

# Sensors, Signals and Noise

## COURSE OUTLINE

- Introduction
- Signals and Noise
- Filtering
- Sensors: SE3 Piezoelectric Force Sensors



# Piezoelectric Force Sensors

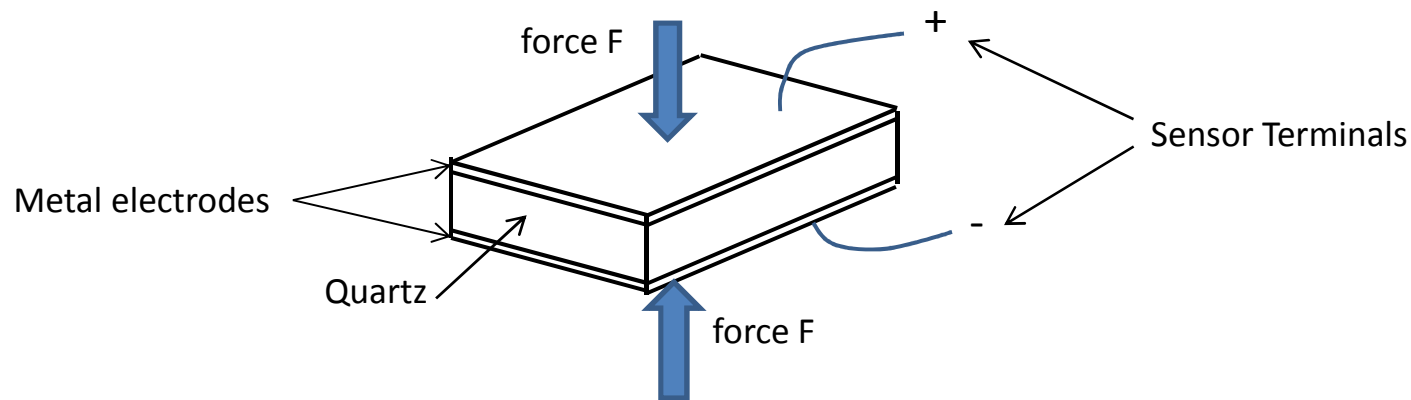
- Piezoelectric Effect and Materials
- Piezoelectric Sensor Devices
- Sensor Equivalent Circuit
- Dynamic Response
- Preamplifiers for Piezoelectric Sensors



# Piezoelectric Effect and Materials

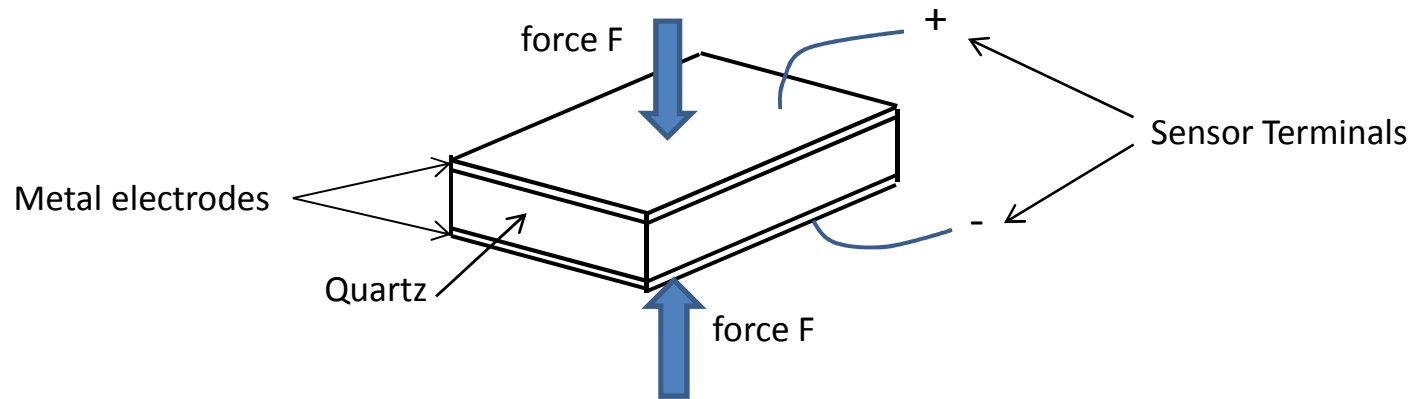


# Piezoelectric Effect



- A quartz crystal cut with parallel faces, when squeezed shows electric charges of opposite polarity on the faces. With metallic electrodes deposited onto the faces, it is a capacitor that in response to external loads generates electrical charges on the plates, hence voltage between them.
- This cross-coupling from mechanical to electrical energy is called **piezoelectric effect** (greek «piezein» = to squeeze or press). It is observed in some insulator materials and works also in the opposite sense: by applying voltage, the material is subject to a force. It can thus be exploited in **sensors** and in **actuators**
- Piezoelectricity is in general a tensorial effect, that is, the equation relating the charge to the force applied depends on the orientation with respect to the crystal structure of
  - a) the force applied and
  - b) the face where charge is observed.

