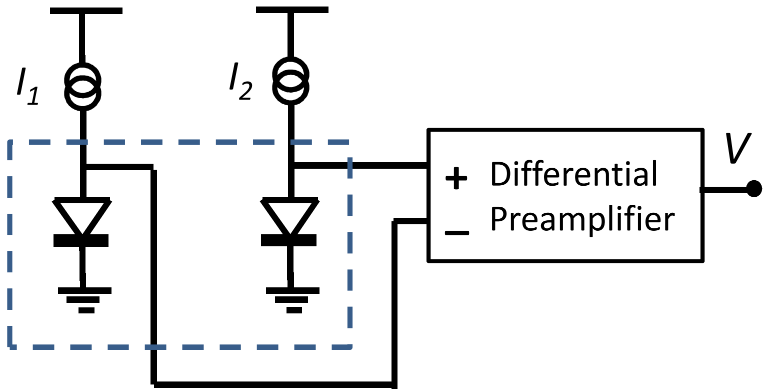
Integrated Dual-Diode Sensor

Power dissipation in the diodes

$$P_1 = I_1 \cdot V_{j1}$$

$$P_2 = I_2 \cdot V_{j2}$$

with $V_{j1} \approx V_{j2} \approx 0{,}6V$



- The bias currents must be low for keeping negligible the self-heating and practically equal the temperature in the diodes; e.g. with I_1=1μA and I_2=10 μA we get P_1=0,6μW and P_2=6μW
- An alternative approach has been devised by noting that the differential measurement can be analyzed making reference to the **density *j*** of the diode current

$$j = j_S \left[\exp\left(qV / m \cdot kT \right) - 1 \right]$$

thus getting

$$V = m \cdot \frac{kT}{q} \ln\left(\frac{j_2}{j_1} \right)$$

Integrated Dual-Diode Sensor

Denoting by A_1 and A_2 the diode areas, it is $I_1 = j_1 A_1$ and $I_1 = j_1 A_1$ and we get

$$V_S = m \cdot \frac{kT}{q} \ln\left(\frac{j_2}{j_1}\right) = m \cdot \frac{kT}{q}\left[\ln\left(\frac{I_2}{I_1}\right) + \ln\left(\frac{A_1}{A_2}\right)\right]$$

It is worth highlighting some features of this integrated sensor

- The **area ratio** A_1/A_2 is controlled and reproducible with great accuracy in the planar technology of **fabrication of integrated devices and circuits**
- If equal diode currents $I_1 = I_2$ are employed, the sensor calibration is set at the factory by the selected ratio A_1/A_2
- With equal diode current, equal dissipation is obtained, hence equal temperature in the two diodes
- The dual-diode sensor can be monolithically integrated with a complete dedicated electronic circuitry for temperature measurement; besides the preamplifier also a main amplifier for producing a high level calibrated analog output (e.g. a voltage output of 10mV/K) or an ADC for having a digital output
- The detector has very good performance, but measurement range limited by the integrated circuit properties, typically from -50°C to 150°C