# Piezoelectric Effect



- A wide variety of piezoelectric sensors is nowadays exploited in different applications. The characteristic general features of piezoelectric sensors will be here highlighted and clarified in the analysis of the simple basic case of **sensors of compression force**.
- In the case of quartz with faces normal to crystallographic x-axis and **compression** force  $F$  normal to the faces, the effect is simply a charge  $Q_s$  proportional to the force, described by a scalar parameter  $T_p$ , the force-to-charge conversion factor. Such sensors are employed also for measuring **acceleration** by coupling a mass to the sensor
- Quartz (and other piezoelectrics) have **excellent elastic** properties: very low internal friction (low energy loss), well proportional strain and stress (relative deformation  $\epsilon$  and force per unit area  $\sigma$ ) with **high Young modulus  $E$**  (about  $100 \text{ Gpa} = 100 \cdot 10^{12} \text{ N/m}^2$ ).

$$\epsilon = \sigma / E$$

They are very stiff and undergo very little strain even with strong applied force.



# Piezoelectric Materials

Various Piezoelectric materials are known and currently employed.

- **Crystals:** Quartz;  $\text{GaPO}_4$  or Tourmaline;  $\text{LiNbO}_3$  Lithium Niobate;  $\text{LiTaO}_3$  Lithium Tantalate
- **Polymers:** PVDF PolyVinylidene Fluoride; **thin films.**
- **Ceramic materials:**  $\text{PbTiO}_3$  ;  $\text{BaTiO}_3$ ; PZT Lead Zirconate-Titanate.  
Ceramics are polycrystalline structures that naturally have randomly oriented microcrystals and do not have piezoelectric properties.  
However, by heating them above their characteristic Curie temperature (in most materials it's in a range from 150 to 300°C) the microcrystal dipoles can be oriented by applying a high electric field. Cooling them down with electric field applied the dipole orientation gets «frozen»; the material becomes permanently polarized and has piezoelectric properties.

Crystals and ceramics have excellent elastic properties with high Young modulus E

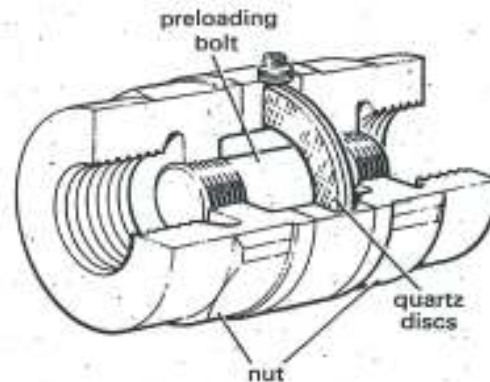


# Piezoelectric Sensor Devices



# Piezoelectric Force Sensors

- A compression force applied to the sensor produces a proportional strain that by piezoelectric effect produces a proportional charge on the capacitor, which gives a proportional voltage. Note that this electrical **signal** is generated by the sensor itself **without power supply**
- Piezoelectric sensors are often mounted in a case, with a cable connected to the metal faces to bring out the electrical signal. Alternatively, an integrated preamplifier circuit can be inserted in the case and connected to the sensor; the preamp output is available to the user.
- For measuring pulling forces the sensor can be «biased», that is, precharged in compression by means of a «nut and bolt» internal arrangement (outlined in the drawing) so that the pulling force is measured as a reduction of compression

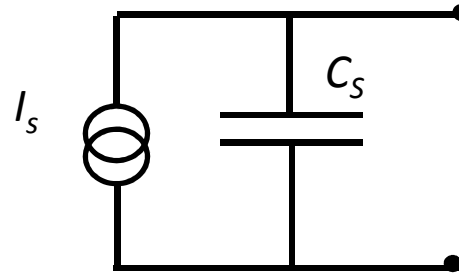
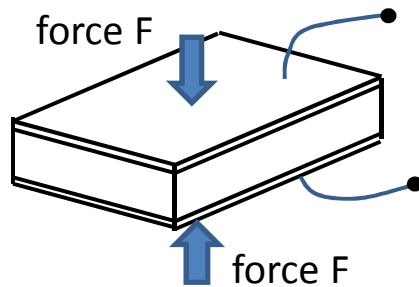




# Sensor Equivalent Circuit



# Force Sensor Equivalent Circuit

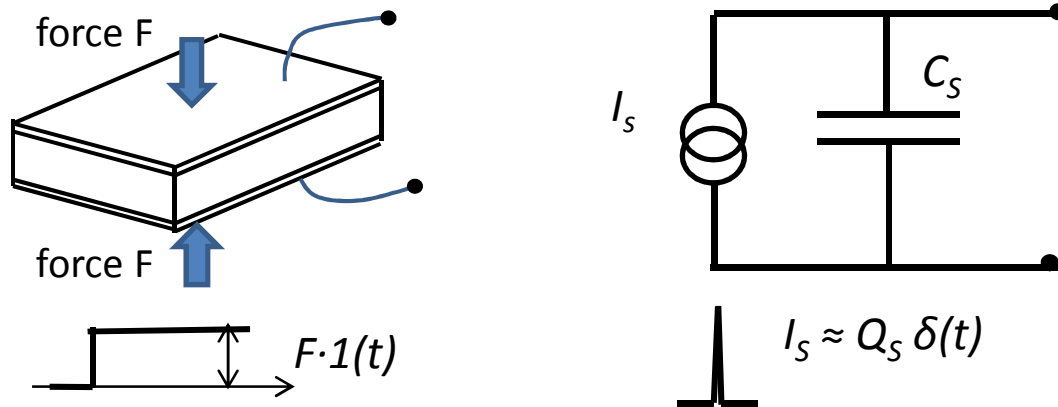


- The sensor equivalent model is simply its capacitance  $C_s$  connected to a current generator  $I_s$ , which produces the signal charge  $Q_s$  that in reality is generated by the piezoelectric effect
- The charge  $Q_s$  is **proportional to the applied force  $F$**

$$Q_s(t) = T_p \cdot F(t)$$

- The force-to-charge conversion factor  $T_p$  depends on the sensor area and material properties. Force sensors in quartz currently employed have  $T_p$  values in the range from 10 to 200 pC/N

# Force Sensor Equivalent Circuit



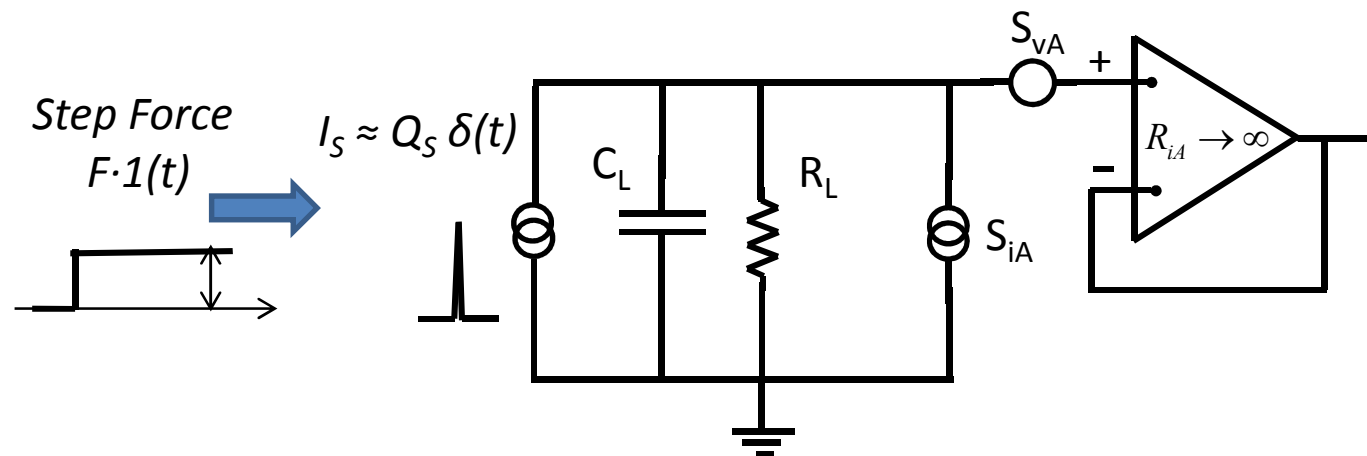
- Since  $Q_s$  is the integral of  $I_s$ , the model current generator is

$$I_s = \frac{dQ_s}{dt} = T_p \frac{dF}{dt} \quad \text{or in Laplace transform} \quad I_s(s) = T_p \cdot sF(s)$$

- For a step force  $F \cdot 1(t)$  applied the current signal is a  $\delta$ -pulse  $I_s = T_p \cdot F \delta(t)$
- The **sensor internal noise is negligible** (with respect to that of the preamplifier circuit connected to the sensor) in all cases where the piezoelectric material is a high quality dielectric materials, such as quartz

# Force Sensor with Buffer Preamplifier

- High-impedance sensors must be connected to preamplifiers with high input resistance\*  $R_{iA} \rightarrow \infty$  for low-noise operation (*hint: review slide sets OPF2 and PD2*)
- Let's consider the **response to step applied force** of a piezoelectric sensor connected to a voltage buffer with high-input-impedance amplifier; that is, with very high resistor load  $R_L$  connected to the sensor equivalent circuit



- $C_L$  total load capacitance =  $C_S$  (sensor cap.) +  $C_{iA}$  (amplifier cap.) +  $C_C$  (connection cap.)
- $R_L$  high load resistance
- $S_{vA}$  amplifier voltage noise
- $S_{iT}$  total current noise =  $S_{iA}$  of amplifier +  $S_{iR}$  of load resistor +  $S_{iS}$  of sensor ( $\approx$ negligible)

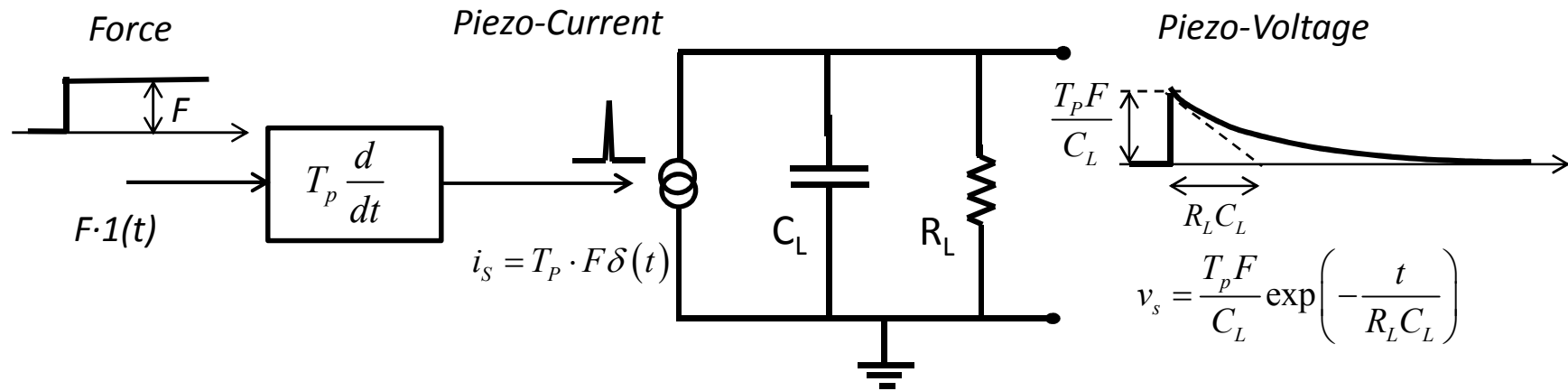
\*  $R_{iA}$  = true physical resistance between the input terminals, not the dynamic input resistance including feedback effects



# Dynamic Response



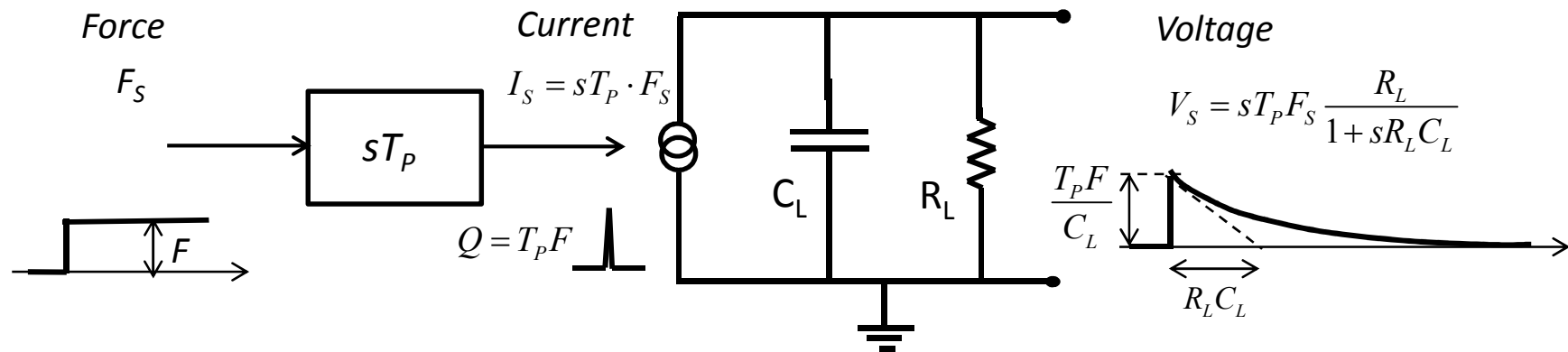
# Transient and Steady Response to Step Input



- A force step  $F$  generates a charge step  $Q = T_p \cdot F$  in  $C_L$ , which in turn produces a voltage step  $Q/C_L = T_p F/C_L$
- The charge and the voltage on the sensor capacitance  $C_L$  then **decay to zero** with time constant  $R_L C_L$  (with high load  $R_L$  it can be very long)
- In time domain, this points out that the sensor gives **zero response to a constant force**, but it responds to **slowly varying forces**, i.e. forces that vary over long time intervals up to about  $R_L C_L$



# Low-Frequency Cutoff of the Response

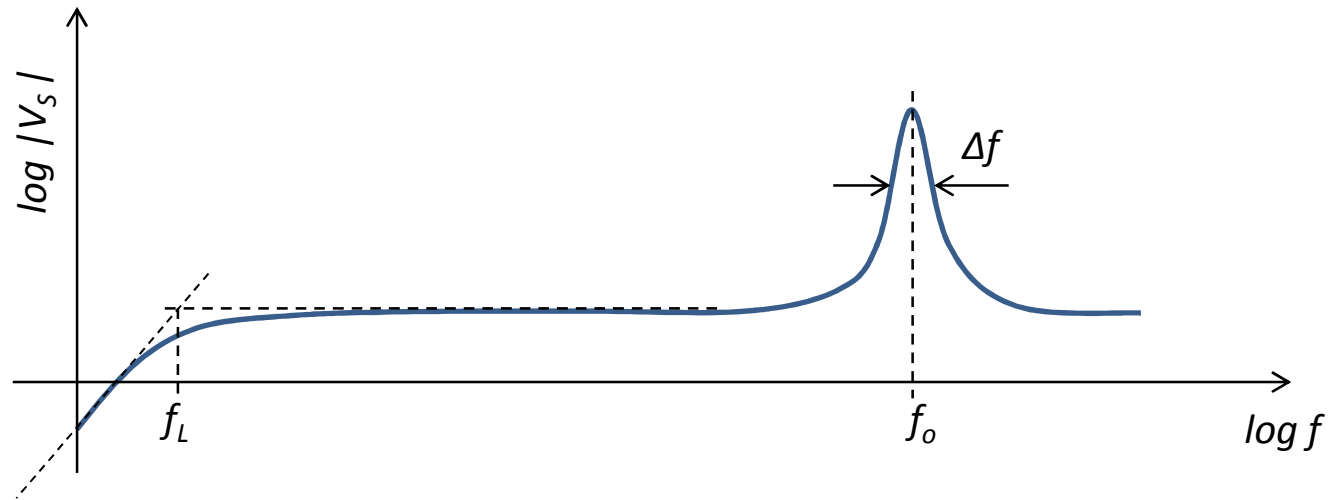


- In transform domain, the intrinsic differentiation of the piezoelectric effect ( $I_S \propto \frac{dF_S}{dt}$ ) causes zero response at zero frequency
- The attenuation versus frequency towards zero is set by the simple pole of the  $R_L C_L$  circuit (20dB/decade cutoff below the pole frequency  $f_L = 1/2\pi R_L C_L$ )

Applied	Force	Piezo-Current	Piezo-Voltage	Piezo-Charge
Any Force	$F_S$	$I_S = sT_p \cdot F_S$	$V_S = sT_P F_S \frac{R_L}{1 + sR_L C_L}$	$Q_S = T_P F_S \frac{sR_L C_L}{1 + sR_L C_L}$
Step Force	$F_S = \frac{F}{s}$	$I_S = T_p \cdot F$	$V_S = T_P F \frac{R_L}{1 + sR_L C_L}$	$Q_S = T_P F \frac{R_L C_L}{1 + sR_L C_L}$



# High Frequency Limit of the Response

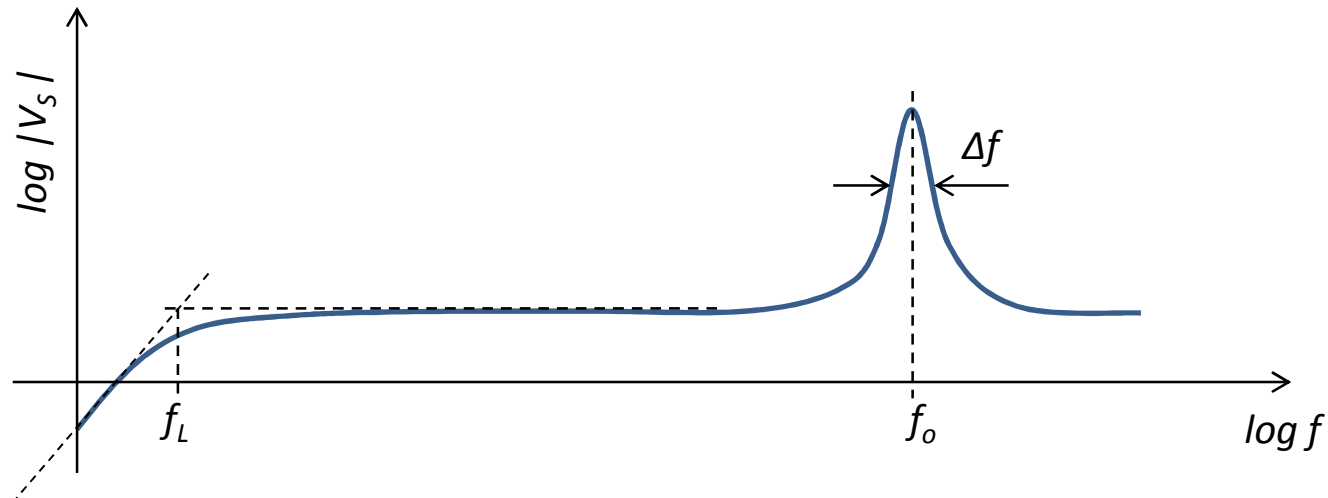


- Piezoelectric sensors intrinsically give high-pass filtering, so they are **unsuitable for static** forces. However, with high load resistors  $R_L$  they can have quite low cutoff frequency  $f_L$  (even below 1Hz) and be **employed with even slowly varying** forces
- Piezoelectric sensors provide uniform force transduction above the low-frequency cutoff  $f_L$  over an extended frequency range, typically up to about 1kHz
- However, at higher frequencies they become unsuitable, because the response shows a peak with amplitude much higher than the flat range, centered at a frequency  $f_o$  (characteristic of the sensor) and with narrow width  $\Delta f \ll f_o$
- The high-frequency response looks similar to that of a resonant LRC circuit with very high quality factor  $Q = f_o / \Delta f$  (typical values from  $\approx 50$  to more than 1000)





# High Frequency Limit of the Response



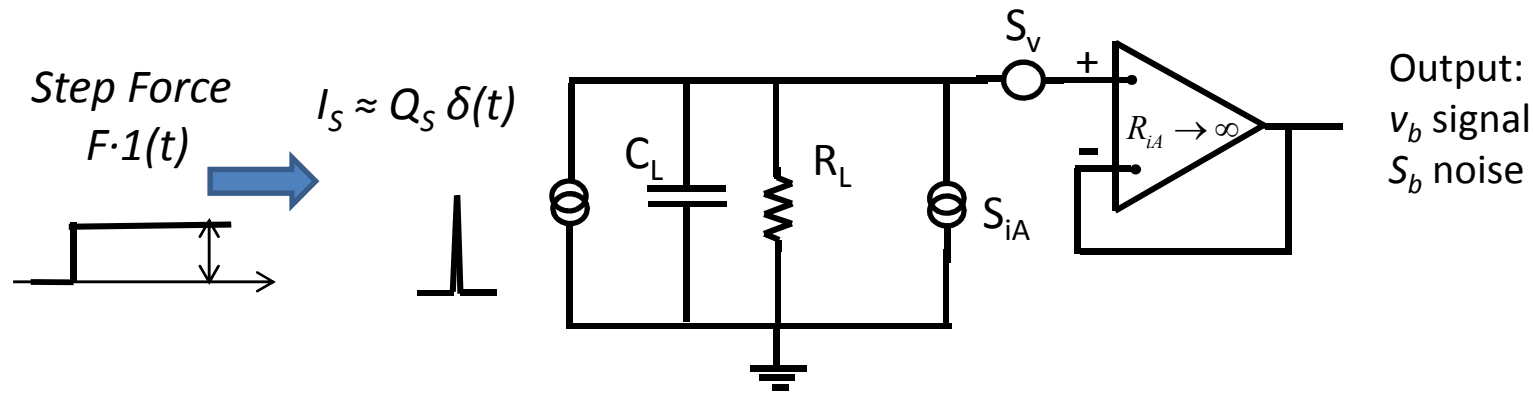
- In fact, the similarity of piezoelectric device to LRC resonator arises because:
  - a) the device is an excellent mechanical resonator and
  - b) the piezoelectric effect cross-couples electrical and mechanical energy
- Piezoelectric sensors (in particular Quartz devices) have good elastic properties, with high Young modulus  $E$  and small mass, hence they have high characteristic vibration frequencies, in the kHz range. Their  $Q$  is high because the internal friction is very low and vibration occurs with very little energy loss.
- The mechanical resonant frequency of quartz devices depends on the device size and orientation of the cut with respect to the crystal axes; very stable values can be obtained. Crystals cut for stable resonance frequency are currently exploited in quartz-controlled oscillators to be employed as high precision clock.



# Preamplifiers for Piezoelectric Sensors



# Buffer Preamplifier



$C_L$  (total load) =  $C_S$  (sensor) +  $C_{iA}$  (amplifier) +  $C_C$  (connection);  $R_L$  high load resistance  
 $S_{iT}$  (current noise) =  $S_{iA}$  (amplifier) +  $S_{iR}$  ( $R_L$  load) +  $S_{iS}$  (sensor);  $S_v$  (voltage noise)

The Signal is

$$v_b(t) = \frac{Q}{C_L} \cdot 1(t) = \frac{Q}{C_S + C_{iA} + C_C} \cdot 1(t)$$

- The amplitude is inversely proportional to the **sum of all the capacitances**
- The amplitude depends **also on the capacitance  $C_C$**  of the connection sensor-preamplifier, which is not well controlled and may change when the connection is rearranged, particularly in case of long connecting wires
- The amplitude depends **also on the capacitance  $C_{iA}$**  of amplifier input, which has variations from sample to sample of the same type of amplifier

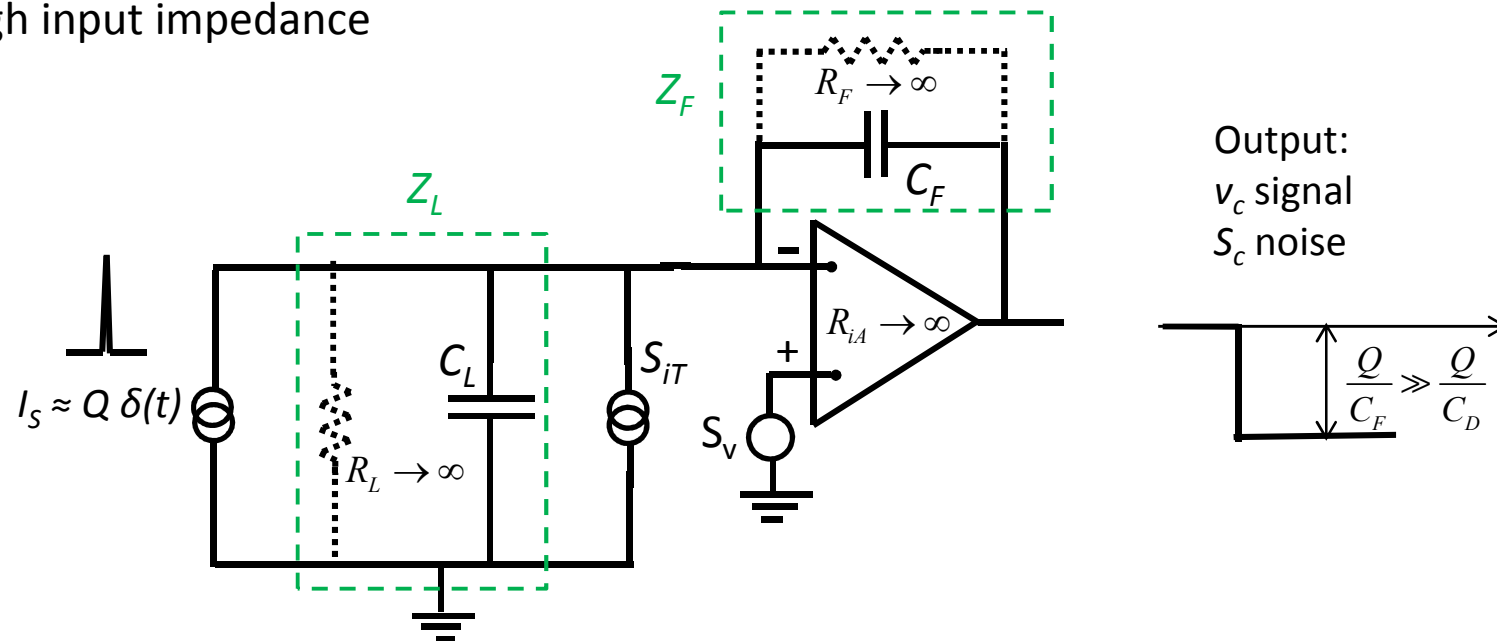
The Noise Spectrum is

$$S_b = S_v + S_{iT} \frac{1}{\omega^2 C_L^2}$$



# Charge Preamplifier or Transimpedance Preamplifier

Alternative configuration: operational integrator based on a low-noise amplifier with high input impedance



- $C_F$  capacitor in feedback. The  **$C_F$  value can be very small and is accurately set** by the capacitor component, because the inherent stray capacitance between output and input pins of the amplifier is negligible. Therefore, **one can work with  $C_F \ll C_L$**
- $R_F$  feedback resistor  $\rightarrow \infty$
- $C_L$  total load capacitance =  $C_S$  (sensor cap.) +  $C_{iA}$  (amplifier cap.) +  $C_C$  (connection cap.)
- $R_L$  total load resistance  $\rightarrow \infty$
- $S_v$  amplifier voltage noise
- $S_{iT}$  (current noise) =  $S_{iA}$  (amplifier noise) +  $S_{iS}$  (sensor noise) +  $S_{iR}$  (load resistor noise)



# Charge Preamplifier or Transimpedance Preamplifier

## Output Signal:

$$\text{in frequency domain } V_c = -QZ_F = -\frac{Q}{j\omega C_F} \quad \text{in time } v_c(t) = -\frac{Q}{C_F} \cdot 1(t)$$

With respect to the buffer, the amplitude is greater by the gain factor  $G_c = C_L/C_F \gg 1$

$$|v_c| = \frac{Q}{C_F} = \frac{C_L}{C_F} \cdot \frac{Q}{C_L} = \frac{C_L}{C_F} \cdot |v_b| = G_c \cdot |v_b|$$

## Advantages:

- The higher signal makes less relevant the noise of the following circuits
- The signal amplitude is ruled by the well controlled and stable  $C_F$  and no more by the other capacitances  $C_S$ ,  $C_{iA}$  and  $C_C$

The noise analysis (see next slide) confirms that these advantages are obtained without degrading the S/N.

The charge amplifier configuration is indeed the solution of choice in most cases met in practice, notwithstanding that it is less simple than the voltage buffer amplifier configuration.



# Charge Preamplifier or Transimpedance Preamplifier

## Output Noise Spectrum :

- the current noise  $S_{iT}$  is processed by the same transfer function as the current signal
- the voltage noise  $S_v$  is processed with the transfer function from non-inverting input to amplifier output.

Denoting by  $Z_L$  the load impedance and by  $Z_F$  the feedback impedance

$$S_c = S_v \left| 1 + \frac{Z_F}{Z_L} \right|^2 + S_{iT} |Z_F|^2$$

in our case  $Z_L \approx 1/j\omega C_L$  and  $Z_F \approx 1/j\omega C_F$  so that

$$S_c = S_v \left| 1 + \frac{C_L}{C_F} \right|^2 + S_{iT} \frac{1}{\omega^2 C_F^2} = \left( \frac{C_L}{C_F} \right)^2 \left[ S_v \left( 1 + \frac{C_F}{C_L} \right)^2 + S_{iT} \frac{1}{\omega^2 C_L^2} \right]$$

if  $C_F/C_L \ll 1$ , with good approximation it is

$$S_c \approx \left( \frac{C_L}{C_F} \right)^2 \left[ S_v + S_{iT} \frac{1}{\omega^2 C_L^2} \right] = \left( \frac{C_L}{C_F} \right)^2 S_b = G_c^2 S_b$$

With respect to the buffer, the signal and noise thus benefit of the same gain  $G_c$  : therefore, the attainable S/N with the charge preamplifier is the same as with the voltage buffer preamplifier

